Cubic Dissection and Related Puzzles

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This document is largely for my own use to document my collection of Cubic Dissection Puzzles. However, I do hope that whoever reads this will find puzzles that they also find interesting. Unless otherwise noted, the puzzles photographed are things that I have made using blank dice and Gorilla glue[1]. The interested reader is referred to following resources:

The Puzzling World of Polyhedral Dissections by Coffin

AP-ART - A Compendium of Geometric Puzzles by Coffin

Puzzles Will Be Played

[1] I normally do not recommend a specific brand. However, I have found that Gorilla Glue is quite strong and dries quickly. The fact that it dries quickly is especially nice as it allows pieces to be made efficiently.

SOMA Cube (3X3X3)

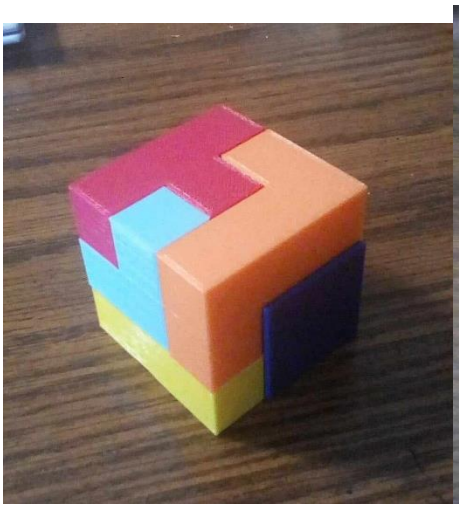
The Soma cube is a stock puzzle. Chances are if you have some pieces that have to be assembled into a 3X3X3 cube, then it is a Soma cube. The first observation that one usually makes is that there are 7 pieces and 8 corners of the cube. The T-shaped piece will either use 0 or 2 corners of the cube. So it is impossible to assemble the cube unless the T-shaped piece occupies 2 corners. If you buy a manufactured Soma Cube, then it is likely to include puzzles to assemble shapes other than the cube.

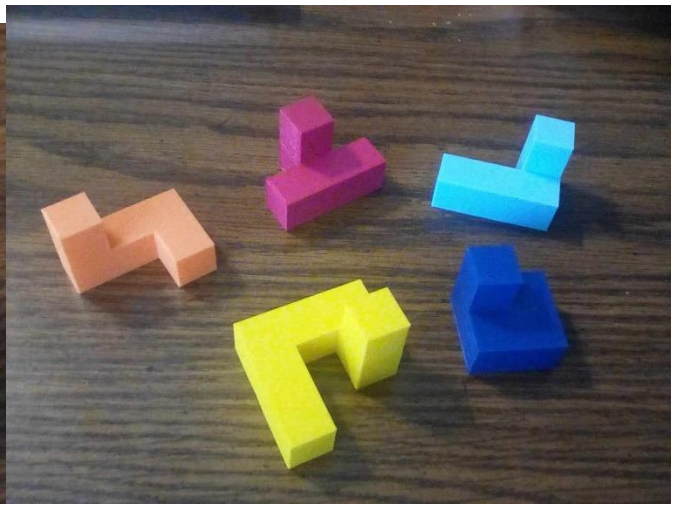
Analysis by Martin Gardner

Pursuing Puzzles Purposefully from Winning Ways

House's Puzzle (3X3X3)

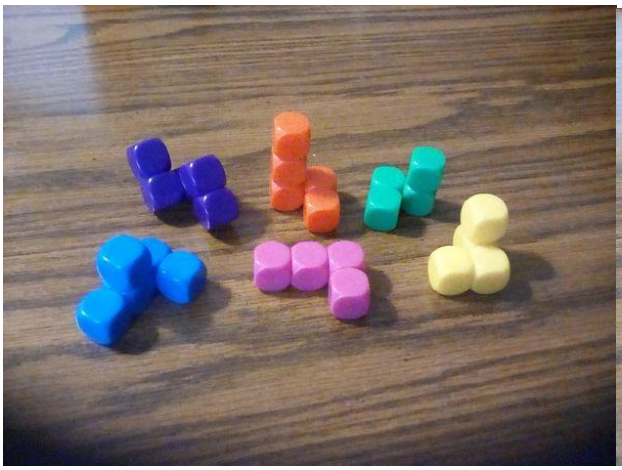
Designed by Chris House. Made from 3D printed plastic. One Solution.

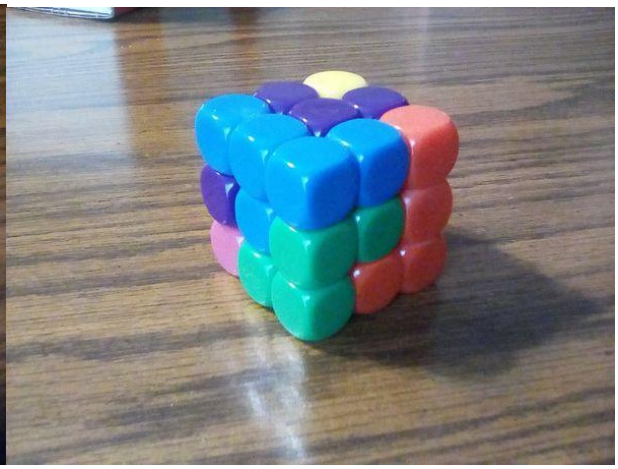




Mikusinski Cube (3X3X3)

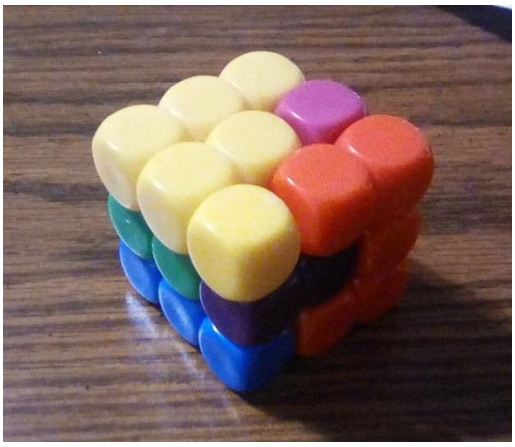
Also known as the Steinhaus Cube. It should be noted that there are other possible ways to make the Mikusinki Cube. Suppose that you make two sets of these pieces, the second set being the mirror of the first. Take one piece of each type from either the first or the second set. If you do the standard problem of making a cube, some of the sets will make for a more difficult solution than others...





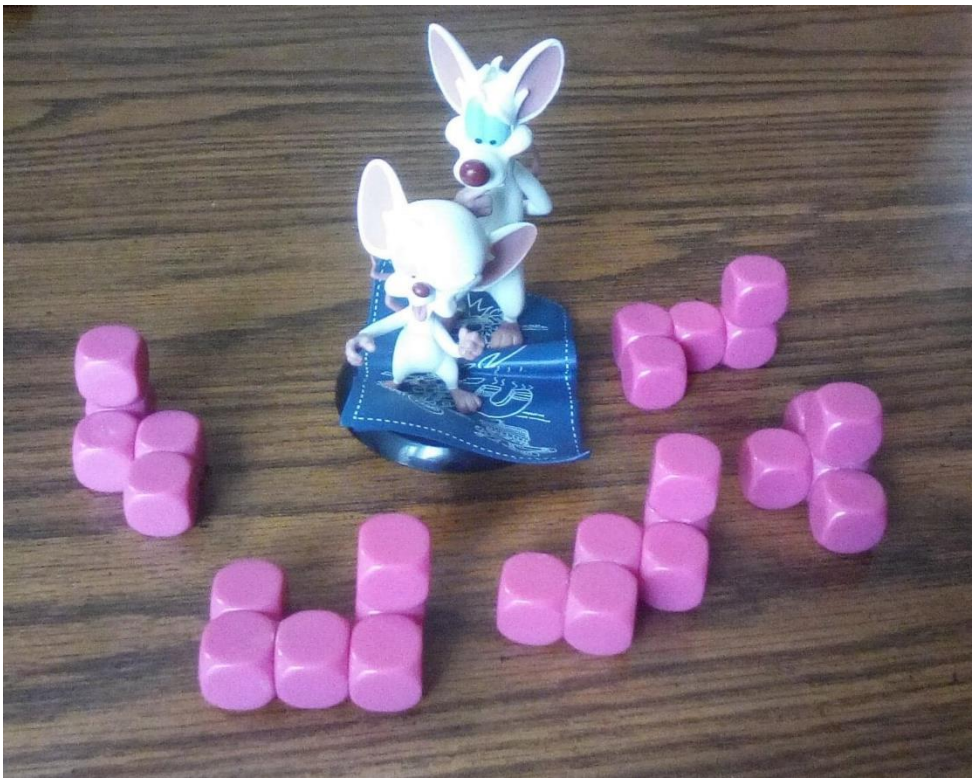
Diabolical Cube (3X3X3)

Diabolical Cube on Wikipedia



Pinky, the Brain (3X3X3)

This is one that I designed and assembled.

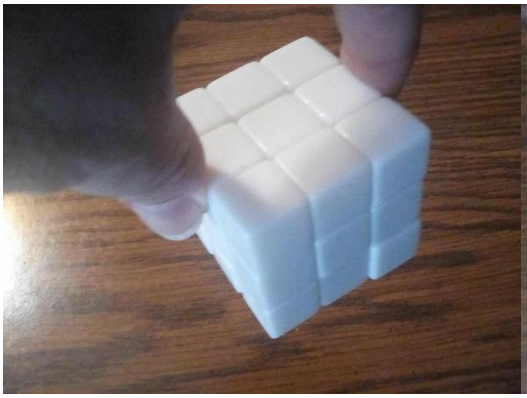


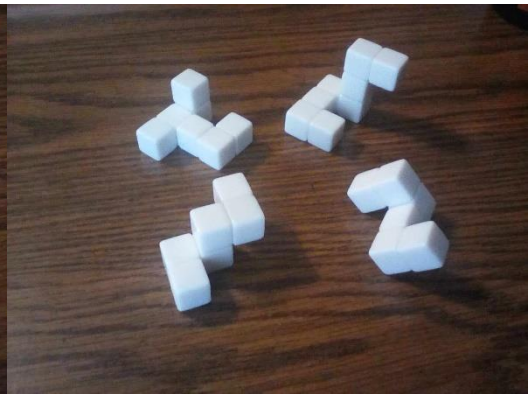
Best 6 (3X3X3)

Designed by Bram Cohen. Up to rotations and reflections, there is a unique solution. We can analyze this puzzle in a similar way to SOMA. Note that the cube has 8 corners. Two of the pieces can occupy at most 2 corners each. The other four pieces can occupy at most 1 corner. So it is easy to see that each piece must occupy the maximum number of corners!


Four-piece Serially Interlocking Cube (3X3X3)

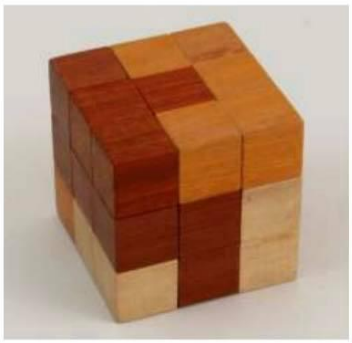
This is a puzzle designed by Stewart Coffin. As the name implies, the four pieces of the puzzle must be placed in sequence to form the cube. This is unlike most such puzzles where the pieces can be placed in any order. Since the pieces interlock, it is possible to pick the solved cube up by one piece on the top (as seen below) and the puzzle not fall apart. This is also very unusual amongst such puzzles.





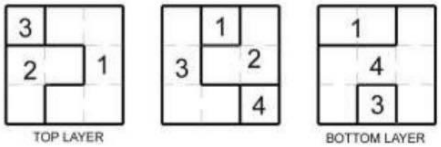
Here is more about the puzzle from Coffin himself:

93. Four-Piece Serially Interlocking Cube. Now, more than sixty years after fashioning the *Mikusiński Cube* from wood scraps, dissections of the 3x3x3 cube continue to fascinate me, but especially those that interlock. Better still, with all dissimilar non-symmetrical pieces. Is such a five-piece version possible? I doubt it, after having searched for years. Perhaps some curious math whiz will come up with an impossibility proof, and perhaps using a computer. I have designed several that come close, but most use a piece or two that is symmetrical, such as a single block key. A four-piece version that satisfies all of these requirements can also be entertaining. Here is one. How many others are possible? Why not explore on your own with hobby store cubes. Children might find recreations like this both enjoyable and educational. Accordingly I omit design details, but the photo with multi-colored pieces gives clues.



As already mentioned in connection with *Convolution* #30, I purposely refrain from publishing some of my design notes, such as those for interlocking dissections of the 3x3x3 or 4x4x4 cube. Let others also have the pleasure of seeking and discovering. By the same token, I feel that some other mathematical recreations of this nature are perhaps better left unpublished (although this *Compendium* may strike some as a glaring contradiction). I have somewhat softened my notions about this in recent years, and this *Compendium* contains several solutions and design details not previously published.

What could I have been thinking when I wrote that in 2014? Here is the plan, but I still think you might have more fun coming up with a new and original design of your very own.

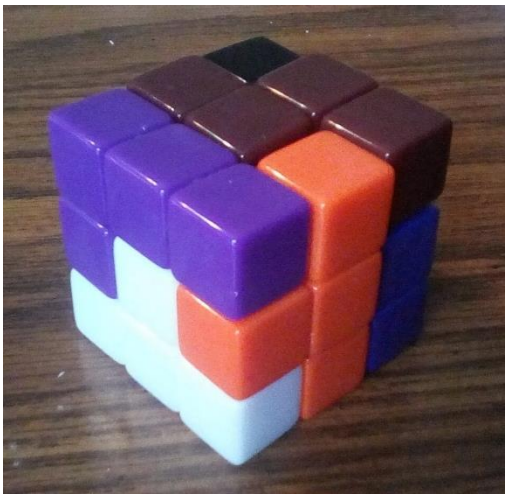


Five-Piece Solid Block Puzzle (3X3X3)

Another by Stewart Coffin.

Coffin's Half-Hour Puzzle (3X3X3)

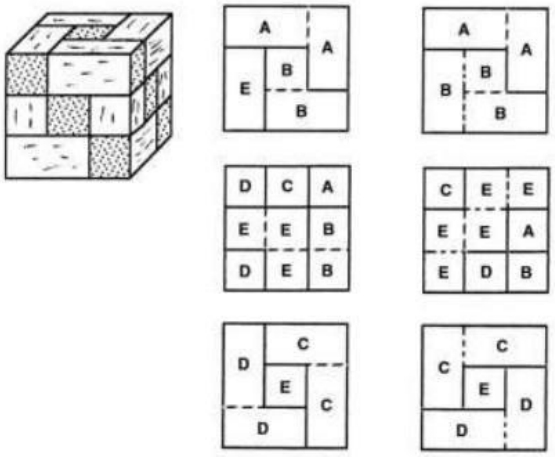
Yet another by Stewart Coffin. This is so called because Coffin believed that a half-hour to solve the puzzle. I must confess, it took me longer than that the first time.



Coffin's Pretty Puzzle (3X3X3)

Another by Coffin. I made this one with wood and stain. Part of the appeal of this puzzle is the pattern when it is solved. As a result, the pieces have individual pieces have multiple colors on them.

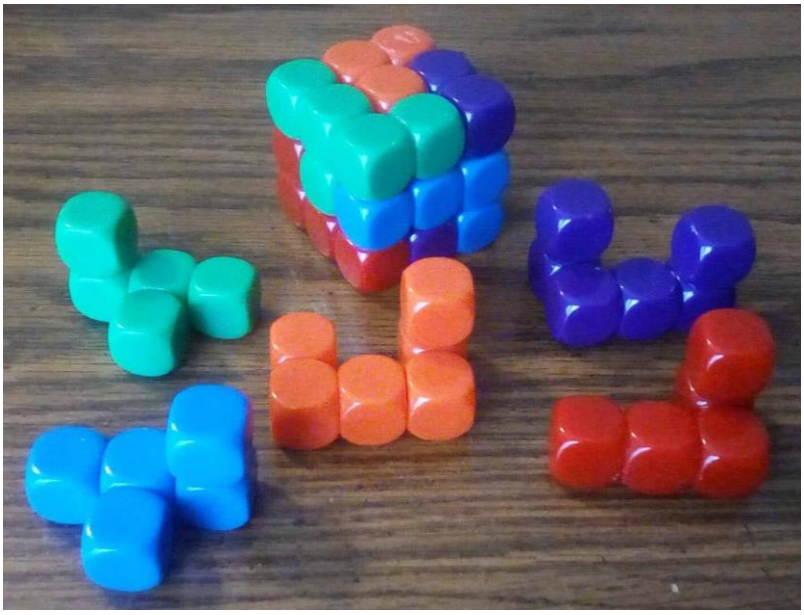
78-D. Pretty Puzzle. This is not just another five piece dissection of the 3x3x3 cube. It rewards the solver with symmetrical patterns of the dissimilar colorful woods on all six faces. Knowing that is an aid to solving. The letters in the drawing indicate the mechanical construction. You can choose your own coloring scheme.



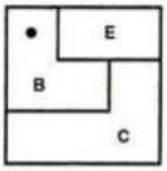
The original design, in the left column, was found to have three solutions. The improved design with only one solution is on the right.

Coffin’s 78-B

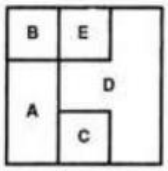
This one is based on Stewart Coffin's "78-B" in his Compendium. Stewart's design had a hidden swivel in one of the joints which allowed you to twist that piece in order to make the puzzle impossible to solve. Because I have neither the technical skill to pull that off nor the sick sense of humor to play such a joke, mine has no such swivel.

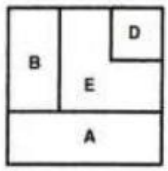


78-B. This was another variation of the above, in which one of the pieces had a swivel joint. The joint used a countersunk screw, with the screw head hidden within a glue joint. The idea was that the jointed piece could be mischievously turned to a wrong shape, and the unsuspecting victim would then seek the cubic 3x3x3 solution in vain. This scheme is not typical of my AP-ART creations, and I can only wonder what accounted for this deviation in 1990. I probably made only one or two of these, and likewise for the others above. But this one, improved, later became an IPP exchange puzzle as *Computer Killer*, #193. In the photo of the pieces, the piece with swivel joint is top right and is shown turned correctly for assembly, with the swivel joint marked in red.



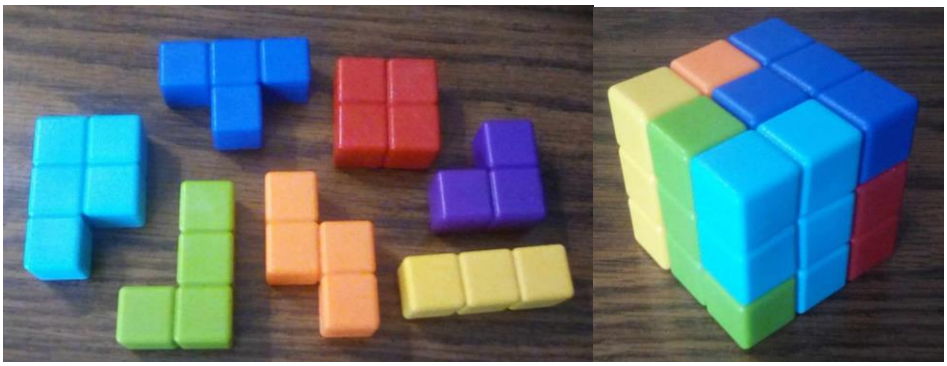
TOP





BOTTOM

Cube Puzzler Go (3X3X3)



Produced by Smart Games. Seven plastic pieces shown above. Comes with a booklet that has 80 different challenges to make cubes.

Cube Puzzler Pro (3X3X3)

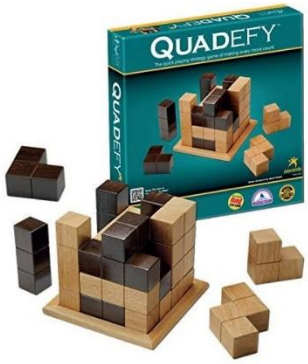
Like the previous puzzle, this is also produced by Smart Games. Not surprisingly, this also consists of seven plastic pieces and a booklet that includes 80 challenges. Unlike the previous puzzle, the individual “cubes” are round orbs. As an unfortunate consequence of this design choice, the puzzle has trouble staying in a “solved’ configuration – even if you solve it in its clear plastic case.

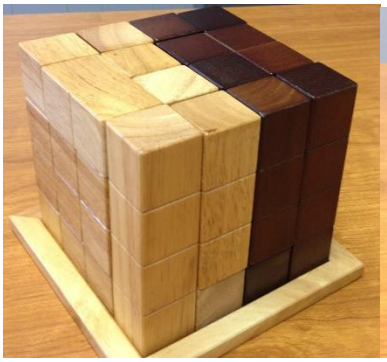
Unknown (3X3X3)

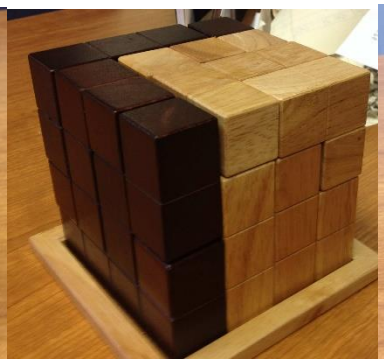
This is one that I assembled. I don’t have a name associated with it, but I doubt that I came up with it in a fever dream. If anyone knows the name of the puzzle, please let me know.

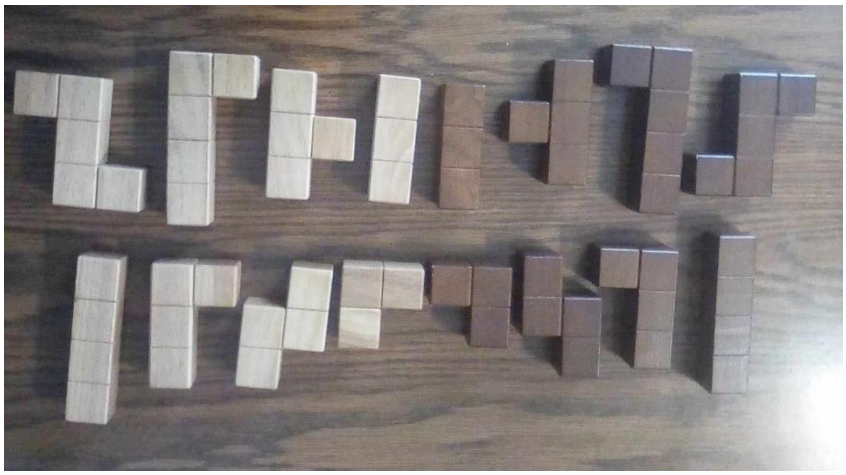
Quadefy (4X4X4)

Quadefy is a 2-player game designed by Mark Fuchs and published by Miranda Enterprises/Hobby World. The light and dark pieces are identical in shape. One player takes the light pieces and the other takes the dark pieces. They alternate placing pieces into a 4X4X4 grid. The first player that plays outside that grid loses. A draw would of course mean placing the pieces into a cube shape. There are several ways to do this, as illustrated below.


QUADEFY

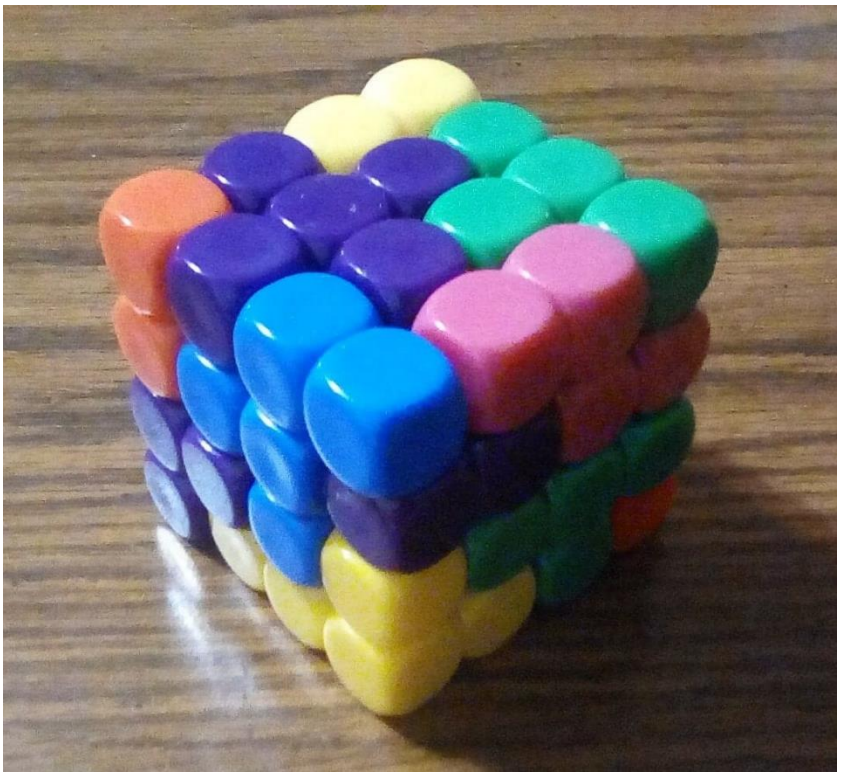






Bedlam Cube (4X4X4)

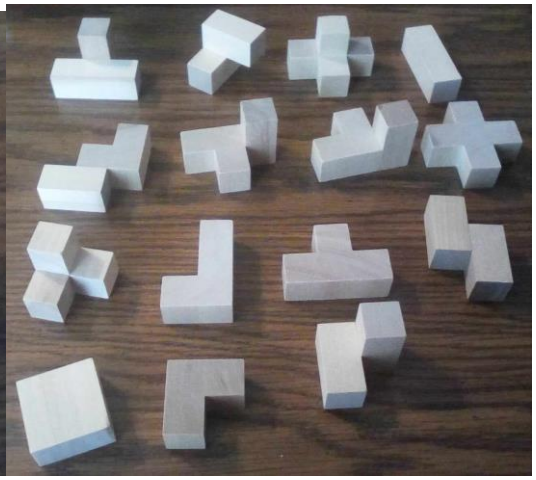
[Bedlam Cube on Wikipedia]

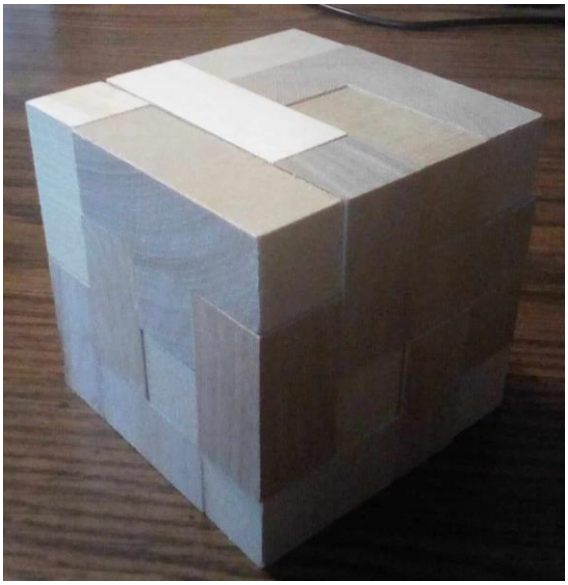


Advanced Soma Cube (4X4X4)

The Advanced Soma Cube (or aSOMA) was designed by Damien Loveland. It consists of 15 wooden pieces and is manufactured by Dee's Invention Corporation. One of the interesting features of this puzzle is that within the set of pieces you have the pieces for a traditional Soma cube and some other puzzles. The manufacturer's webpage contain puzzles to assemble into other shapes than the cube.

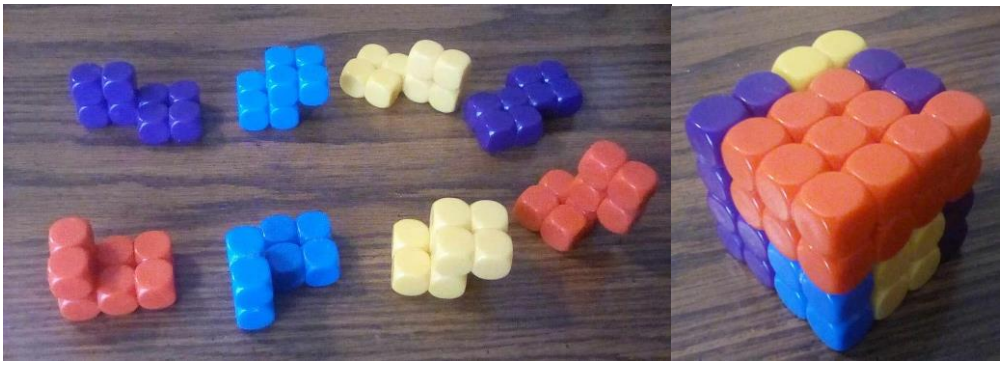
Advanced Soma Cube Website





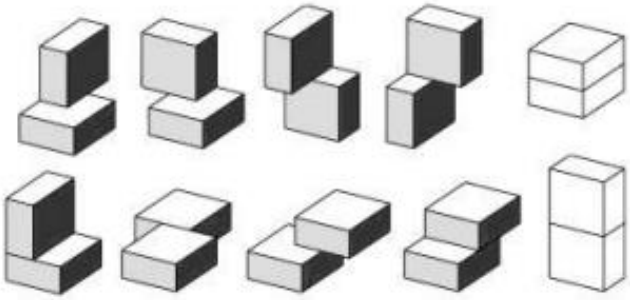
Patio Blocks (4X4X4)

Another by Stewart Coffin. Like the Mikusinski Cube, there is not a unique set of pieces to this puzzle. Start with a set of 2X2X1 blocks and join them together in all possible ways (up to rotation) to yield 10 pieces. Then remove the two that are rectangular solids. The remaining 8 pieces cannot be assembled to form a cube. However, if you remove any one of the pieces and replace it with a duplicate of another remaining piece, then you can form a cube.

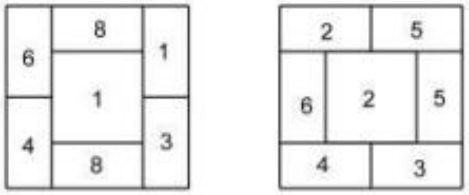


Here is what Coffin has to say about the puzzle:

82. Patio Block. The idea for this one came to me from a publication by Rik van Grol and from a similar design by Kevin Holmes. Here is a great opportunity for recreation that demands very little shop work. Start by joining 1x2x2 blocks all ten possible ways. Now put aside the two that are rectangular solids. Try fitting the other eight into a 4x4x4 box until you become convinced that it is impossible. Try eliminating one and duplicating another until they not only fit but do so with interesting symmetry. I purposely omit the design so that readers may have the pleasure of rediscovering it, the pieces are so easy to make and so much fun to play with. An IPP exchange.

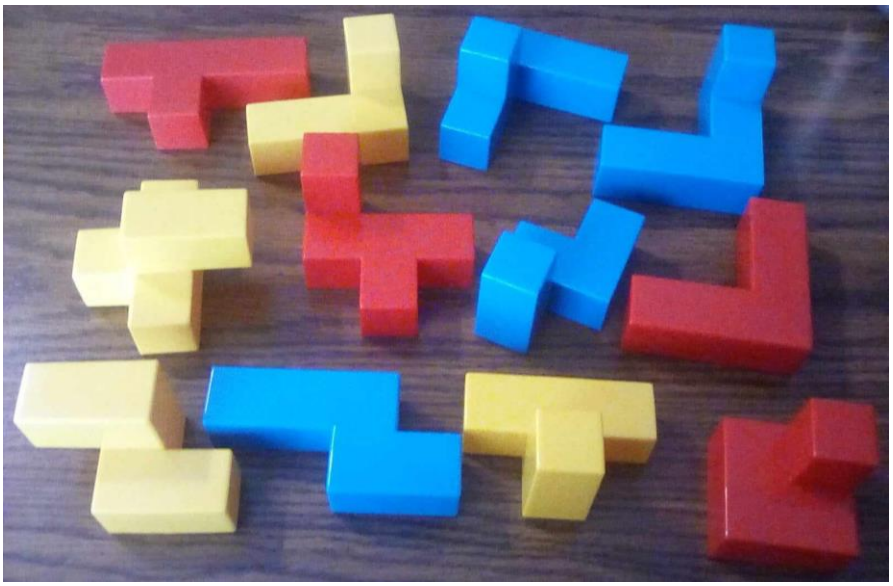


In editing, I have decided, why be so coy? In my version the zigzag piece on lower right is the one that is duplicated, and the two-step piece just to the left of it is omitted. But evidently I have not saved my design notes, so I do not now know if this is necessarily the only way, or if other possibilities exist. So here is another opportunity for investigation in recreational mathematics. And of course play around also with the full set of ten pieces.



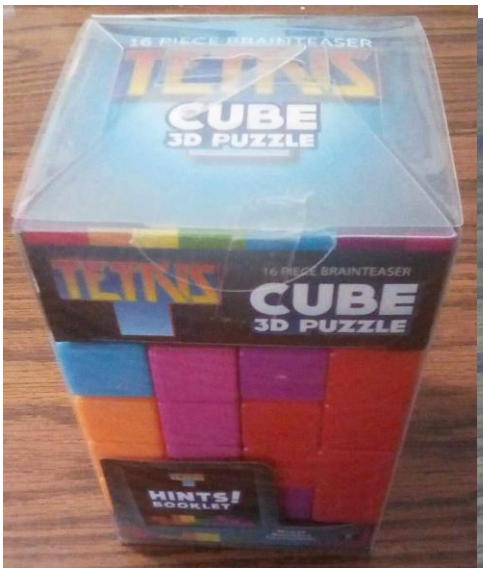
Tetris Cube “Imagination” (4X4X4)

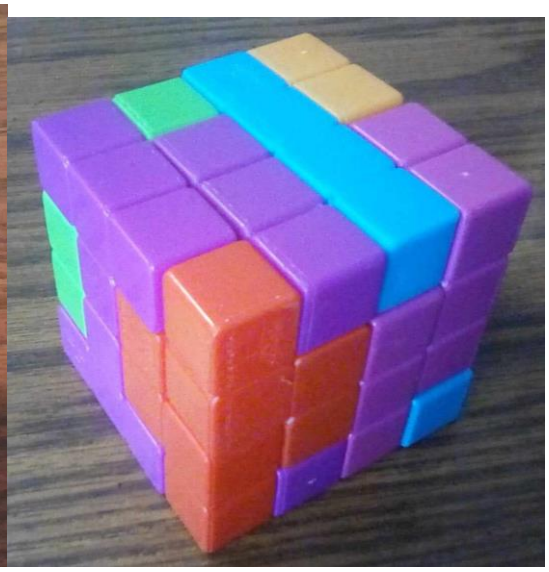
Note: There are two puzzles on my list known as the “Tetris Cube”. In order to differentiate them, I have put the manufacturer’s name in Quotes. 12 distinct plastic pieces.

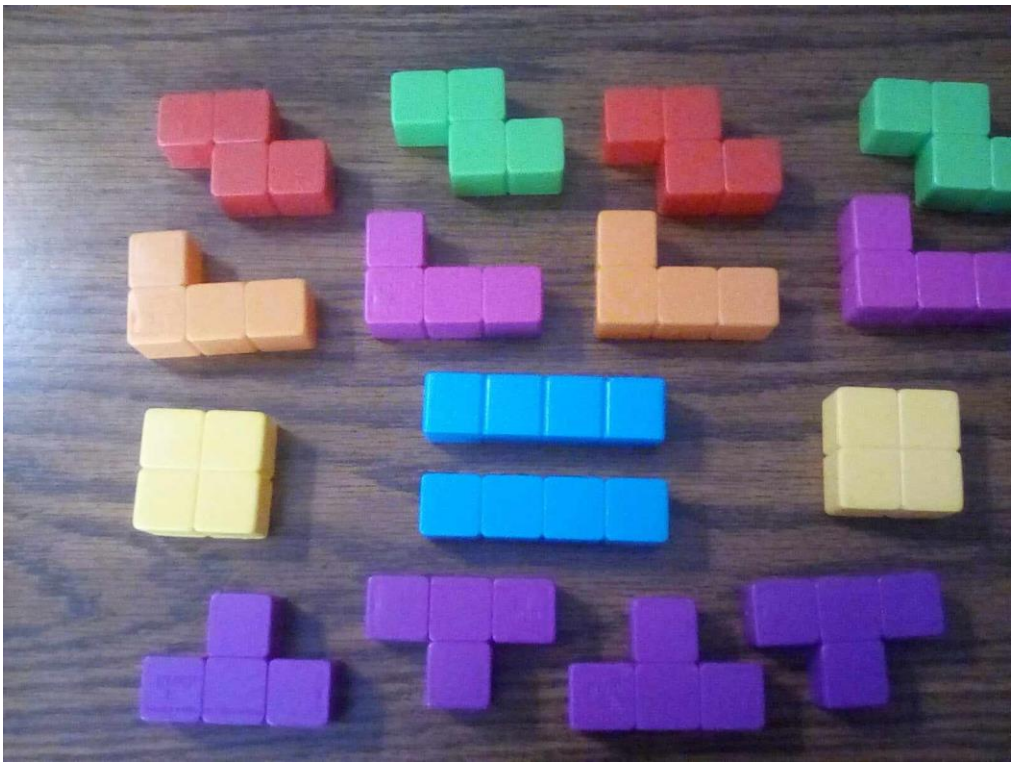


Tetris Cube “Masterpieces” (4X4X4)

Produced by Masterpieces Corporation. 16 plastic pieces. Unlike the previous “Tetris Cube” puzzle, these are flat pieces that appear in the Tetris game.

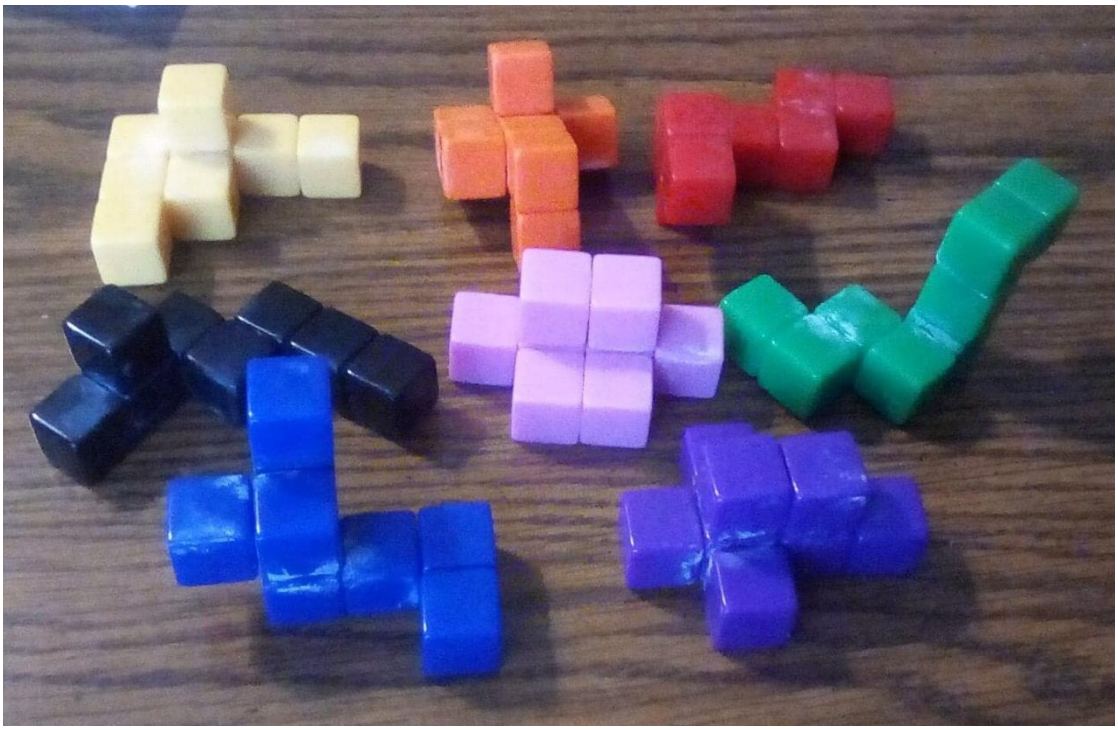


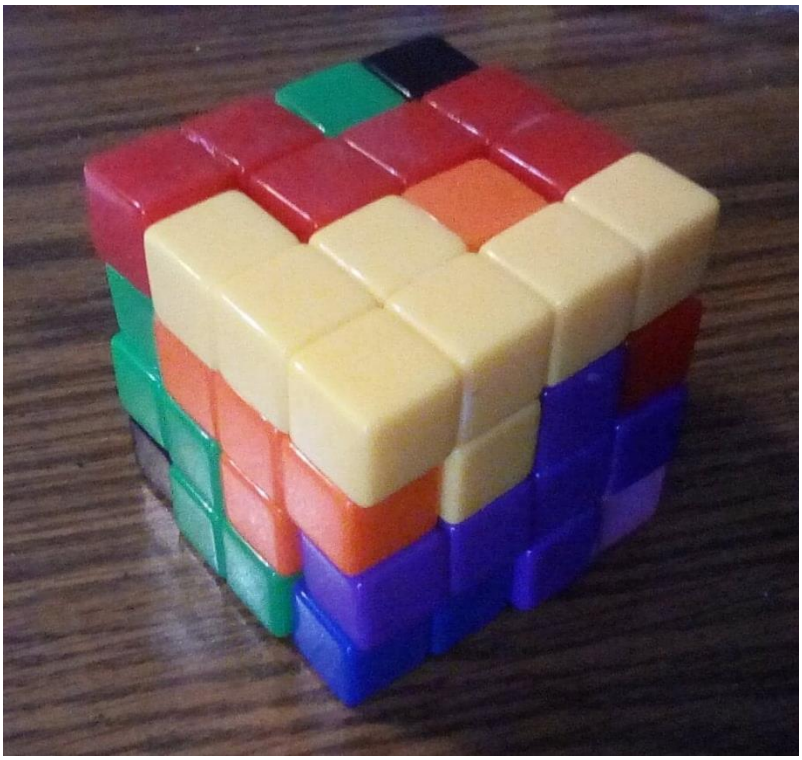




Beeler's Cube (4X4X4)

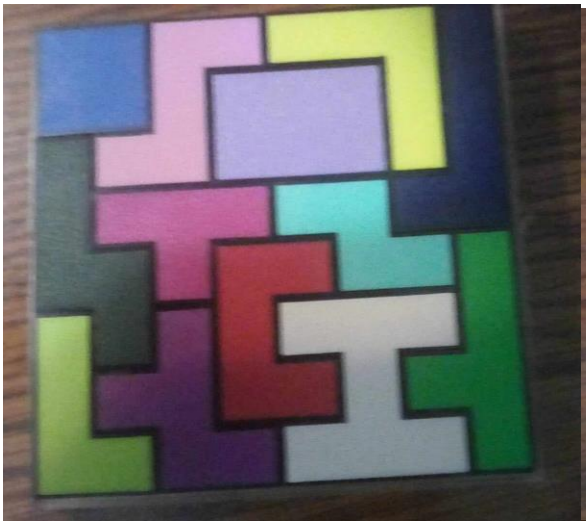
This is one that I designed and made from blank dice and gorilla glue. Eight pieces, each made from eight cubes.

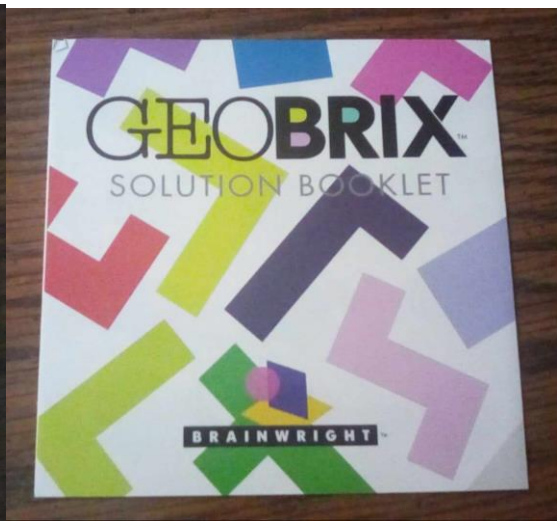




Geobrix (4X4X4)

Published by Brainwright out of plastic pieces. The pieces only have their top faces colored (the rest is black). As a result, the assembled cube unfortunately does not display the same level of color as the 8X8X1 flat solution.





Stacked Sticks (4X4X4)

This is another puzzle suggested to me by John Devost. The pieces interlock and it is somewhat difficult to assemble or dissassemble, even if you know the trick (it separates into two halves). Unlike most of the puzzles I have put together, I made this from 4 in by 1in by 1 in wood blocks glued together. In order to get the pieces to slide together, it was necessary to do quite a bit of sanding (which would have been impossible with plastic dice).

Einstein Letter Block Puzzle (4X4X5)

Designed by Martin Watson and published by Professor Puzzle. This puzzle has 16 wooden pieces that resemble the letters in “Albert Einstein”.