Chapter 7
Rotating Objects
Circular Motion
and Gravitation

Rotational Motion

Why learn about rotational motion?

- Gears
- Tools
- Wheels
- Orbital motion
- Roller coasters

For rotational motion, we look at displacement using angles.

$\theta = s/r$, where s is the arc length and r is the radius.

θ is measured in radians or degrees.

360 degrees is 2π radians.

1 revolution = 2π radians

1 radian is about 57.3 degrees.

See figures on page 203

Rotational motion

Earlier we did linear motion:

position	x
linear displacement	$\Delta \mathbf{x}$
linear velocity	$\mathbf{v}$
linear acceleration	$\mathbf{a}$

Now we look at rotational motion

angular position	θ
angular displacement	$\Delta\theta$
angular velocity	ω
angular acceleration	α

Relations between angular and linear quantities.

$\Delta\theta = \Delta s/r$

$\omega = v/r$

Tangential speed v_t

$v_t = r\omega$ or $\omega = v_t/r$

Tangential acceleration

$a_t = r\alpha$ or $\alpha = a_t/r$

do quick quizzes 7.4, 7.5

Work example 7.4

The rotational quantities behave exactly like their linear counterparts.

Linear motion equations:

$$v = v_0 + at$$

$$\Delta x = v_0 t + ½\, at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Become:

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \omega_0 t + ½\, \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

Centripetal Acceleration

- $a_c = v^2/r$
- This is the acceleration you get when traveling in a circle.

- Substituting $\omega = v/r$
- $a_c = \omega^2 r$

- Total acceleration includes the tangential and the centripetal accelerations.

$$a = \sqrt{a_t^2 + a_c^2}$$

quick quizzes 7.6, 7.7

Centripetal acceleration is a reaction to an external force.

The force that causes an object to travel in a circular path.

examples: a string tied to a swinging ball

friction between the road and tires

gravity between a planet and a moon

$F_c = ma_c = mv^2/r$

a_c points towards the center of the circle for any curved path.

This is an application of Newton's 2nd Law.

Subbing in v^2/r for the 'a' in F = ma.

Work examples 7.6, 7.7,

Gravitation.

- Earlier we saw how to measure the force of gravity between 2 objects.

$$F = G\frac{m_1 m_2}{r^2}$$

This is the general form of the force of gravity between two objects.

Near the surface of the Earth, we can just use F = mg .

See table 7.1

Gravitational Potential Energy

Related to force of gravity and vertical displacement.

Earlier we used PE = mgh

Using the general form of the gravitational force, we can use:

$$PE = -G\frac{M_E m}{r}$$

The minus sign, indicates the potential is from an attractive force.

PE = mgh when the change in heights are relatively small compares to Earth's radius.

$$PE = -G\frac{M_E m}{r}$$ when the change in heights are not small compared to the Earth's radius.

This is because the gravitational force varies when the altitude varies considerably.

Remember that we can set the zero of potential energy line to be wherever we want.

Using the general form of gravitation, a convenient place is when two objects are infinitely separated.

Let’s graph the function of potential energy to explain then.

This is important in calculating an object’s escape velocity.

Also see figure 7.20

Escape velocity.

- You can figure out what velocity is needed to escape the gravitational pull of a planet.
- This is the initial velocity needed for an object to reach a distance of infinity away from the planet.
- Using conservation of energy:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_E m}{R_E} = 0$$

Escape velocity becomes: $v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$

Kepler’s Laws

Law that quantify orbital motion.

Work for all types of “satellites”.

Planets orbiting the sun.

Moons orbiting planets.

Manmade satellites orbiting planets.

Will use planets and the sun in the description.

Kepler’s Laws

1st Law: Planets orbit in elliptical orbits with the sun being located at one of the focal points.

2nd Law: Lines drawn from the sun to the planet sweeps outs equal areas over equal time intervals.

See figures on page 227

3rd Law: The square of the orbital period is proportional to the cube of the average distance from the planet to sun.

Kepler's 3rd Law

Gravity is what is forcing the circular motion of the orbiting object.

Results in a centripetal acceleration. (mv^2/r)

$$\frac{m_p v^2}{r} = G\frac{M_s m_p}{r^2}$$

Substituting $v = 2\pi r/T$ and then doing some algebra we get:

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right) r^3$$

Kepler's 3rd Law

Useful to calculate the mass of a planet or star.

If r and T are known, then GM can be found.

Since G is a universal constant, we can find M.

Important for determining satellite altitudes. See example 7.13

Sample problem

Tarzan swings on a vine.

Tarzan has mass of 85 kg.

Vine has a length of 10 m.

Speed at the bottom of the swing is 8.0 m/s

Vine can withstand a tension of 1000 N before it snaps.

Does the vine break?

Use $\Sigma F = ma$

a gets replaced by v^2/r

$T - mg = mv^2/r$

$T = mg + mv^2/r$

$T = (85kg)g + (85kg)\ (8m/s)^2/(10m)$

$T = 1377\ N$

vine breaks