Physics 2010: Sample Problems for Exam 1

1. An object is dropped from rest at a certain height. When it is 42.6 m above the ground, it is traveling at 22.4 m/s downward. (a) How fast is the object moving when it hits the ground? (b) What was its height when it was dropped? (c) How long was it in the air before it hit the ground?

\[ v_1^2 = v_0^2 - 2g(y_1-y_0) \]
\[ v_1 = -22.4 \text{ m/s} \]
\[ y_1 = 42.6 \text{ m} \]
\[ y_0 = 0 \]
\[ t_0 = 0 \]

\[ v_2 = \sqrt{v_1^2 - 2g(y_2-y_0)} \]
\[ = \sqrt{(-22.4 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0 - 42.6 \text{ m})} \]
\[ = \sqrt{501.76 \text{ m}^2/\text{s}^2 + 834.96 \text{ m}^2/\text{s}^2} \]
\[ = 1336.72 \text{ m}^2/\text{s}^2 \]
\[ v_2 = -36.56 \text{ m/s} \]

\[ v_1^2 = \frac{1}{2}gh \]
\[ = \frac{1}{2} \cdot 9.80 \text{ m/s}^2 \cdot 42.6 \text{ m} \]
\[ = 201.76 \text{ m}^2/\text{s}^2 \]
\[ h = \frac{v_1^2}{2g} = \frac{1}{2} \cdot 9.80 \text{ m/s}^2 \cdot 201.76 \text{ m}^2/\text{s}^2 \]
\[ = 42.6 \text{ m} + 25.6 \text{ m} = 68.2 \text{ m} \]

\[ v_2 = v_0 - gt \]
\[ = 0 - 9.80 \text{ m/s} \cdot 3.73 \text{ s} \]
\[ t_2 = -\frac{v_2}{g} = -\frac{-36.56 \text{ m/s}}{9.80 \text{ m/s}^2} = 3.73 \text{ s} \]
2. A ball is throw horizontally off of the top of a 122 m tall building. It hits a second building a horizontal distance of 214 m from where the ball was thrown at a height of 96.2 m above the ground. (a) How long in time did it take for the ball to travel this distance? (b) How fast was the ball thrown?

\[ y_0 = 122 \text{ m}, \quad t_0 = 0 \]
\[ \vec{v}_0 = \vec{v}_{0x} = ?, \quad \vec{v}_{0y} = 0 \]
\[ t_1 = \, ? \]
\[ y_1 = 96.2 \text{ m} \]

\[ x_0 = 0 \quad x_1 = 214 \text{ m} \]

\[ a) \quad y_f = y_0 + \vec{v}_{0y} (t_1 - t_0) - \frac{1}{2} g (t_1 - t_0)^2 \]
\[ y_f - y_0 = -\frac{1}{2} g t_1^2, \quad \frac{1}{2} g t_1^2 = y_0 - y_f \]
\[ t_1^2 = \frac{y_0 - y_f}{\frac{1}{2} g} = \frac{2(y_0 - y_f)}{g} \]
\[ t_1 = \sqrt{\frac{2(y_0 - y_f)}{g}} = \sqrt{\frac{2(122 \text{ m} - 96.2 \text{ m})}{9.80 \text{ m/s}^2}} \]
\[ = \boxed{2.29 \text{ s}} \]

\[ b) \quad \vec{v}_0 = \vec{v}_{0x}, \quad x_1 = x_0 + \vec{v}_{0x} (t_1 - t_0) = \vec{v}_{0x} t_1 \]
\[ \vec{v}_{0x} = \frac{x_1}{t_1} = \frac{214 \text{ m}}{2.294 \text{ s}} = \boxed{93.3 \text{ m/s}} \]
3. A plane is traveling horizontally at a constant speed of 56.7 m/s. It drops a package which lands 612 m “downstream” from where it was dropped. (a) How long did it take to reach the ground? (b) How high is the plane? (c) At what speed does the package hit the ground?

\[ y = y_0 + v_{oy} (t-t_0) - \frac{1}{2} g (t-t_0)^2 \]

\[ 0 = h - \frac{1}{2} g t^2, \quad h = \frac{1}{2} g t^2 = \frac{1}{2} \left(9.80 \frac{m}{s^2}\right)(10.8 s)^2 \]

\[ = 571 \text{ m} \]

(c) \[ v_x = v_{ox} = 56.7 \text{ m/s} \quad \text{(given)} \]

\[ v_y = v_{oy} - g (t-t_0) = -g t = -\left(9.80 \frac{m}{s^2}\right)(10.8 s) \]

\[ = -105.8 \text{ m/s} \]

\[ \vec{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(56.7 \frac{m}{s})^2 + (-105.8 \frac{m}{s})^2} = 120 \text{ m/s} \]
4. Two objects are connected by a cord with negligible mass via an ideal pulley (i.e., frictionless) similar to an Atwood machine. The heavier object of mass of 6.22 kg \((m_2)\) drops from rest 16.7 cm in 2.88 s. (a) What is the acceleration of this mass? (b) What is the mass of the lighter object \((m_1)\)? (c) What is the tension \((T)\) of the cord?

\[ m_2 = 6.22 \text{ kg} \]

\[ \begin{align*}
  a) & \quad y = y_0 + v_0 (t - t_0) + \frac{1}{2} a (t - t_0)^2 \\
  v_{0y} &= 0, \quad t_0 = 0 \\
  y - y_0 &= -\frac{1}{2} a t^2 \\
  \frac{1}{2} a t^2 &= y_0 - y \\
  a &= \frac{2(y_0 - y)}{t^2} \\
  &\quad = \frac{2(0.167 \text{ m} - 0)}{(2.88 \text{ s})^2} \\
  &\quad = 0.0403 \text{ m/s}^2
\end{align*} \]

\[ \begin{align*}
  b) & \quad m_1 : \sum F_y = T - m_1 g = m_1 a \\
  & \quad m_2 : \sum F_y = m_2 g - T = m_2 a
\end{align*} \]

(1) \[ T = m_1 g + m_2 a = m_1 (g + a) \]

(2) \[ T = m_2 g - m_2 a = m_2 (g - a) \]

\[ m_1 (g + a) = m_2 (g - a) \]

\[ m_1 = m_2 \frac{g - a}{g + a} \]

\[ m_1 = m_2 \frac{9.80 \text{ m/s}^2 - 0.0403 \text{ m/s}^2}{9.80 \text{ m/s}^2 + 0.0403 \text{ m/s}^2} \]

\[ m_1 = 6.17 \text{ kg} \]

\[ T = 60.7 \text{ N} \]