General Physics Labs I (PHYS-2011)
EXPERIMENT MEAS–1:
The Science of Measurements

1 Introduction

The science known as **physics** represents the *foundation* of all of the **physical sciences**, which includes astronomy, geology, chemistry, and their various subfields. Physics is written in the language of mathematics and is based upon logical thought processes. It is often convenient to talk about physics in two components: *theoretical physics* and *experimental physics*. In this course, we will be learning about the experimental component of physics and focus on the following areas of classical physics:

- **Classical Mechanics** – the physics of motion.
- **Thermodynamics** – the physics of heat involving solids, liquids, and gases.
- **Fluid Mechanics** – the physics of fluid flow.

In this introductory *General Physics I Laboratory* class, we will be exploring the science of making, analyzing, and reporting scientific measurements. In order to carry out this work, we will exploring various topics used in the scientific method as described in the following sections.

2 Units of Measure

There are three different unit systems that are used in science and engineering. In the list below, the first two are commonly called the **metric system**.

- **International Standard (SI) units** (once called mks [for meter-kilogram-second] units). This is the unit system used by most scientists.
- **cgs** (for centimeter-gram-second) **units**. This unit system is still used in some areas of science (*e.g.*, astronomy and thermodynamics).
- **English units** (foot-slug-second), also called American, British, or Empirical units. This unit system is considered <u>archaic</u> by the scientific community. The United States is the only technologically advanced country that still uses this system (though American scientists do not use it). Strangely, American engineers still use the English system.

There are 3 basic units in each unit system that relates to 3 independent concepts in physics: **length**, **mass**, and **time**. For the SI unit system, these 3 concepts are measured in units of:

- **Length [L]:** meter — [m].
- **Mass [M]:** kilogram ("kilo" mean 1000, or 1000-grams) — [kg].
- **Time [T]:** seconds — [s].

Since physics often deals with very large and very small numbers for the measurement of units, the metric system contain *prefixes* for units as shown in the table below.

Metric Prefix†	**Numeric Multiplier**	**Multiplier Name**
yotta- (Y-)	10^{24}	septillion
zetta- (Z-)	10^{21}	sextillion
exa- (E-)	10^{18}	quintillion
peta- (P-)	10^{15}	quadrillion
tera- (T-)	10^{12}	trillion
giga- (G-)	10^{9}	billion
mega- (M-)	10^{6}	million
kilo- (k-)	10^{3}	thousand
hecto- (h-)	10^{2}	hundred
deka- (da-)	10	ten
deci- (d-)	10^{-1}	tenth
centi- (c-)	10^{-2}	hundredth
milli- (m-)	10^{-3}	thousandth
micro- (μ-)	10^{-6}	millionth
nano- (n-)	10^{-9}	billionth
pico- (p-)	10^{-12}	trillionth
femto- (f-)	10^{-15}	quadrillionth
atto- (a-)	10^{-18}	quintillionth
zepto- (z-)	10^{-21}	sextillionth
yocto- (y-)	10^{-24}	septillionth

In the previous table, the prefix name (see the column marked with †), has an abbreviation in parentheses associated with the name that can be associated with the abbreviation for the unit. For instance, centimeter is written in abbreviation form as 'cm' and microjoule is written in abbreviation form as μJ.

In order to solve problems in physics, one needs to express all parameters given in the same unit system — this is accomplished with **conversion of units**:

$$v = 5.0\ \frac{\text{mi}}{\text{hr}} = \left(\frac{5.0\ \text{mi}}{\text{hr}}\right)\left(\frac{1.6\ \text{km}}{\text{mi}}\right)\left(\frac{10^3\ \text{m}}{\text{km}}\right)\left(\frac{1\ \text{hr}}{3600\text{s}}\right) = 2.2\ \frac{\text{m}}{\text{s}}\ .$$

Note in the example above that there are 1.6 km in one mile [mi], 10^3 m in one kilometer [km], and 3600 s (seconds) in one hour [hr]. Also note that the conversion fractions have been set up such that the units cancel until we wind up with the units we want (SI units).

3 Scientific Notation

In physics you often find numbers that are both very large and very small. To handle such numbers, scientists express numbers using **scientific notation**:

$$m \times 10^n\ .$$

- **Rule #1:** m is called the **mantissa** of the number and can be a positive or negative real number, where the absolute value of m ranges anywhere from (and equal to) 1.0 up to (but not including) 10:

$$1.0 \leq |m| < 10.$$

- **Rule #2:** n is called the **exponent** of the number and must be a positive or negative integer that ranges from $-\infty$ to $+\infty$:

$$-\infty, ..., -3, -2, -1, 0, 1, 2, 3, ..., \infty\ .$$

In terms of scientific notation, the numbers 232 and 0.0232 are expressed as

$$232 = 2.32 \times 10^2 \qquad \text{and} \qquad 0.0232 = 2.32 \times 10^{-2}$$

When carrying out calculations using scientific notation, one needs to handle the mantissas and the exponents separately:

- Multiplication:

$$\begin{aligned}(4.6\times10^{16})\,(2.0\times10^{2}) &= (4.6\times2.0)\times10^{16+2}=9.2\times10^{18}\\(-5.0\times10^{8})\,(6.0\times10^{-10}) &= (-5.0\times6.0)\times10^{8+(-10)}=-30.\times10^{8-10}\\&= -30.\times10^{-2}=-3.0\times10^{-1}=-0.30\end{aligned}$$

- Division:

$$\begin{aligned}\frac{(6.3\times10^{8})}{(3.0\times10^{4})} &= \frac{6.3}{3.0}\times10^{8-4}=2.1\times10^{4}\\\frac{(6.3\times10^{8})}{(3.0\times10^{-4})} &= \frac{6.3}{3.0}\times10^{8-(-4)}=2.1\times10^{8+4}=2.1\times10^{12}\end{aligned}$$

- Raising to a power:

$$\begin{aligned}(200)^2 &= (2\times10^2)^2=(2)^2\times10^{2\times2}=4\times10^4\\(1600)^{1/2} &= (16\times10^2)^{1/2}=(4.0^2)^{1/2}\times(10^2)^{1/2}\\&= 4.0^{2\cdot1/2}\times10^{2\cdot1/2}=4.0\times10=40.\end{aligned}$$

  Note that $\sqrt{x}\equiv x^{1/2}$, $\sqrt[3]{x}\equiv x^{1/3}$, etc. Hence, the square root is the same as raising a number or variable to the one-half power. The "$\equiv$" symbol means "defined to be."

Note that "as a rule-of-thumb," numbers smaller than <u>0.01</u> and larger than <u>9,999</u> should be written using scientific notation

4 Significant Digits

Measurements of data or results of calculations should <u>never</u> be written out as more digits than are significant. The significance of a measurement is typically limited by the precision of the measuring tool you use, by your ability to use that tool, or even by the nature of what you are trying to measure. *You must learn to express the results of your measurements so that the precision (or lack of precision) is clearly indicated.*

The precision of a measurement will be based on the "smallest" marker on your measuring device. For instance, if you are making length measurements with a meter stick whose markings are subdivided into centimeter marks, the precision of your length measurement will be to within 1 cm (*i.e.*, ±0.01 m). If a stop watch is subdivided into one-tenth of a second markers, then your precision is to within ±0.1 s.

When carrying out calculations, the result you write down should not exceed the significance of your input numbers, even if your calculator display a lot of digits (*i.e.*, *calculators do not keep track of significant digits – it is up to you to keep track of significant digits*).

Multiplication and division have a separate set of rules than addition and subtraction concerning significant digits. In multiplication and division, the number of significant figures (or digits) in the final result should be equal to that factor with the least number of significant digits:

$$\begin{aligned}\frac{(3.0379624 \times 10^{-24})\,(\underline{2.6} \times 10^{-2})}{(3.14156 \times 10^{-6})} &= 2.514261 \times 10^{-20} \\ &= \underline{2.5} \times 10^{-20}\end{aligned}$$

In addition and subtraction, the vertical column containing the least significant digit limits the result:

$$\begin{array}{rcll} 37.2697\underline{2} & & & (7 \text{ s.d.}) \\ 25.4\underline{3} & & & (4 \text{ s.d.}) \\ .83\underline{7} & & & (3 \text{ s.d.}) \\ 101.2\underline{2} & & & (5 \text{ s.d.}) \\ 3.\underline{1} & & & (2 \text{ s.d.}) \\ \hline 167.\underline{8}5672 & = & 167.\underline{9} & (4 \text{ s.d.}) \end{array}$$

Here we rounded the least significant digit up by one since the digit just to the right of it is 5 or above.

To add or subtract numbers written in scientific notation, one must first re-express the numbers such that they all have the same power of 10. Then the addition or subtraction is carried out following the technique above:

$$\begin{array}{rlcrlcl} 3.7697 & \times 10^{-4} & = & 376.9\underline{7} & \times 10^{-6} & & \\ -2.892 & \times 10^{-6} & = & -2.89\underline{2} & \times 10^{-6} & & \\ \cline{4-5} & & & 374.0\underline{7}8 & \times 10^{-6} & = & 3.740\underline{8} \times 10^{-4} \end{array}$$

The significant digits of powers and roots are treated the same as multiplication and division. When an expression has both addition/subtraction and multiplication/division, both rules will have to be used in the order defined by the equation set-up.

Often the result of a calculation will have trailing "zeros." One will need to figure out how many of the trailing zeros are significant. One follows the rules described above to determine whether or not a trailing zero is significant. For instance, if a certain length measurement has a precision of 0.1 cm, then one would write that measured length as "38.0 cm." The underline indicates that the "0" must not be dropped! To drop it would imply a less precise measurement. If a number is written as an integer (*i.e.*, no decimal point shown), trailing zeros after a non-zero integer may or may not be significant. In these cases, **always underline the last zero that is significant**. If a "whole" number is written as a *real* number (*i.e.*, one with a decimal point), then one should assume that all of the trailing zeros after the non-zero number(s) are significant. If one or more are not significant, then one should underline the last significant zero. For instance: 76,000 (2 s.d. – significant digits), 76,000 (4 s.d.), 800. (3 s.d.), 800 (2 s.d.). **Please note that your calculator will drop trailing zeros written after a decimal point (*i.e.*, the right side of a decimal point). However if any of those trailing zeros are significant, it is up to you to make sure that you write these significant trailing zeros on your data sheets and final Lab Report.**

How do we handle "leading" zeros? The answer to this question depends on whether the leading zeros are before or after a decimal point. Leading zeros prior to a non-zero number on the left side of the decimal point are never significant and should not be written out – for instance if we have the following number, 000876.98, we would never write out the leading zeros, instead this number would be written as 876.98, assuming the '.98' is significant. If the zeros follow a decimal point and precede a non-zero integer (*e.g.*, .000789), even though the leading zeros are not significant, they must still be written to properly express the value of the number. However note that in this example, we should express this number in scientific notation (*e.g.*, 7.89×10^{-4}) following the "rule-of-thumb" note at the end of §3, Scientific Notation.

Final notes: If one makes unit conversions involving prefixes (millimeters to meters, or kilograms to grams, for example), these will never limit the precision of the results, since the prefixes are defined to be exact. For example, 1 m = 100 cm, exactly. Also, when one has a measurement that must be an integer (6 coins, for example), it is generally assumed that the integer is an exact value. Thus, for example, if one takes the average of 8 measurements, one assumes there were exactly 8 measurements. These numbers also never limit the precision of results. Also, one sometimes deals with the combination of multiple measurements. In this case, it is sometimes a question whether one should be rounding intermediate results. One way to deal with this is to retain all of the digits in the intermediate results, but then express the final answer with the appropriate number of significant figures, hence avoiding any *round-off* errors. This retention of the digits in intermediate calculations is known as retaining **guard digits**.

5 Errors and Uncertainties

5.1 Types of Errors

- **Blunders:** These errors can be controlled with enough sleep! If you spot inconsistencies in data points, then a "blunder" may have occurred. The following examples can be considered blunders:
  - Typographical errors in the data.
  - Using wrong data in the analysis.
  - Using wrong equations for the analysis.
- **Random Errors:** These errors do not occur in a definite pattern and can't be controlled. Possible examples include:
  - Electronic fluctuations in the measuring equipment due to power surges or defective battery.
  - Cosmic ray damage on detectors.

Random errors are not associated with a definite pattern and are not avoidable. Even in the most carefully designed experiment done by the best experimenters there are random effects that influence results. In our lab exercises, we will be using a variety of measuring devices such as rulers, vernier calipers, thermometers, etc. We will see that each measuring device has a minimum degree of precision with which they can make a measurement. If a ruler is marked in millimeters, different experimenters may measure different lengths for the same object if they look down on the ruler from a different perspective, and round to different millimeter marks. It becomes even more likely that different measurements will occur if the observer tries to divide up the space between markings in their mind, trying to obtain a reading to the nearest tenth of a millimeter when the ruler is marked only to the nearest millimeter.

No matter how many measurements are made of the same physical quantity, the results will be spread over a range of values. These variations are due to unnoticed variations in technique or unknown changes in the environment. Here there is not necessarily anything wrong with your technique or with your instrument. You are trying to make a measurement with the maximum precision possible with your instrument, and the precise circumstances of each measurement are not exactly repeatable.

When a measurement is subject to random errors, the best way to improve the measurement is to make a number of independent measurements. The best value we can give is then the arithmetic average from all the individual measurements. Even this improved value, however, should be accompanied by an estimate of uncertainties due to random error (see §5.2 below). Usually, in this course, we will not have the luxury to repeat an experiment multiple times. As a result, we will estimate this uncertainty by examining the measurement devices and estimating an intrinsic uncertainty.

- **Systematic Errors:** Systematic errors generally lead to results that are consistently "off" in some manner. The following conditions can lead to systematic errors:
  - Faulty calibration of equipment.
  - Bias from observer or experimenter.
  - A defective technique used by the experimenter.
  - A defect in the design of the experiment.
  - A defective measuring tool.
  - Somebody pulled the plug or the battery goes dead.

  Systematic errors can arise from defects in the design of an experiment, or in the measuring device, or faulty procedure on the part of the observer. Often, they lead to results which are consistently off in some particular manner. A scale which reads 100 grams when no mass is placed on it will consistently indicate too large a mass. Many "meter sticks" are manufactured with centimeters on one side and inches on the other. If a student uses the inches side to measure distance in centimeters they will always obtain too small a result. The effects of systematic errors are impossible to predict, and difficult to minimize. Certainly, the experimenter should be very careful with their procedures to at least eliminate the possibility of outright mistakes in their measurements. This is something that can be avoided at the outset.

5.2 Uncertainties

The term *error* signifies a deviation of the result from some "true" value. However, since we often cannot know the "true" value of a measurement prior to the experiment, we can only determine *estimates* of the errors inherent to the experiment. The difference between two measurements is called the **discrepancy** between the results. The discrepancy arises due to the fact that we can only determine the results to a certain **uncertainty**. The uncertainty of a measurement is typically written as a number followed by the ± of the uncertainty. For instance, if a mass measurement has a value of 25.6 kg and an uncertainty of 0.2 kg, we would write this measurement as 25.6 ± 0.2 kg.

5.2.1 There are two classes of uncertainties:

- The most prominent type: Those which result from fluctuations in repeated measurements of data from which the results are calculated.

- The secondary type: Those which result from the fact that we may not always know the appropriate theoretical formula for expressing the result.

5.2.2 Uncertainty with Respect to an Average:

If we are reasonably sure that errors in a measurement are dominated by random errors, there are more sophisticated methods to evaluate uncertainties in measurements. The best way to obtain a good final number is to make a number of independent measurements, as many as feasible. The best number we can give is the arithmetic average of all the measurements. For N measurements of a quantity L, this arithmetic average, $\overline{L}$, is obtained from all the individual measurements, L_i, as follows:

$$\overline{L} = \frac{1}{N}\left(L_1 + L_2 + L_3 + \ldots + L_N\right) = \frac{1}{N}\sum_{i=1}^{N} L_i \ .$$

We can also calculate an estimate of the uncertainty associated with the average $\overline{L}$, based upon the spread of the individual measurements. This spread is measured quantitatively by $\Delta\overline{L}$, which is calculated (approximately) from the "root mean square" deviation RMS by the following equations:

$$\begin{aligned} \mathrm{RMS} &= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\overline{L} - L_i\right)^2} \\ \Delta\overline{L} &= \mathrm{RMS}/\sqrt{N} \ . \end{aligned}$$

Our measurement is then expressed as $\overline{L} \pm \Delta\overline{L}$. Some of you may be familiar with the first equation, and know how to calculate it directly with your calculator. Don't forget to divide by $\sqrt{N}$ when calculating $\Delta\overline{L}$. Those who are not familiar with these calculations can simply perform them by following the above equations.

5.3 Accuracy versus Precision

Accuracy is how close an experiment comes to the "true" value. It is a measure of the correctness of the result. For an experimenter, it is a measure of how skilled the experimenter is.

Meanwhile, **precision** of an experiment is a measure of how exactly the result is determined without reference to what the results means. It is a measure of the precision of the instruments being used in the experiment.

6 Comparison of Measurements

This semester, you may be asked to compare your measurements, either with measurements made by your classmates (or multiple measurements made on the same object by you), or with accepted values of those measurements. For comparison of measurements with your classmates, one calculates a **% difference**:

$$\% \text{ difference } = \frac{|\text{ measurement 1 } - \text{measurement 2 }|}{\text{average of 2 measurements}} \times 100\% \;.$$

In this case neither measurement is necessarily better. Such a calculation can be another way to estimate the uncertainty associated with the measurements. This is another way to estimate the precision to which the quantities have been measured.

For comparison of measurements with accepted values, one calculates a **% error**:

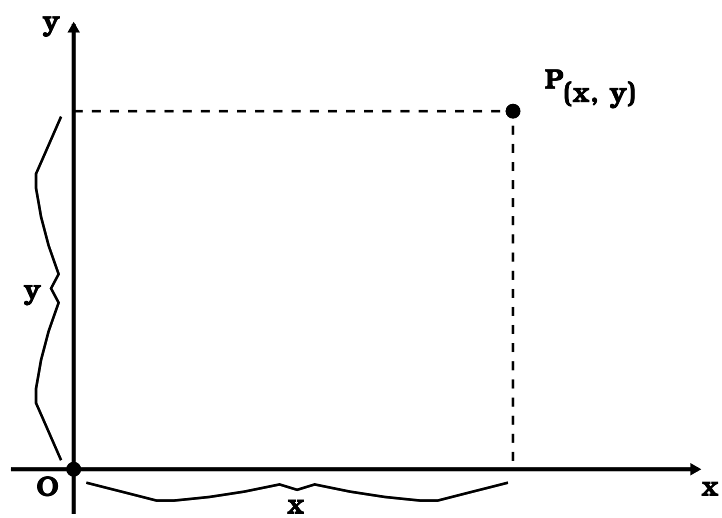
$$\% \text{ error } = \frac{|\text{ measurement } - \text{accepted value }|}{\text{accepted value}} \times 100\% \;.$$

This is a way to evaluate the accuracy, or correctness of your result.

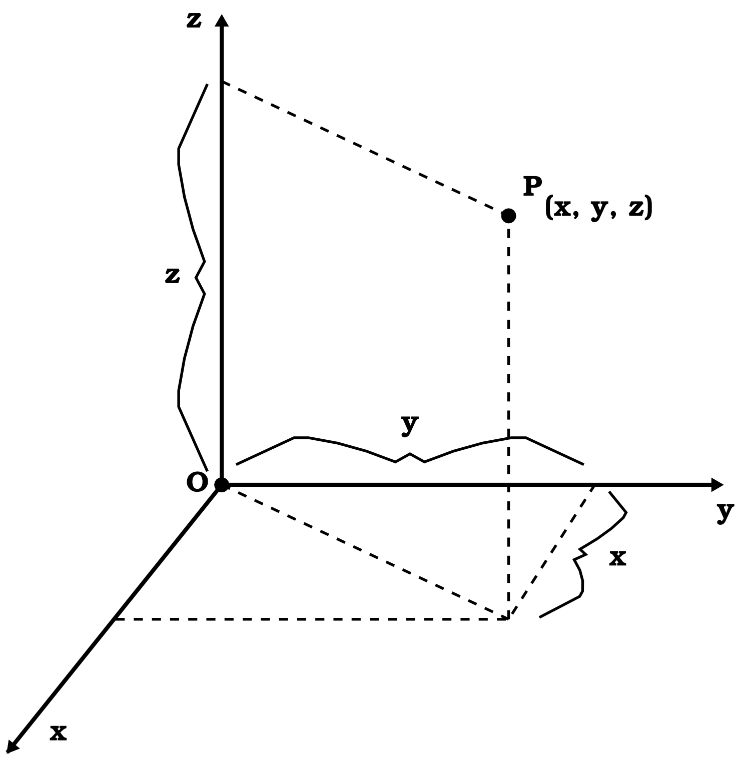
7 Plotting Your Data

After one has completed making measurements, one often needs to display this data in either data tables or by making plots of this data (sometimes you do both of these options). In order to plot your data, you will need to decide on which coordinate system works best to display your data.

- **Cartesian** or **orthogonal coordinates** (x, y, z).
  - 2–D (x, y):

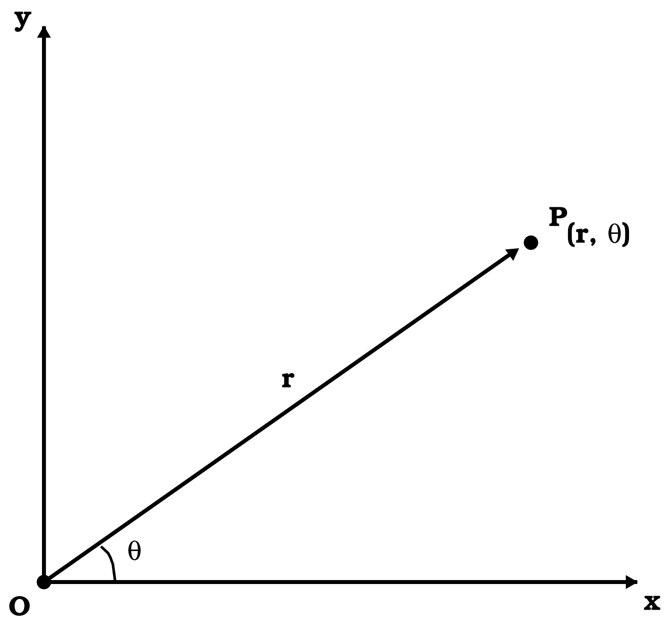


- 3–D (x, y, z): (Note that the 3 axes in Cartesian 3-D space have a specific orientation that follows the **right-hand rule**: With your right hand thumb extended perpendicularly away from your hand, follow the rotation of the x-axis with your fingers curving towards the y-axis. Then, the direction your thumb points is the direction that the z-axis points. Note that the data point $P(x, y, z)$ on the plot on the next page is y units to the right of the origin, z units above the origin, and x units out of the page from the origin.



- **Polar coordinates** (r, θ) can also be used in 2–D situations. In 3–D, polar coordinates become either **spherical coordinates** (r, θ, ϕ) or **cylindrical coordinates** (r, θ, z). Here, we will just focus on 2-D polar coordinates. (See the figure on the next page.)
  - r is called the **radius vector** as is the distance that a point is from the *origin.*
  - θ (Greek letter "theta") is the angle that the radius vector, r, makes with the $+x$-axis (*i.e.*, the **reference axis**). Note that when the radius vector r rotates in the counterclockwise (CCW) direction with respect to the reference axis, θ is positive ($\theta > 0$), and when r rotates in the clockwise (CW) direction with respect to the reference axis, θ is negative ($\theta < 0$).

A little later in this course, you will carry out an experiment where you will learn to create graphs (also called "plots") of data obtained from a hypothetical experimental set-up (see Experiment MEAS-4: Graphs and Graphical Analysis).



8 Example Exercises

The following exercise is an example of the MEAS-1 lab worksheet you will be given in lab today. Solutions are shown after the questions/problems. Try to answer/solve each question/problem prior to looking at the answer/solution.

Math Lab Worksheet
(10 points total)

1. How many significant digits are there for each number listed:

(a) 3.4056×10^{-12}	________ s.d.	(f) 0.0007906	________ s.d.
(b) 2004	________ s.d.	(g) 2004.0	________ s.d.
(c) 86,000	________ s.d.	(h) $86{,}0\underline{0}0$	________ s.d.
(d) -45.67×10^{-5}	________ s.d.	(i) $000897.8\underline{0}$	________ s.d.
(e) 4000.	________ s.d.	(j) 4000	________ s.d.

2. The following length measurements have been made for a wooden block: 13.2 cm, 13.4 cm, 13.1 cm, 13.2 cm, and 13.3 cm. Calculate the average of these measurements with the associated uncertainty for this average.

Answer: ____________________

3. Add the following numbers following the addition/subtraction rule for significant digits:

$$\begin{array}{r} 69.85926 \\ 206.831 \\ 2.7986 \\ 398.21 \\ 0.369 \\ \hline \end{array}$$

Answer: ____________________

4. Evaluate the following expression following the multiplication/division rules for significant digits (don't forget the units and remember 'guard digits'):

$$\frac{6.987 \times 10^{-12}\ \mathrm{N}}{(-1.22 \times 10^{-8}\ \mathrm{m})\,(8.314159\ \mathrm{m})}\ .$$

Answer: ____________________

5. The mass density of any object is defined as its mass divided by its volume. A silver bar has mass 23.52 kg and volume 2.24×10^{-3} m^3. What is the density of silver? (Don't forget units and significant digits!)

Answer: ____________________

6. The radius of a circle is measured to be 3.44×10^{-3} m. Find the area of this circle. (*Hint:* $A = \pi r^2$ for a circle, where $\pi = 3.14159$.)

Answer: ____________________

7. Solve the following radical:

$$\sqrt[3]{8.742 \times 10^{6}}\ .$$

Answer: ____________________

8. A student measures the length and width of a rectangular surface, recording a length and a width in units of centimeters, and calculates the area of the rectangle to be 753 cm^2. Unfortunately, the student did not notice that the measuring stick was numbered in inches, NOT in centimeters. Given that one cm = 0.3937 inches, what is the actual area of the rectangle in cm^2?

   Answer: ____________________

9. A student measures the density of glass in a laboratory experiment and finds the value $\rho_{\text{glass}} = 3.08\ \text{gm/cm}^3$. Compare this result to the accepted value of $\rho_\circ = 2.58\ \text{gm/cm}^3$. (**Show all work!**)

   Answer: ____________________

10. Two students each measure the index of refraction of a glass prism, and get the values $n_1 = 1.81$ and $n_2 = 1.42$ respectively. Compare the two measurements.

    Answer: ____________________

Math Lab Worksheet Solutions

1. How many significant digits are there for each number listed:

(a) 3.4056×10^{-12}	5	s.d.	(f) 0.0007906	4	s.d.
(b) 2004	4	s.d.	(g) 2004.0	5	s.d.
(c) 86,000	2	s.d.	(h) $86{,}0\underline{0}0$	4	s.d.
(d) -45.67×10^{-5}	4	s.d.	(i) $000897.8\underline{0}$	5	s.d.
(e) 4000.	4	s.d.	(j) 4000	1	s.d.

2. The following length measurements have been made for a wooden block: 13.2 cm, 13.4 cm, 13.1 cm, 13.2 cm, and 13.3 cm. Calculate the average of these measurements with the associated uncertainty for this average.

$$\begin{aligned}
\overline{L} = \frac{1}{5}\sum_{i=1}^{5} L_i &= \frac{1}{5}\left(13.2\text{ cm } + 13.4\text{ cm } + 13.1\text{ cm } + 13.2\text{ cm } + 13.3\text{ cm}\right) \\
&= 66.2\text{ cm}/5 = 13.\underline{2}4\text{ cm } = 13.2\text{ cm} \\
\text{RMS} &= \sqrt{\frac{1}{5-1}\sum_{i=1}^{5}\left(\overline{L} - L_i\right)^2} = \sqrt{\frac{1}{4}\sum_{i=1}^{5}\sigma_i^2} \\
\sum_{i=1}^{5}\sigma_i^2 &= (13.2\text{ cm } - 13.2\text{ cm})^2 + (13.4\text{ cm } - 13.2\text{ cm})^2 + \\
&\quad (13.1\text{ cm } - 13.2\text{ cm})^2 + (13.2\text{ cm } - 13.2\text{ cm})^2 + \\
&\quad (13.3\text{ cm } - 13.2\text{ cm})^2 = 0.06\text{ cm}^2 \\
\text{RMS} &= \sqrt{\frac{1}{4}\left(0.06\text{ cm}^2\right)} = 0.1\underline{2}25\text{ cm } = 0.12\text{ cm} \\
\Delta\overline{L} &= \text{RMS}/\sqrt{N} = 0.12\text{ cm}/5 = 0.02\text{ cm}\ .
\end{aligned}$$

Answer: 13.2 ± 0.02 cm

3. Add the following numbers following the addition/subtraction rule for significant digits:

$$\begin{array}{r}
69.85926 \\
206.831 \\
2.7986 \\
398.21 \\
0.369 \\
\hline
678.0\underline{6}786
\end{array}$$

Answer: 678.07

4. Evaluate the following expression following the multiplication/division rules for significant digits (don't forget the units and remember 'guard digits'):

$$\frac{6.987 \times 10^{-12}\ \text{N}}{(-1.22 \times 10^{-8}\ \text{m})\,(8.314159\ \text{m})} = \frac{6.987 \times 10^{-12}\ \text{N}}{-1.014327 \times 10^{-7}\ \text{m}^2} = -6.89 \times 10^{-5}\ \text{N/m}^2\ .$$

Answer: $\underline{\ -6.89 \times 10^{-5}\ \text{N/m}^2\ }$

5. The mass density of any object is defined as its mass divided by its volume. A silver bar has mass 23.52 kg and volume 2.24×10^{-3} m^3. What is the density of silver? (Don't forget units and significant digits!)

$$\rho = \frac{m}{V} = \frac{23.52\ \text{kg}}{2.24 \times 10^{-3}\ \text{m}^3} = 1.05 \times 10^4\ \text{kg/m}^3\ .$$

Answer: $\underline{\ 1.05 \times 10^4\ \text{kg/m}^3\ }$

6. The radius of a circle is measured to be 3.44×10^{-3} m. Find the area of this circle. (*Hint:* $A = \pi\, r^2$ for a circle, where $\pi = 3.14159$.)

$$A = \pi\, r^2 = (3.14159.)\,(3.44 \times 10^{-3}\ \text{m})^2 = 3.72 \times 10^{-5}\ \text{m}^2.$$

Answer: $\underline{\ 3.72 \times 10^{-5}\ }$

7. Solve the following radical:

$$\sqrt[3]{8.742 \times 10^6} = (8.742)^{1/3} \times 10^{6/3} = 2.060 \times 10^2 = 206.0\ .$$

Answer: $\underline{\ 206.0\ }$

8. A student measures the length and width of a rectangular surface, recording a length and a width in units of centimeters, and calculates the area of the rectangle to be 753 cm^2. Unfortunately, the student did not notice that the measuring stick was numbered in inches, NOT in centimeters. Given that one cm = 0.3937 inches, what is the actual area of the rectangle in cm^2?

$$\begin{aligned} A &= 753\ \text{in}^2\ * (1\ \text{cm}/0.3937\ \text{in})^2 \\ &= 753\ \text{in}^2\ * 6.4516\ \text{cm}^2/\text{in}^2 \\ &= 4.8\underline{5}8 \times 10^3\ \text{cm}^2 = 4.86 \times 10^3\ \text{cm}^2 \end{aligned}$$

Answer: $4.86 \times 10^3\ \mathrm{cm}^2$

9. A student measures the density of glass in a laboratory experiment and finds the value $\rho_{\mathrm{glass}} = 3.08\ \mathrm{gm/cm^3}$. Compare this result to the accepted value of $\rho_\circ = 2.58\ \mathrm{gm/cm^3}$. (**Show all work!**)

$$\begin{aligned} \%\ \text{error} &= \frac{|\rho - \rho_\circ|}{\rho_\circ} \times\ 100\,\% = \frac{\left|3.08\ \mathrm{gm/cm^3} - 2.58\ \mathrm{gm/cm^3}\right|}{2.58\ \mathrm{gm/cm^3}} \times\ 100\,\% \\ &= \frac{0.50\ \mathrm{gm/cm^3}}{2.58\ \mathrm{gm/cm^3}} \times\ 100\,\% = 0.19 \times 100\,\% \ = 19.\,\%. \end{aligned}$$

Answer: 19. %

10. Two students each measure the index of refraction of a glass prism, and get the values $n_1 = 1.81$ and $n_2 = 1.42$ respectively. Compare the two measurements.

$$\begin{aligned} \%\ \text{difference} &= \frac{|n_2 - n_1|}{n_{\mathrm{ave}}} \times\ 100\,\% = \frac{|1.42 - 1.81|}{(1.42 + 1.81)/2} \times\ 100\,\% \\ &= \frac{|-0.39|}{1.615} \times\ 100\,\% = 0.24 \times 100\,\% \ = 24.\,\%. \end{aligned}$$

Answer: 24. %