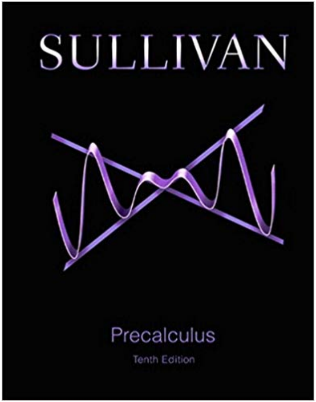
Precalculus 1 (Algebra)

Chapter 1. Graphs

1.2. Graphs of Equations in Two Variables; Intercepts; Symmetry—Exercises, Examples, Proofs



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For $(x,y) = (0,1)$ the equation $x^2 + 4y^2 = 4$ becomes $(0)^2 + 4(1)^2 \stackrel{?}{=} 4$, which is a true statement and so $\boxed{\text{point } (0,1) \text{ is on the graph of } x^2 + 4y^2 = 4}$.

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For $(x,y)=(2,1/2)$ the equation $x^2+4y^2=4$ becomes $(2)^2+4(1/2)^2 \stackrel{?}{=} 4$, which is a false statement since the left hand side is 5 and so $\boxed{\text{point } (2,1/2) \text{ is not on the graph of } x^2+4y^2=4}$. □

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Solution. To find the x-intercept, we set $y = 0$ and find: $0 = x - 6$ or $x = 6$. So $\boxed{\text{the } x\text{-intercept is the point } (6, 0)}$. To find the y-intercept, we set $x = 0$ and find: $y = (0) - 6$ or $y = -6$. So $\boxed{\text{the } y\text{-intercept is the point } (0, -6)}$. Since the graph is known to be a line, we can now graph it knowing only the intercepts:

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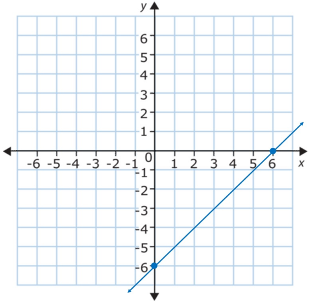

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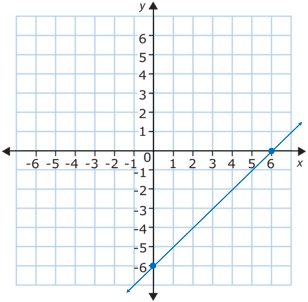

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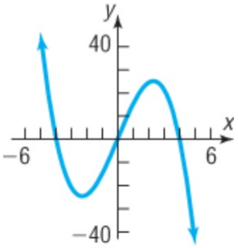

Solution. To find the x-intercept(s), by definition we look for points at which the graph crosses or touches the x-axis. We see from the graph that this happens at the points $(-4, 0)$, $(0, 0)$, and $(4, 0)$, so

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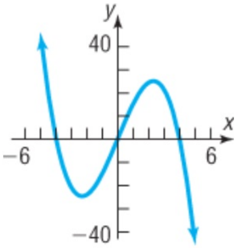

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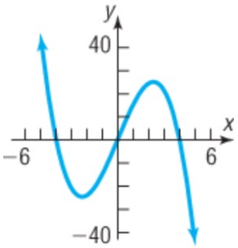

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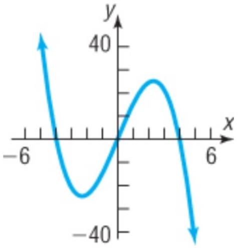
Page 17 Number 48 (continued 2)



Solution (continued). A graph is, by definition, symmetric with respect to the x-axis if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. Geometrically, this means that the if we rotate the graph about the x-axis then the result will lie on top of the original graph (the graph is a "mirror image" of itself about the x-axis). This does not happen here since, for example, the points in the first quadrant do not have corresponding points on the graph in the fourth quadrant. So the graph is $\boxed{\text{not symmetric with respect to the } x\text{-axis}}$.

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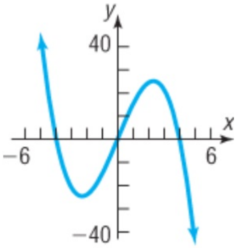

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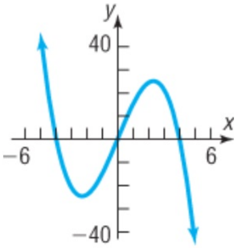

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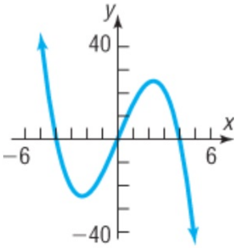

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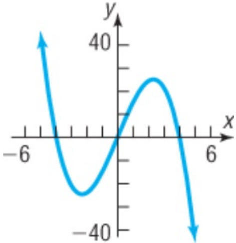

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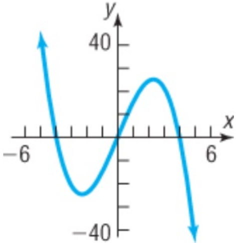

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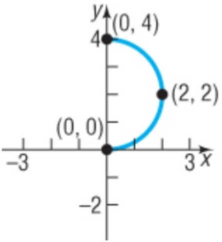

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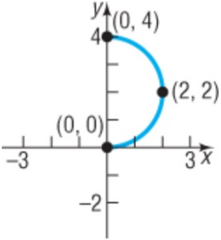

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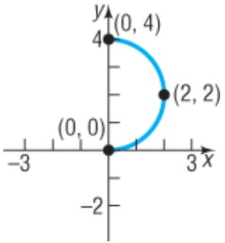
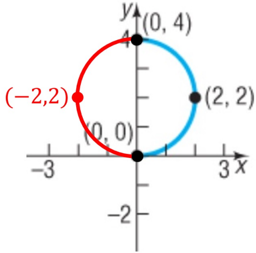
Page 17 Number 56 (continued)

Solution (continued). In particular, since the point $(2, 2)$ is on the given graph then the point $(-2, 2)$ must be on the "complete" graph. More generally, each point on the semicircle in the first quadrant must have a corresponding point in the second quadrant and so the complete graph must have a semicircle in the second quadrant as follows:



Original Graph



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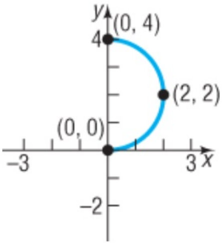

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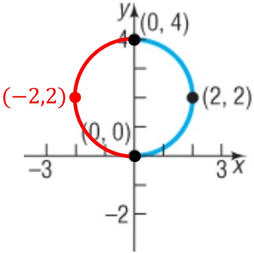

Complete Graph

Page 18 Number 72

Page 18 Number 72. List the intercepts and test for symmetry:

$$y = \frac{x^4 + 1}{2x^5}.$$

Solution. First, notice that we cannot have $x = 0$, or else we would have division by 0. To find the x-intercept(s), if any, of the graph of an equation, we let $y = 0$ in the equation and solve for x. This gives $y = \dfrac{x^4 + 1}{2x^5} = 0$. The only way a quotient can equal 0 is if the numerator is 0 (and the denominator is not), so we need $x^4 + 1 = 0$ or $x^4 = -1$. But since any real number raised to an even power is nonnegative, then there is no such x and $\boxed{\text{there are no } x\text{-intercepts}}$.

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Solution (continued). To test for symmetry with respect to the x-axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $-y = \dfrac{x^4 + 1}{2x^5}$ or $y = -\dfrac{x^4 + 1}{2x^5}$. This new equation is *not* equivalent to the original equation because, for example, with $x = 1$ in the original equation we get $y = 1$, but in this new equation with $x = 1$ we get $y = -1$. So $\boxed{\text{the graph is not symmetric with respect to the } x\text{-axis}}$.

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Page 11 Example 3

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Solution. We make a table of x and y values and graph:

x	$y = x^2$	x	$y = x^2$
-4	$(-4)^2 = 16$	4	$4^2 = 16$
-3	$(-3)^2 = 9$	3	$3^2 = 9$
-2	$(-2)^2 = 4$	2	$2^2 = 4$
-1	$(-1)^2 = 1$	1	$1^2 = 1$
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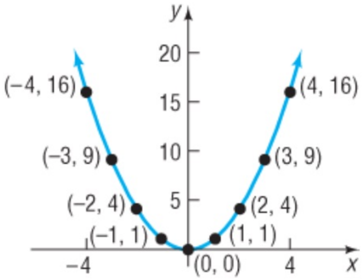

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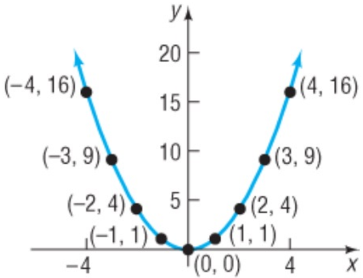

Page 15 Example 10

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Solution. We make a table of x and y values and graph:

x	$y = x^3$	x	$y = x^3$
-2	$(-2)^3 = -8$	2	$2^3 = 8$
-1	$(-1)^3 = -1$	1	$1^3 = 1$
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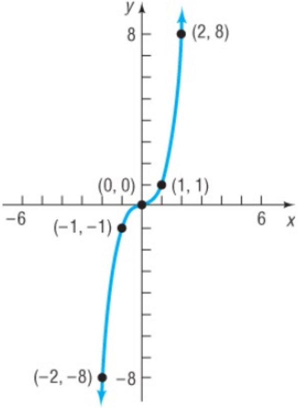

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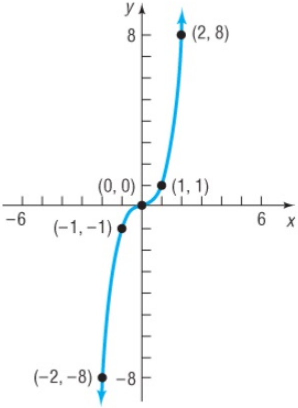

Page 16 Example 12

Page 16 Example 12. Plot points for x taking on the values -2, -1, $-1/2$, $1/2$, 1, and 2 for the equation $y = 1/x$ and guess the graph of $y = 1/x$.

Solution. We make a table of x and y values and graph:

x	$y = 1/x$	x	$y = 1/x$
-2	$1/(-2) = -1/2$	2	$1/(2) = 1/2$
-1	$1/(-1) = -1$	1	$1/(1) = 1$
$-1/2$	$1/(-1/2) = -2$	$1/2$	$1/(1/2) = 2$

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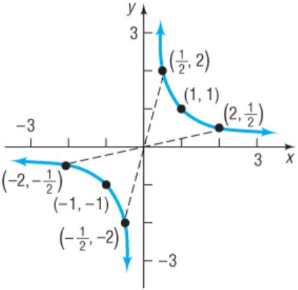

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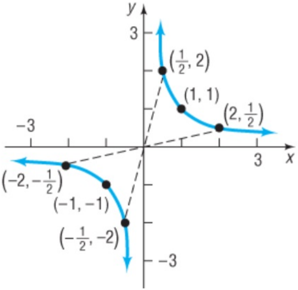

Page 18 Example 84

Page 18 Example 84. The solar electric generating systems at Kramer Junction, California, use parabolic troughs to heat a heat-transfer fluid to a high temperature. This fluid is used to generate steam that drives a power conversion system to produce electricity. For troughs 7.5 feet wide, an equation for the cross section is $16y^2 = 120x - 225$. Find the intercepts of the graph of this equation. Test the equation for symmetry with respect to the x-axis, the y-axis, and the origin.

Solution. To find the x-intercept(s), if any, of the graph of an equation, we let $y = 0$ in the equation and solve for x. This gives $16(0)^2 = 120x - 225$ or $0 = 120x - 225$ or $x = 225/120 = 15/8$. So $\boxed{\text{the } x\text{-intercept is } (15/8, 0)}$.

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Solution. To find the x-intercept(s), if any, of the graph of an equation, we let $y = 0$ in the equation and solve for x. This gives $16(0)^2 = 120x - 225$ or $0 = 120x - 225$ or $x = 225/120 = 15/8$. So $\boxed{\text{the } x\text{-intercept is } (15/8, 0)}$.

To find the y-intercept(s), if any, of the graph of an equation, we let $x = 0$ in the equation and solve for y. This gives $16y^2 = 120(0) - 225$ or $16y^2 = -225$ or $y^2 = -225/16$. But every real number squared is nonnegative, so there is no such y and $\boxed{\text{there is no } y\text{-intercept}}$.

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To find the y-intercept(s), if any, of the graph of an equation, we let $x = 0$ in the equation and solve for y. This gives $16y^2 = 120(0) - 225$ or $16y^2 = -225$ or $y^2 = -225/16$. But every real number squared is nonnegative, so there is no such y and $\boxed{\text{there is no } y\text{-intercept}}$.

Page 18 Example 84 (continued 1)

Solution. To test for symmetry with respect to the x-axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $16(-y)^2 = 120x - 225$ or $16y^2 = 120x - 225$. This new equation is equivalent to the original equation, so $\boxed{\text{the graph is symmetric with respect to the } x\text{-axis}}$.

Page 18 Example 84 (continued 1)

Solution. To test for symmetry with respect to the x-axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $16(-y)^2 = 120x - 225$ or $16y^2 = 120x - 225$. This new equation is equivalent to the original equation, so $\boxed{\text{the graph is symmetric with respect to the } x\text{-axis}}$.

To test for symmetry with respect to the y-axis, we replace x by $-x$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ gives the new equation $16y^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$.

Page 18 Example 84 (continued 1)

Solution. To test for symmetry with respect to the x-axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $16(-y)^2 = 120x - 225$ or $16y^2 = 120x - 225$. This new equation is equivalent to the original equation, so $\boxed{\text{the graph is symmetric with respect to the } x\text{-axis}}$.

To test for symmetry with respect to the y-axis, we replace x by $-x$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ gives the new equation $16y^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. This new equation is not equivalent to the original equation since, for example, with $x = 2$ in the original equation we get $16y^2 = 120(2) - 225 = 15$ or $y^2 = 15/16$, but with $x = 2$ in the new equation we get $16y^2 = -120(2) - 225 = -465$ or $y^2 = -465/16$ which has no solution since any real number squared is nonnegative. So $\boxed{\text{the graph is not symmetric with respect to the } y\text{-axis}}$.

Page 18 Example 84 (continued 1)

Solution. To test for symmetry with respect to the x-axis, we replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing y with $-y$ gives the new equation $16(-y)^2 = 120x - 225$ or $16y^2 = 120x - 225$. This new equation is equivalent to the original equation, so $\boxed{\text{the graph is symmetric with respect to the } x\text{-axis}}$.

To test for symmetry with respect to the y-axis, we replace x by $-x$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ gives the new equation $16y^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. This new equation is not equivalent to the original equation since, for example, with $x = 2$ in the original equation we get $16y^2 = 120(2) - 225 = 15$ or $y^2 = 15/16$, but with $x = 2$ in the new equation we get $16y^2 = -120(2) - 225 = -465$ or $y^2 = -465/16$ which has no solution since any real number squared is nonnegative. So $\boxed{\text{the graph is not symmetric with respect to the } y\text{-axis}}$.

Page 18 Example 84 (continued 2)

Solution. To test for symmetry with respect to the origin, we both replace x with $-x$ and replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ and replacing y with $-y$ gives the new equation $16(-y)^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. As just shown, this new equation is not equivalent to the original equation (consider $x = 2$ in each equation, as above). So

$\boxed{\text{the graph is not symmetric with respect to the origin}}$. □

Page 18 Example 84 (continued 2)

Solution. To test for symmetry with respect to the origin, we both replace x with $-x$ and replace y by $-y$ in the equation and see if we get an equivalent equation. Replacing x with $-x$ and replacing y with $-y$ gives the new equation $16(-y)^2 = 120(-x) - 225$ or $16y^2 = -120x - 225$. As just shown, this new equation is not equivalent to the original equation (consider $x = 2$ in each equation, as above). So $\boxed{\text{the graph is not symmetric with respect to the origin}}$. □