Chapter 6
Momentum and Collisions
Momentum
Momentum is the product of object’s mass and its velocity.

\[ p = m \times v \]

Momentum is a vector quantity.
(direction is important)

Momentum is in the same direction as the object’s velocity.

The units of momentum are: \( \text{kg m/s} \)
Examples

A 3000 kg truck moving with velocity 30 m/s
\[ p = (3000 \text{ kg})(30 \text{ m/s}) = 9 \times 10^4 \text{ kg m/s} \]

A 0.14 kg baseball with velocity 40 m/s
\[ p = (0.14 \text{ kg})(40 \text{ m/s}) = 5.6 \text{ kg m/s} \]

A 6 kg bowling ball with velocity 4 m/s
\[ p = (6 \text{ kg})(4 \text{ m/s}) = 24 \text{ kg m/s} \]
Momentum and Impulse

\[ F_{\text{net}} = ma \quad \text{and:} \quad a = \Delta v/\Delta t \]

\[ F_{\text{net}} = m \Delta v/\Delta t \]

Use momentum \( p = mv \)

So \( F_{\text{net}} = \Delta p/\Delta t \)

Impulse \( I = F\Delta t \)

So \( I = \Delta p \)

Impulse is equal to the change in momentum.
Impulse – Momentum Theorem

\[ F \Delta t = \Delta p = m v_f - m v_i \]

The impulse of the force acting on an object, equals the \textit{change in momentum} of that object.

Work out exercise 6.1 and example 6.2
Impulse from non-constant force

The magnitude of the impulse delivered by a force over the time interval $\Delta t$, is equal to the area under the force vs. time graph as in Figure 6.1a.

This is equivalent to the area $F_{\text{ave}} \Delta t$ as shown in figure 6.1b.

$$F_{\text{ave}} \Delta t = \Delta p$$
Conservation of momentum

When a collision occurs in an isolated system, the individual momentums of objects may change, but the total momentum vector of the whole system is a constant.

\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

There could be more than two particles. If that is the case, we just include as many initial and final momentum terms as we need.
Conservation of momentum

When there is no net total force acting on a system, the total momentum of the system remains constant with time.

Note that when looking at a system or group of particles, because of the law of equal and opposite forces, internal forces will not change the total momentum of the system.
Archer example

An archer stands on frictionless ice and shoots a 0.5kg arrow horizontally at 50 m/s. The combined mass of the archer and the bow is 60 kg. With what velocity will the archer move after shooting the arrow?

Use conservation of momentum

\[ m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \]
Archer

Initially the total momentum is zero. The initial velocities of the archer and the arrow are both zero.

So after the arrow is shot, the total momentum must still be zero.
(Labeled the archer as 1 and the arrow as 2).

\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

\[ (60\text{kg})0\text{m/s} + (0.5\text{kg})0\text{m/s} = (60\text{kg})v_{1f} + (0.5\text{kg})(50\text{m/s}) \]

\[ 0 = (60\text{kg})v_{1f} + (0.5\text{kg})(50\text{m/s}) \]

\[ (60\text{kg})v_{1f} = -(0.5\text{kg})(50\text{m/s}) \]

\[ v_{1f} = -0.417 \text{ m/s} \]

If the arrow is shot to the right, the archer moves to the left.
Notice the minus sign for the velocity of the archer.

This just means that in this case, the velocity of the archer and the of the arrow are in opposite directions.

If the arrow’s velocity is picked to be positive, the archer’s velocity must be negative.

This is an example of “recoil”.
Collisions

When objects collide, momentum is always conserved.

Kinetic energy is not always conserved. Some of the kinetic energy is turned into sound, work needed to deform the object, heat, internal energy.
Momentum and kinetic energy are both conserved. The objects leave each other with the same magnitude of relative velocity as they had before they hit each other. There is no deformation of the objects during the collision.

Examples:

- Perfectly elastic collisions

Momentum and kinetic energy are both conserved.

The objects leave each other with the same magnitude of relative velocity as they had before they hit each other.

There is no deformation of the objects during the collision.

Examples:

- Pool balls (approximately)
- Proton-proton collision
Collisions

• **Perfectly Inelastic collisions**
  Momentum is conserved, but some, or even all, of the kinetic energy will be lost.

  The objects stick together after the collision. They move with the same velocity.
  
  Examples:
  Two cars hitting each other and the bumpers lock up.

  Football player being tackled (wrapped up) by another player.

  Person catching a ball.

  Shooting a ball into a ballistic pendulum.
Collisions

Nearly all collisions in the real world fall somewhere in between being perfectly elastic and perfectly inelastic.

When you hit a baseball the ball doesn’t stick to the bat, so the collision isn’t perfectly inelastic.

Some of the kinetic energy is converted to sound and is used to momentarily deform the ball and bat, so the collision isn’t perfectly elastic either.

https://www.youtube.com/watch?v=QFIEIybC7rU
Perfectly Inelastic Collisions

Since the two objects have the same velocity after the collision, we can rewrite the conservation of energy as:

\[ m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f \]

Solving for the final velocity gives us:

\[ v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} \]

Remember to treat \( v_{1i} \) and \( v_{2i} \) as vectors. Their direction matters.
Inelastic Collision.

Shoot a 0.05 kg bullet at 300 m/s into a 5kg wooden block. The bullet gets stuck in the block, so they move together afterwards. Find the final velocity of the bullet/block.

Cons. of Mom. says: \( m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f \)

Let the bullet be mass 1 and the block be mass2.

\[
v_f = \frac{(0.05 \text{ kg})(300 \text{ m/s}) + (5 \text{ kg})(0 \text{ m/s})}{0.05 \text{ kg} + 5 \text{ kg}} = 2.97 \text{ m/s}
\]
Perfectly Elastic Collision

Momentum is conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Kinetic Energy is conserved:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Combining these two equations, if I gave you the initial velocities of the objects you could solve for the two final velocities simultaneously.
Elastic Collisions

Do a bunch of algebra and you will get:

\[ v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2v_{2i}}{m_1 + m_2} \]

and

\[ v_{2f} = \frac{2m_1v_{1i} + (m_2 - m_1)v_{2i}}{m_1 + m_2} \]
Some visual examples of elastic collisions:


https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html
Glancing Collisions

Remember that momentum is a vector. We can expand our work into two dimensions.

All you need to realize is that:

the momentum in the x-direction is conserved...

and

the momentum in the y-direction is conserved.

Look at momentum in the two dimensions separately!!!!!!

pages 186 and 187
Rocket Propulsion

Rockets work using the law of conservation of momentum.
Remember how the archer could propel himself across the lake by shooting an arrow?

Rockets work in a similar manner. When expelling gas in one direction, in order for momentum to be conserved, the rocket must move in the opposite direction.
Rockets

A rocket starts with an initial mass. This includes the fuel that will be used.

As the rocket burns the fuel, the total mass of the rocket is reduced.

Therefore the mass of the rocket that is being propelled changes with time.

If we want to show how to calculate the final speed of the rocket we need to know how much fuel is burnt at that time.

Can be solved by applying calculus to conservation of momentum.
Rocket

Skipping the derivation we get the equation:

\[ v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right) \]

\( v_e \) is the exhaust velocity of the gas relative to the rocket.

\( M_i \) and \( M_f \) are the initial and final masses of the rocket.
Momentum examples:

**Ballistic Pendulum**

A 0.05 kg bullet with velocity 150 m/s is shot into a 3 kg ballistic pendulum.

Find how high the pendulum rises after the bullet gets stuck inside.
Ballistic Pendulum

First use conservation of momentum to find the final velocity of the bullet/pendulum.

This is an inelastic collision.

\[ m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f \]

\[ (0.05\text{kg})(150\text{m/s}) + (3\text{kg})(0\text{m/s}) = (3.05\text{kg})v_f \]

\[ v_f = 2.5 \text{ m/s} \]
Ballistic Pendulum

Now that we know the ballistic pendulum with the bullet in it begins to swing with a speed of 2.5 m/s, we use conservation of energy to find how high it swings.

\[
\frac{1}{2} mv^2 = mg\Delta y
\]
\[
\frac{1}{2} m (2.5 \text{m/s})^2 = mg\Delta y
\]
\[
\Delta y = 0.32 \text{ m}
\]
Two disks with equal mass experience a glancing collision. Orange disk is traveling 5 m/s to the right, while the green disk is stationary.

After the collision, the orange disk has a velocity that is 37 degrees to the away from its initial velocity’s direction.

The green disk moves at an angle of 53 degrees away from the orange disks initial line of motion.

Determine the speeds of the two disks.
Figure P6.38

Before the collision

5.00 m/s

After the collision

$v_{ef}$

$37.0^\circ$

$v_{gf}$

$53.0^\circ$