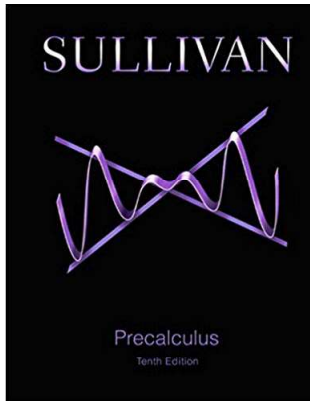


Precalculus 1 (Algebra)

Chapter 1. Graphs

1.2. Graphs of Equations in Two Variables; Intercepts; Symmetry—Exercises, Examples, Proofs



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Page 17 Number 18

Page 17 Number 18. Determine which of the points $(0, 1)$, $(2, 0)$, $(2, 1/2)$ are on the graph of the equation $x^2 + 4y^2 = 4$.

Solution. We test each of the pairs (x, y) to see if they make the equation a true statement.

For $(x, y) = (0, 1)$ the equation $x^2 + 4y^2 = 4$ becomes $(0)^2 + 4(1)^2 \stackrel{?}{=} 4$, which is a true statement and so

point $(0, 1)$ is on the graph of $x^2 + 4y^2 = 4$.

For $(x, y) = (2, 0)$ the equation $x^2 + 4y^2 = 4$ becomes $(2)^2 + 4(0)^2 \stackrel{?}{=} 4$, which is a true statement and so

point $(2, 0)$ is on the graph of $x^2 + 4y^2 = 4$.

For $(x, y) = (2, 1/2)$ the equation $x^2 + 4y^2 = 4$ becomes $(2)^2 + 4(1/2)^2 \stackrel{?}{=} 4$, which is a false statement since the left hand side is 5 and so point $(2, 1/2)$ is not on the graph of $x^2 + 4y^2 = 4$. \square

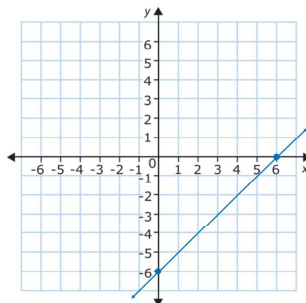
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Page 17 Number 20

Page 17 Number 20. Find the intercepts and graph the equation $y = x - 6$. HINT: The graph is a line.

Solution. To find the x -intercept, we set $y = 0$ and find: $0 = x - 6$ or $x = 6$. So the x -intercept is the point $(6, 0)$. To find the y -intercept, we set $x = 0$ and find: $y = (0) - 6$ or $y = -6$. So

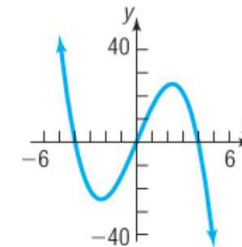
the y -intercept is the point $(0, -6)$. Since the graph is known to be a line, we can now graph it knowing only the intercepts:



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Page 17 Number 48

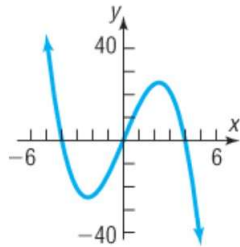
Page 17 Number 48. Find the intercepts and indicate whether the graph is symmetric with respect to the x -axis, the y -axis, or the origin:



Solution. To find the x -intercept(s), by definition we look for points at which the graph crosses or touches the x -axis. We see from the graph that this happens at the points $(-4, 0)$, $(0, 0)$, and $(4, 0)$, so the x -intercepts are $(-4, 0)$, $(0, 0)$, and $(4, 0)$.

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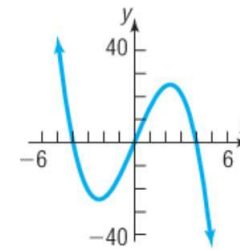
Page 17 Number 48 (continued 1)



Solution (continued). To find the y -intercept(s), by definition we look for points at which the graph crosses or touches the y -axis. We see from the graph that this happens at the point $(0, 0)$, so

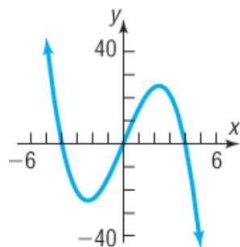
the y -intercept is $(0, 0)$.

Page 17 Number 48 (continued 2)



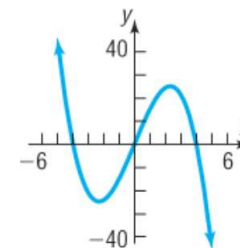
Solution (continued). A graph is, by definition, symmetric with respect to the x -axis if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. Geometrically, this means that if we rotate the graph about the x -axis then the result will lie on top of the original graph (the graph is a “mirror image” of itself about the x -axis). This does not happen here since, for example, the points in the first quadrant do not have corresponding points on the graph in the fourth quadrant. So the graph is not symmetric with respect to the x -axis.

Page 17 Number 48 (continued 3)



Solution (continued). A graph is, by definition, symmetric with respect to the y -axis if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. Geometrically, this means that if we rotate the graph about the y -axis then the result will lie on top of the original graph (the graph is a “mirror image” of itself about the y -axis). This does not happen here since, for example, the points in the first quadrant do not have corresponding points on the graph in the second quadrant. So the graph is not symmetric with respect to the y -axis.

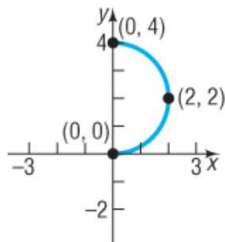
Page 17 Number 48 (continued 4)



Solution (continued). A graph is, by definition, symmetric with respect to the origin if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. Geometrically, this means that if we rotate the graph about the x -axis and then rotate it about the y -axis then the result will lie on top of the original graph. This *does* happen here since the points in the first quadrant have corresponding points on the graph in the third quadrant, and the points in the second quadrant have corresponding points on the graph in the fourth quadrant. So the graph is symmetric with respect to the origin. \square

Page 17 Number 56

Page 17 Number 56. Draw a complete graph so that it is symmetric with respect to the y -axis:

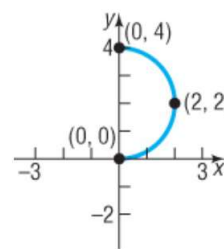


Solution. A graph is, by definition, symmetric with respect to the y -axis if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. Geometrically, this means that if we rotate the graph about the y -axis then the result will lie on top of the original graph (the graph is a “mirror image” of itself about the y -axis).

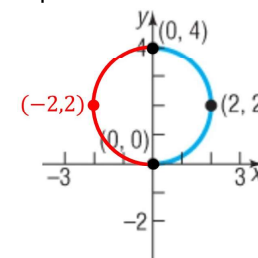
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Page 17 Number 56 (continued)

Solution (continued). In particular, since the point $(2, 2)$ is on the given graph then the point $(-2, 2)$ must be on the “complete” graph. More generally, each point on the semicircle in the first quadrant must have a corresponding point in the second quadrant and so the complete graph must have a semicircle in the second quadrant as follows:



Original Graph



Complete Graph

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Page 18 Number 72

Page 18 Number 72. List the intercepts and test for symmetry:

$$y = \frac{x^4 + 1}{2x^5}$$

Solution. First, notice that we cannot have $x = 0$, or else we would have division by 0. To find the x -intercept(s), if any, of the graph of an equation, we let $y = 0$ in the equation and solve for x . This gives $y = \frac{x^4 + 1}{2x^5} = 0$. The only way a quotient can equal 0 is if the numerator is 0 (and the denominator is not), so we need $x^4 + 1 = 0$ or $x^4 = -1$. But since any real number raised to an even power is nonnegative, then there is no such x and there are no x -intercepts.

To find the y -intercept(s), if any, of the graph of an equation, we let $x = 0$ in the equation and solve for y . But we cannot have $x = 0$ as mentioned above, so there are no y -intercepts.

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Page 18 Number 72 (continued 1)

Page 18 Number 72. List the intercepts and test for symmetry:

$$y = \frac{x^4 + 1}{2x^5}$$

Solution (continued). To test for symmetry with respect to the x -axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $-y = \frac{x^4 + 1}{2x^5}$ or $y = -\frac{x^4 + 1}{2x^5}$. This new equation is *not* equivalent to the original equation because, for example, with $x = 1$ in the original equation we get $y = 1$, but in this new equation with $x = 1$ we get $y = -1$. So the graph is not symmetric with respect to the x -axis.

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Page 18 Number 72 (continued 2)

Page 18 Number 72. List the intercepts and test for symmetry:

$$y = \frac{x^4 + 1}{2x^5}.$$

Solution (continued). To test for symmetry with respect to the y -axis, we replace x by $-x$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ gives the new equation $y = \frac{(-x)^4 + 1}{2(-x)^5}$ or $y = \frac{x^4 + 1}{-2x^5}$. This new equation is *not* equivalent to the original equation because, for example, with $x = 1$ in the original equation we get $y = 1$, but in this new equation with $x = 1$ we get $y = -1$. So the graph is not symmetric with respect to the y -axis.

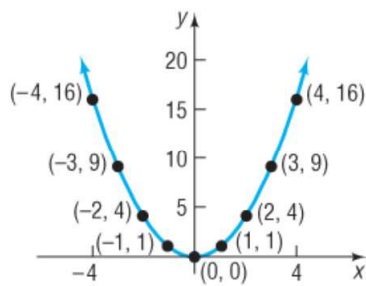
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Page 11 Example 3

Page 11 Example 3. Plot points for x an integer between -4 and 4 for the equation $y = x^2$ and guess the graph of $y = x^2$.

Solution. We make a table of x and y values and graph:

x	$y = x^2$	x	$y = x^2$
-4	$(-4)^2 = 16$	4	$4^2 = 16$
-3	$(-3)^2 = 9$	3	$3^2 = 9$
-2	$(-2)^2 = 4$	2	$2^2 = 4$
-1	$(-1)^2 = 1$	1	$1^2 = 1$
0	$(0)^2 = 0$		



□

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Page 18 Number 72 (continued 3)

Page 18 Number 72. List the intercepts and test for symmetry:

$$y = \frac{x^4 + 1}{2x^5}.$$

Solution (continued). To test for symmetry with respect to the origin, we both replace x by $-x$ and replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ and replacing y with $-y$ gives the new equation $-y = \frac{(-x)^4 + 1}{2(-x)^5}$ or $-y = \frac{x^4 + 1}{-2x^5}$ or $y = \frac{x^4 + 1}{2x^5}$. This new equation is equivalent to the original equation, so the graph is symmetric with respect to the origin. □

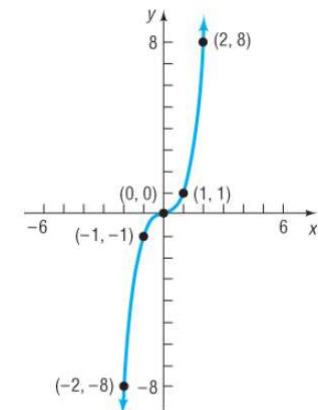
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Page 15 Example 10

Page 15 Example 10. Plot points for x an integer between -2 and 2 for the equation $y = x^3$ and guess the graph of $y = x^3$.

Solution. We make a table of x and y values and graph:

x	$y = x^3$	x	$y = x^3$
-2	$(-2)^3 = -8$	2	$2^3 = 8$
-1	$(-1)^3 = -1$	1	$1^3 = 1$
0	$(0)^3 = 0$		



□

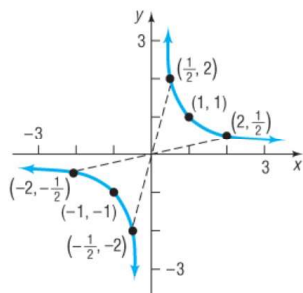
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Page 16 Example 12

Page 16 Example 12. Plot points for x taking on the values -2 , -1 , $-1/2$, $1/2$, 1 , and 2 for the equation $y = 1/x$ and guess the graph of $y = 1/x$.

Solution. We make a table of x and y values and graph:

x	$y = 1/x$	x	$y = 1/x$
-2	$1/(-2) = -1/2$	2	$1/(2) = 1/2$
-1	$1/(-1) = -1$	1	$1/(1) = 1$
$-1/2$	$1/(-1/2) = -2$	$1/2$	$1/(1/2) = 2$



□

Page 18 Example 84

Page 18 Example 84. The solar electric generating systems at Kramer Junction, California, use parabolic troughs to heat a heat-transfer fluid to a high temperature. This fluid is used to generate steam that drives a power conversion system to produce electricity. For troughs 7.5 feet wide, an equation for the cross section is $16y^2 = 120x - 225$. Find the intercepts of the graph of this equation. Test the equation for symmetry with respect to the x -axis, the y -axis, and the origin.

Solution. To find the x -intercept(s), if any, of the graph of an equation, we let $y = 0$ in the equation and solve for x . This gives $16(0)^2 = 120x - 225$ or $0 = 120x - 225$ or $x = 225/120 = 15/8$. So the x -intercept is $(15/8, 0)$.

To find the y -intercept(s), if any, of the graph of an equation, we let $x = 0$ in the equation and solve for y . This gives $16y^2 = 120(0) - 225$ or $16y^2 = -225$ or $y^2 = -225/16$. But every real number squared is nonnegative, so there is no such y and there is no y -intercept.

Page 18 Example 84 (continued 1)

Solution. To test for symmetry with respect to the x -axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $16(-y)^2 = 120x - 225$ or $16y^2 = 120x - 225$. This new equation is equivalent to the original equation, so the graph is symmetric with respect to the x -axis.

To test for symmetry with respect to the y -axis, we replace x by $-x$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ gives the new equation $16y^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. This new equation is not equivalent to the original equation since, for example, with $x = 2$ in the original equation we get $16y^2 = 120(2) - 225 = 15$ or $y^2 = 15/16$, but with $x = 2$ in the new equation we get $16y^2 = -120(2) - 225 = -465$ or $y^2 = -465/16$ which has no solution since any real number squared is nonnegative. So the graph is not symmetric with respect to the y -axis.

Page 18 Example 84 (continued 2)

Solution. To test for symmetry with respect to the origin, we both replace x with $-x$ and replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ and replacing y with $-y$ gives the new equation $16(-y)^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. As just shown, this new equation is not equivalent to the original equation (consider $x = 2$ in each equation, as above). So the graph is not symmetric with respect to the origin.

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