Precalculus 1 (Algebra)

Chapter 1. Graphs

1.3. Lines—Exercises, Examples, Proofs

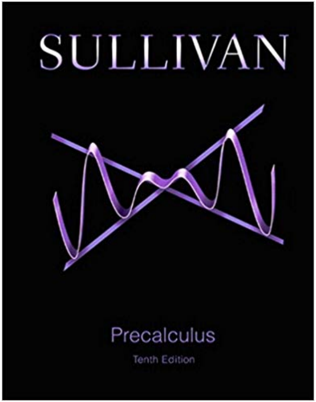
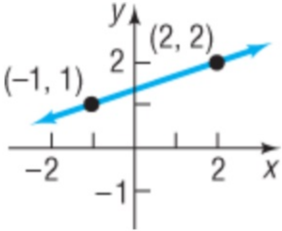


Table of contents

Page 30 Number 16

Page 30 Number 16. Find the slope of the line and interpret the slope.

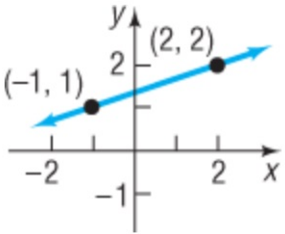


Solution. Two points on the line are $P = (x_1, y_1) = (-1, 1)$ and $Q = (x_2, y_2) = (2, 2)$. So by the definition of slope, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (1)}{(2) - (-1)} = \boxed{\frac{1}{3} = m}.$$

Page 30 Number 16

Page 30 Number 16. Find the slope of the line and interpret the slope.



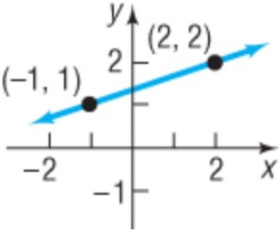
Solution. Two points on the line are $P = (x_1, y_1) = (-1, 1)$ and $Q = (x_2, y_2) = (2, 2)$. So by the definition of slope, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (1)}{(2) - (-1)} = \boxed{\frac{1}{3} = m}.$$

Since the slope is positive, then the line is going "uphill" as viewed from left to right. For each 1 unit that the line "rises," it "runs" to the right by an amount of 3 units.

Page 30 Number 16

Page 30 Number 16. Find the slope of the line and interpret the slope.



Solution. Two points on the line are $P = (x_1, y_1) = (-1, 1)$ and $Q = (x_2, y_2) = (2, 2)$. So by the definition of slope, we have

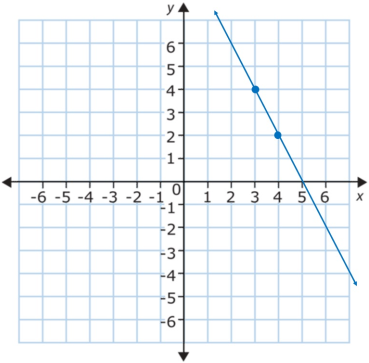
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (1)}{(2) - (-1)} = \boxed{\frac{1}{3} = m}.$$

Since the slope is positive, then the line is going "uphill" as viewed from left to right. For each 1 unit that the line "rises," it "runs" to the right by an amount of 3 units. □

Page 30 Number 18

Page 30 Number 18. Plot the pair of points $(4, 2)$ and $(3, 4)$ and determine the slope of the line containing them. Graph the line.

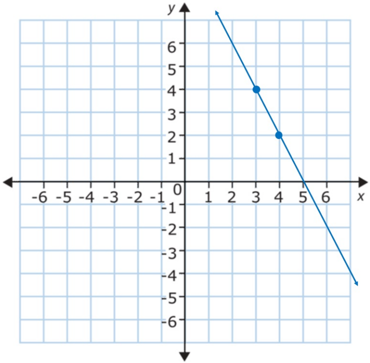
Solution. Two points determine a line, so graphing the two points allows us to graph the line as:



Page 30 Number 18

Page 30 Number 18. Plot the pair of points $(4, 2)$ and $(3, 4)$ and determine the slope of the line containing them. Graph the line.

Solution. Two points determine a line, so graphing the two points allows us to graph the line as:



Page 30 Number 18 (continued)

Page 30 Number 18. Plot the pair of points $(4, 2)$ and $(3, 4)$ and determine the slope of the line containing them. Graph the line.

Solution (continued). Two points on the line are $P = (x_1, y_1) = (4, 2)$ and $Q = (x_2, y_2) = (3, 4)$. So by the definition of slope, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4) - (2)}{(3) - (4)} = \frac{2}{-1} = \boxed{-2 = m}.$$

□

Page 30 Number 36

Page 30 Number 36. The slope of a line is $m = 4/3$ and a point on the line is $(-3, 2)$. Use this information to locate three additional points on the line.

Solution. We use the fact that the slope of a line is $m = \Delta y/\Delta x$. Since $m = 4/3 = \Delta y/\Delta x$ then we can take, say,

Page 30 Number 36

Page 30 Number 36. The slope of a line is $m = 4/3$ and a point on the line is $(-3, 2)$. Use this information to locate three additional points on the line.

Solution. We use the fact that the slope of a line is $m = \Delta y/\Delta x$. Since $m = 4/3 = \Delta y/\Delta x$ then we can take, say,

(1) $\Delta x = 3$ and $\Delta y = 4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 3, 2 + 4) = (0, 6)$,

Page 30 Number 36

Page 30 Number 36. The slope of a line is $m = 4/3$ and a point on the line is $(-3, 2)$. Use this information to locate three additional points on the line.

Solution. We use the fact that the slope of a line is $m = \Delta y/\Delta x$. Since $m = 4/3 = \Delta y/\Delta x$ then we can take, say,
(1) $\Delta x = 3$ and $\Delta y = 4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 3, 2 + 4) = (0, 6)$,
(2) $\Delta x = -3$ and $\Delta y = -4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get
$(x + \Delta x, y + \Delta y) = (-3 + (-3), 2 + (-4)) = (-6, -2)$.

Page 30 Number 36

Page 30 Number 36. The slope of a line is $m = 4/3$ and a point on the line is $(-3, 2)$. Use this information to locate three additional points on the line.

Solution. We use the fact that the slope of a line is $m = \Delta y/\Delta x$. Since $m = 4/3 = \Delta y/\Delta x$ then we can take, say,
(1) $\Delta x = 3$ and $\Delta y = 4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 3, 2 + 4) = (0, 6)$,
(2) $\Delta x = -3$ and $\Delta y = -4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get
$(x + \Delta x, y + \Delta y) = (-3 + (-3), 2 + (-4)) = (-6, -2)$.
(3) $\Delta x = 6$ and $\Delta y = 8$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 6, 2 + 8) = (3, 10)$. □

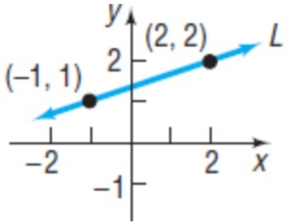
Page 30 Number 36

Page 30 Number 36. The slope of a line is $m = 4/3$ and a point on the line is $(-3, 2)$. Use this information to locate three additional points on the line.

Solution. We use the fact that the slope of a line is $m = \Delta y/\Delta x$. Since $m = 4/3 = \Delta y/\Delta x$ then we can take, say,
(1) $\Delta x = 3$ and $\Delta y = 4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 3, 2 + 4) = (0, 6)$,
(2) $\Delta x = -3$ and $\Delta y = -4$ to get a new point by starting at $(x, y) = (-3, 2)$ to get
$(x + \Delta x, y + \Delta y) = (-3 + (-3), 2 + (-4)) = (-6, -2)$.
(3) $\Delta x = 6$ and $\Delta y = 8$ to get a new point by starting at $(x, y) = (-3, 2)$ to get $(x + \Delta x, y + \Delta y) = (-3 + 6, 2 + 8) = (3, 10)$. □

Page 30 Number 42

Page 30 Number 42. Find an equation of the line L. (This is the same line as in Page 30 Number 16.)

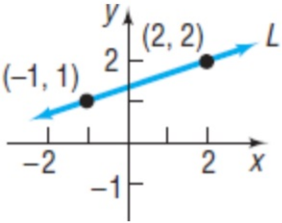


Solution. Two points on the line are $P = (x_1, y_1) = (-1, 1)$ and $Q = (x_2, y_2) = (2, 2)$. So by the definition of slope, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (1)}{(2) - (-1)} = \boxed{\frac{1}{3} = m}.$$

Page 30 Number 42

Page 30 Number 42. Find an equation of the line L. (This is the same line as in Page 30 Number 16.)



Solution. Two points on the line are $P = (x_1, y_1) = (-1, 1)$ and $Q = (x_2, y_2) = (2, 2)$. So by the definition of slope, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (1)}{(2) - (-1)} = \boxed{\frac{1}{3} = m}.$$

Page 30 Number 42 (continued)

Solution (continued). Using point $P = (x_1, y_1) = (-1, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{3}(x - (-1))$ or $\boxed{y - 1 = \frac{1}{3}(x + 1)}$. If we use the point $Q = (2, 2)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (2) = \frac{1}{3}(x - (2))$ or $y - 2 = \frac{1}{3}(x - 2)$. Notice that if we add 1 to both sides of this equation we get $(y - 2) + 1 = \frac{1}{3}(x - 2) + 1$ or

$$y - 1 = \frac{1}{3}x - \frac{2}{3} + 1 = \frac{1}{3}x + \frac{1}{3} = \frac{1}{3}(x + 1).$$

So we get the same equation (as we should) regardless of which point we use. □

Page 30 Number 42 (continued)

Solution (continued). Using point $P = (x_1, y_1) = (-1, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{3}(x - (-1))$ or $\boxed{y - 1 = \frac{1}{3}(x + 1)}$. If we use the point $Q = (2, 2)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (2) = \frac{1}{3}(x - (2))$ or $y - 2 = \frac{1}{3}(x - 2)$. Notice that if we add 1 to both sides of this equation we get $(y - 2) + 1 = \frac{1}{3}(x - 2) + 1$ or

$$y - 1 = \frac{1}{3}x - \frac{2}{3} + 1 = \frac{1}{3}x + \frac{1}{3} = \frac{1}{3}(x + 1).$$

So we get the same equation (as we should) regardless of which point we use. □

Page 31 Number 50

Page 31 Number 50. Find an equation for the line with slope $m = 1/2$ containing the point $(3, 1)$.

Solution. Using point $P = (x_1, y_1) = (3, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{2}(x - (3))$ or $y - 1 = \frac{1}{2}x - \frac{3}{2}$.

Page 31 Number 50

Page 31 Number 50. Find an equation for the line with slope $m = 1/2$ containing the point $(3, 1)$.

Solution. Using point $P = (x_1, y_1) = (3, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{2}(x - (3))$ or $y - 1 = \frac{1}{2}x - \frac{3}{2}$. We can write this in other ways. For example, we have $y = \frac{1}{2}x - \frac{3}{2} + 1 = \frac{1}{2}x - \frac{1}{2}$. We can multiply both sides of this by 2 to get $2y = x - 1$ or $2y - x = -1$ or $x - 2y = 1$.

Page 31 Number 50

Page 31 Number 50. Find an equation for the line with slope $m = 1/2$ containing the point $(3, 1)$.

Solution. Using point $P = (x_1, y_1) = (3, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{2}(x - (3))$ or $y - 1 = \frac{1}{2}x - \frac{3}{2}$. We can write this in other ways. For example, we have $y = \frac{1}{2}x - \frac{3}{2} + 1 = \frac{1}{2}x - \frac{1}{2}$. We can multiply both sides of this by 2 to get $2y = x - 1$ or $2y - x = -1$ or $x - 2y = 1$. So an equation of the line is

$$\boxed{y - 1 = \frac{1}{2}x - \frac{3}{2} \text{ or } y = \frac{1}{2}x - \frac{1}{2} \text{ or } x - 2y = 1.}$$

□

Page 31 Number 50

Page 31 Number 50. Find an equation for the line with slope $m = 1/2$ containing the point $(3, 1)$.

Solution. Using point $P = (x_1, y_1) = (3, 1)$, we have from the point-slope form that $y - y_1 = m(x - x_1)$ or $y - (1) = \frac{1}{2}(x - (3))$ or $y - 1 = \frac{1}{2}x - \frac{3}{2}$. We can write this in other ways. For example, we have $y = \frac{1}{2}x - \frac{3}{2} + 1 = \frac{1}{2}x - \frac{1}{2}$. We can multiply both sides of this by 2 to get $2y = x - 1$ or $2y - x = -1$ or $x - 2y = 1$. So an equation of the line is

$$\boxed{y - 1 = \frac{1}{2}x - \frac{3}{2} \text{ or } y = \frac{1}{2}x - \frac{1}{2} \text{ or } x - 2y = 1.}$$ □

Page 31 Number 54

Page 31 Number 54. Find an equation for the line with slope $m = -2$ with y-intercept $(0, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. We have slope $m = -2$ and y-intercept $(0, b) = (0, -2)$ so that $b = -2$. By the slope-intercept form of an equation of a line we have $y = mx + b$ or $y = -2x + (-2)$. So $\boxed{\text{the slope-intercept form is } y = -2x - 2}$.

Page 31 Number 54

Page 31 Number 54. Find an equation for the line with slope $m = -2$ with y-intercept $(0, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. We have slope $m = -2$ and y-intercept $(0, b) = (0, -2)$ so that $b = -2$. By the slope-intercept form of an equation of a line we have $y = mx + b$ or $y = -2x + (-2)$. So $\boxed{\text{the slope-intercept form is } y = -2x - 2}$. Rearranging we have $\boxed{\text{a general form of } 2x + y = -2}$. □

Page 31 Number 54

Page 31 Number 54. Find an equation for the line with slope $m = -2$ with y-intercept $(0, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. We have slope $m = -2$ and y-intercept $(0, b) = (0, -2)$ so that $b = -2$. By the slope-intercept form of an equation of a line we have $y = mx + b$ or $y = -2x + (-2)$. So $\boxed{\text{the slope-intercept form is } y = -2x - 2}$. Rearranging we have $\boxed{\text{a general form of } 2x + y = -2}$. □

Page 31 Number 74

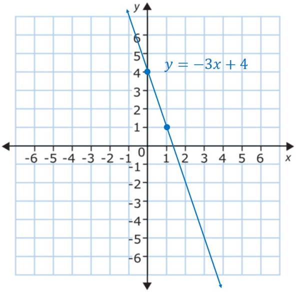
Page 31 Number 74. Find the slope, y-intercept, and graph the line $y = -3x + 4$.

Solution. Since the line is already in slope intercept form $y = mx + b$ then we have $\boxed{\text{slope } m = -3 \text{ and } y\text{-intercept } (0, 4)}$.

Page 31 Number 74

Page 31 Number 74. Find the slope, y-intercept, and graph the line $y = -3x + 4$.

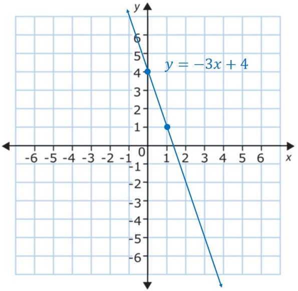
Solution. Since the line is already in slope intercept form $y = mx + b$ then we have $\boxed{\text{slope } m = -3 \text{ and } y\text{-intercept } (0, 4)}$. Since the slope is $m = -3 = \Delta y / \Delta x$, then we can take $\Delta x = 1$ and $\Delta y = -3$ to get a second point on the line of $(x + \Delta x, y + \Delta y) = (0 + 1, 4 + (-3)) = (1, 1)$. So the graph of the line is:



Page 31 Number 74

Page 31 Number 74. Find the slope, y-intercept, and graph the line $y = -3x + 4$.

Solution. Since the line is already in slope intercept form $y = mx + b$ then we have $\boxed{\text{slope } m = -3 \text{ and } y\text{-intercept } (0, 4)}$. Since the slope is $m = -3 = \Delta y / \Delta x$, then we can take $\Delta x = 1$ and $\Delta y = -3$ to get a second point on the line of $(x + \Delta x, y + \Delta y) = (0 + 1, 4 + (-3)) = (1, 1)$. So the graph of the line is:



□

Page 31 Number 94

Page 31 Number 94. Find the intercepts of the graph of $3x - 2y = 6$ and graph it.

Solution. This is a general form of the equation of a line $Ax + By = C$ where $A = 3$, $B = -2$, and $C = 6$. So $\boxed{\text{the } x\text{-intercept is } (C/A, 0) = (6/3, 0) = (2, 0)}$ and $\boxed{\text{the } y\text{-intercept is } (0, C/B) = (0, 6/(-2)) = (0, -3)}$. Notice that we can also find the intercepts by setting y and then x equal to 0.

Page 31 Number 94

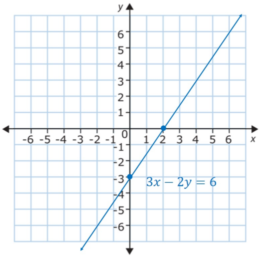
Page 31 Number 94. Find the intercepts of the graph of $3x - 2y = 6$ and graph it.

Solution. This is a general form of the equation of a line $Ax + By = C$ where $A = 3$, $B = -2$, and $C = 6$. So $\boxed{\text{the } x\text{-intercept is } (C/A, 0) = (6/3, 0) = (2, 0)}$ and $\boxed{\text{the } y\text{-intercept is } (0, C/B) = (0, 6/(-2)) = (0, -3)}$. Notice that we can also find the intercepts by setting y and then x equal to 0. We graph the intercepts and find the graph is:

Page 31 Number 94

Page 31 Number 94. Find the intercepts of the graph of $3x - 2y = 6$ and graph it.

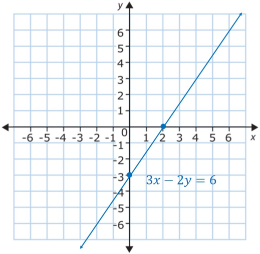
Solution. This is a general form of the equation of a line $Ax + By = C$ where $A = 3$, $B = -2$, and $C = 6$. So $\boxed{\text{the } x\text{-intercept is } (C/A, 0) = (6/3, 0) = (2, 0)}$ and $\boxed{\text{the } y\text{-intercept is } (0, C/B) = (0, 6/(-2)) = (0, -3)}$. Notice that we can also find the intercepts by setting y and then x equal to 0. We graph the intercepts and find the graph is:



Page 31 Number 94

Page 31 Number 94. Find the intercepts of the graph of $3x - 2y = 6$ and graph it.

Solution. This is a general form of the equation of a line $Ax + By = C$ where $A = 3$, $B = -2$, and $C = 6$. So $\boxed{\text{the } x\text{-intercept is } (C/A, 0) = (6/3, 0) = (2, 0)}$ and $\boxed{\text{the } y\text{-intercept is } (0, C/B) = (0, 6/(-2)) = (0, -3)}$. Notice that we can also find the intercepts by setting y and then x equal to 0. We graph the intercepts and find the graph is:



$\square$

Page 31 Number 64

Page 31 Number 64. Find an equation for the line parallel to the line $x - 2y = -5$ which contains the point $(0, 0)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. First, we can write $x - 2y = -5$ in the slope-intercept form as $y = mx + b = \frac{1}{2}x + \frac{5}{2}$, so the slope of line $x - 2y = -5$ is $m = 1/2$. By Theorem 1.3.E, "Criterion for Parallel Lines," we know that the desired line must also have slope $m = 1/2$. Since the desired line contains the point $(x_1, y_1) = (0, 0)$, then from the point-slope form of the equation of a line we need $y - y_1 = m(x - x_1)$ or $y - (0) = (1/2)(x - (0))$. So the desired line $\boxed{\text{in slope intercept form is } y = (1/2)x}$.

$\boxed{\text{A general form is } (1/2)x - y = 0 \text{ or } x - 2y = 0}$. □

Page 31 Number 64

Page 31 Number 64. Find an equation for the line parallel to the line $x - 2y = -5$ which contains the point $(0, 0)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. First, we can write $x - 2y = -5$ in the slope-intercept form as $y = mx + b = \dfrac{1}{2}x + \dfrac{5}{2}$, so the slope of line $x - 2y = -5$ is $m = 1/2$. By Theorem 1.3.E, "Criterion for Parallel Lines," we know that the desired line must also have slope $m = 1/2$. Since the desired line contains the point $(x_1, y_1) = (0, 0)$, then from the point-slope form of the equation of a line we need $y - y_1 = m(x - x_1)$ or $y - (0) = (1/2)(x - (0))$. So the desired line $\boxed{\text{in slope intercept form is } y = (1/2)x}$.

$\boxed{\text{A general form is } (1/2)x - y = 0 \text{ or } x - 2y = 0}$. □

Theorem 1.3.F

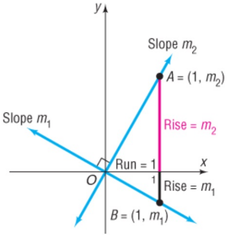
Theorem 1.3.F. Criterion for Perpendicular Lines. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Proof. Let m_1 and m_2 be the slopes of the two lines. We can rigidly translate the lines so that their point of intersection is at the origin (this simplifies the equations of the lines).

Theorem 1.3.F

Theorem 1.3.F. Criterion for Perpendicular Lines. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Proof. Let m_1 and m_2 be the slopes of the two lines. We can rigidly translate the lines so that their point of intersection is at the origin (this simplifies the equations of the lines).

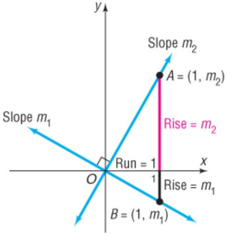


The point $A = (1, m_2)$ is on the line having slope m_2, and the point $(1, m_1)$ is on the line having slope m_1 (consider the rise/run interpretation of slope to see this).

Theorem 1.3.F

Theorem 1.3.F. Criterion for Perpendicular Lines. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Proof. Let m_1 and m_2 be the slopes of the two lines. We can rigidly translate the lines so that their point of intersection is at the origin (this simplifies the equations of the lines).



The point $A = (1, m_2)$ is on the line having slope m_2, and the point $(1, m_1)$ is on the line having slope m_1 (consider the rise/run interpretation of slope to see this).

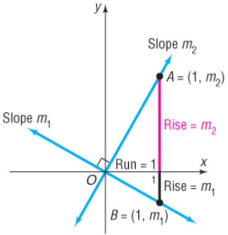
First, suppose that the lines are perpendicular. Then triangle OAB is a right triangle. So by the Pythagorean Theorem (Theorem A.2.A),

$$d(O, A)^2 + d(O, B)^2 = d(A, B)^2.$$

Theorem 1.3.F

Theorem 1.3.F. Criterion for Perpendicular Lines. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Proof. Let m_1 and m_2 be the slopes of the two lines. We can rigidly translate the lines so that their point of intersection is at the origin (this simplifies the equations of the lines).



The point $A = (1, m_2)$ is on the line having slope m_2, and the point $(1, m_1)$ is on the line having slope m_1 (consider the rise/run interpretation of slope to see this).

First, suppose that the lines are perpendicular. Then triangle OAB is a right triangle. So by the Pythagorean Theorem (Theorem A.2.A),

$$d(O, A)^2 + d(O, B)^2 = d(A, B)^2.$$

Theorem 1.3.F (continued 1)

Proof (continued). Since $O = (0,0)$, $A = (1, m_2)$, and $B = (1, m_1)$, then we have the distances

$$\begin{aligned} d(O,A)^2 &= (1-0)^2 + (m_2 - 0)^2 = 1 + m_2^2, \\ d(O,B)^2 &= (1-0)^2 + (m_1 - 0)^2 = 1 + m_1^2, \\ d(A,B)^2 &= (1-1)^2 + (m_2 - m_1)^2 = m_2^2 - 2m_1 m_2 + m_1^2. \end{aligned}$$

So $d(O,A)^2 + d(O,B)^2 = d(A,B)^2$ implies

$$(1 + m_2^2) + (1 + m_1^2) = m_2^2 - 2m_1 m_2 + m_1^2$$

$$\text{or } 2 + m_2^2 + m_1^2 = m_2^2 - 2m_1 m_2 + m_1^2$$

or $2 = -2m_1 m_2$ or $m_1 m_2 = -1$, as claimed.

Theorem 1.3.F (continued 1)

Proof (continued). Since $O = (0,0)$, $A = (1, m_2)$, and $B = (1, m_1)$, then we have the distances

$$\begin{aligned} d(O,A)^2 &= (1-0)^2 + (m_2-0)^2 = 1 + m_2^2, \\ d(O,B)^2 &= (1-0)^2 + (m_1-0)^2 = 1 + m_1^2, \\ d(A,B)^2 &= (1-1)^2 + (m_2-m_1)^2 = m_2^2 - 2m_1m_2 + m_1^2. \end{aligned}$$

So $d(O,A)^2 + d(O,B)^2 = d(A,B)^2$ implies

$$(1 + m_2^2) + (1 + m_1^2) = m_2^2 - 2m_1m_2 + m_1^2$$

$$\text{or } 2 + m_2^2 + m_1^2 = m_2^2 - 2m_1m_2 + m_1^2$$

or $2 = -2m_1m_2$ or $m_1m_2 = -1$, as claimed.

Theorem 1.3.F (continued 2)

Proof (continued). Second, suppose that $m_1 m_2 = -1$. (We basically reverse the previous argument. This is Exercise 1.3.130.) Then $2 = -2m_1 m_2$ or

$$2 + m_2^2 + m_1^2 = m_2^2 - 2m_1 m_2 + m_1^2$$

$$\text{or } (1 + m_2^2) + (1 + m_1^2) = m_2^2 - 2m_1 m_2 + m_1^2.$$

So $d(O, A)^2 + d(O, B)^2 = d(A, B)^2$. By the Converse of the Pythagorean Theorem (Theorem A.2.B, which states that we have a right triangle only if the square of one side is the sum of the squares of the other two sides), triangle OAB is a right triangle with right angle at point O. Therefore the two lines are perpendicular, as claimed. □

Theorem 1.3.F (continued 2)

Proof (continued). Second, suppose that $m_1 m_2 = -1$. (We basically reverse the previous argument. This is Exercise 1.3.130.) Then $2 = -2m_1 m_2$ or

$$2 + m_2^2 + m_1^2 = m_2^2 - 2m_1 m_2 + m_1^2$$

$$\text{or } (1 + m_2^2) + (1 + m_1^2) = m_2^2 - 2m_1 m_2 + m_1^2.$$

So $d(O,A)^2 + d(O,B)^2 = d(A,B)^2$. By the Converse of the Pythagorean Theorem (Theorem A.2.B, which states that we have a right triangle only if the square of one side is the sum of the squares of the other two sides), triangle OAB is a right triangle with right angle at point O. Therefore the two lines are perpendicular, as claimed. □

Page 31 Number 68

Page 31 Number 68. Find an equation for the line perpendicular to the line $y = 2x - 3$ which contains the point $(1, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. First, $y = 2x - 3$ is in the slope-intercept form $y = mx + b$, so the slope of given line is $m = 2$. By Theorem 1.3.F, "Criterion for Perpendicular Lines." we know that the desired line must have slope $\dfrac{-1}{m} = \dfrac{-1}{2}$.

Page 31 Number 68

Page 31 Number 68. Find an equation for the line perpendicular to the line $y = 2x - 3$ which contains the point $(1, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. First, $y = 2x - 3$ is in the slope-intercept form $y = mx + b$, so the slope of given line is $m = 2$. By Theorem 1.3.F, "Criterion for Perpendicular Lines." we know that the desired line must have slope $\dfrac{-1}{m} = \dfrac{-1}{2}$. Since the desired line contains the point $(x_1, y_1) = (1, -2)$, then from the point-slope form of the equation of a line we need $y - y_1 = m(x - x_1)$ or $y - (-2) = (-1/2)(x - (1))$ or $y = (-1/2)(x - 1) - 2 = -(1/2)x + 1/2 - 2 = -(1/2)x - 3/2$. So the desired line $\boxed{\text{in slope intercept form is } y = -(1/2)x - 3/2}$.

$\boxed{\text{A general form is } (1/2)x + y = -3/2 \text{ or } x + 2y = -3}$. □

Page 31 Number 68

Page 31 Number 68. Find an equation for the line perpendicular to the line $y = 2x - 3$ which contains the point $(1, -2)$. Express your answer using the slope-intercept form and a general form of the equation of a line.

Solution. First, $y = 2x - 3$ is in the slope-intercept form $y = mx + b$, so the slope of given line is $m = 2$. By Theorem 1.3.F, "Criterion for Perpendicular Lines." we know that the desired line must have slope $\dfrac{-1}{m} = \dfrac{-1}{2}$. Since the desired line contains the point $(x_1, y_1) = (1, -2)$, then from the point-slope form of the equation of a line we need $y - y_1 = m(x - x_1)$ or $y - (-2) = (-1/2)(x - (1))$ or $y = (-1/2)(x - 1) - 2 = -(1/2)x + 1/2 - 2 = -(1/2)x - 3/2$. So the desired line $\boxed{\text{in slope intercept form is } y = -(1/2)x - 3/2}$.

$\boxed{\text{A general form is } (1/2)x + y = -3/2 \text{ or } x + 2y = -3}$. □

Page 31 Number 106

Page 31 Number 106. Determine whether the lines $y = \frac{1}{2}x - 3$ and $y = -2x + 4$ are parallel, perpendicular, or neither.

Solution. Let the first line L_1 have equation $y = \frac{1}{2}x - 3$ and let the second line L_2 have equation $y = -2x + 4$. Both of the given lines are in the slope-intercept form $y = mx + b$, so the respective slopes of the given lines are $m_1 = 1/2$ and $m_2 = -2$. Since $1/2 = m_1 \neq m_2 = -2$ then by Theorem 1.3.E, "Criterion for Parallel Lines," the lines are not parallel. Since $(m_1)(m_2) = (1/2)(-2) = -1$ then by Theorem 1.3.F, "Criterion for Perpendicular Lines," $\boxed{\text{the lines are perpendicular}}$. □

Page 31 Number 106

Page 31 Number 106. Determine whether the lines $y = \frac{1}{2}x - 3$ and $y = -2x + 4$ are parallel, perpendicular, or neither.

Solution. Let the first line L_1 have equation $y = \frac{1}{2}x - 3$ and let the second line L_2 have equation $y = -2x + 4$. Both of the given lines are in the slope-intercept form $y = mx + b$, so the respective slopes of the given lines are $m_1 = 1/2$ and $m_2 = -2$. Since $1/2 = m_1 \neq m_2 = -2$ then by Theorem 1.3.E, "Criterion for Parallel Lines," the lines are not parallel. Since $(m_1)(m_2) = (1/2)(-2) = -1$ then by Theorem 1.3.F, "Criterion for Perpendicular Lines," $\boxed{\text{the lines are perpendicular}}$. □

Page 32 Number 114

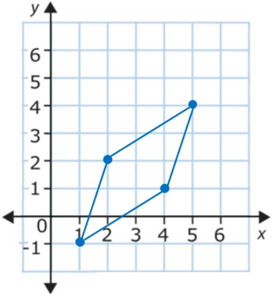
Page 32 Number 114. Use slopes to show that the quadrilateral whose vertices are $(1, -1)$, $(4, 1)$, $(2, 2)$, and $(5, 4)$ is a parallelogram.

Solution. The points are

Page 32 Number 114

Page 32 Number 114. Use slopes to show that the quadrilateral whose vertices are $(1,-1)$, $(4,1)$, $(2,2)$, and $(5,4)$ is a parallelogram.

Solution. The points are

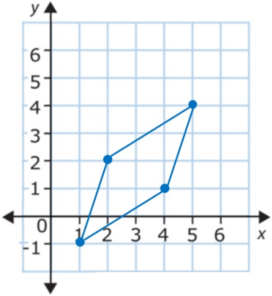


The slope of the line segment joining $(2,2)$ and $(5,4)$ is $m_1 = \dfrac{4-2}{5-2} = \dfrac{2}{3}$. The slope of the line segment joining $(1,-1)$ and $(4,1)$ is $m_2 = \dfrac{1-(-1)}{4-1} = \dfrac{2}{3}$. The slope of the line segment joining $(1,-1)$ and $(2,2)$ is $m_3 = \dfrac{2-(-1)}{2-1} = \dfrac{3}{1} = 3$. The slope of the line segment joining $(4,1)$ and $(5,4)$ is $m_4 = \dfrac{4-1}{5-4} = \dfrac{3}{1} = 3$.

Page 32 Number 114

Page 32 Number 114. Use slopes to show that the quadrilateral whose vertices are $(1, -1)$, $(4, 1)$, $(2, 2)$, and $(5, 4)$ is a parallelogram.

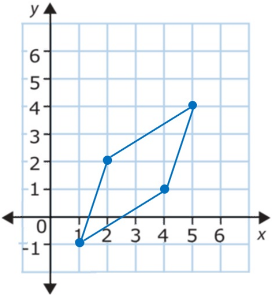
Solution. The points are



The slope of the line segment joining $(2, 2)$ and $(5, 4)$ is $m_1 = \frac{4-2}{5-2} = \frac{2}{3}$. The slope of the line segment joining $(1, -1)$ and $(4, 1)$ is $m_2 = \frac{1-(-1)}{4-1} = \frac{2}{3}$. The slope of the line segment joining $(1, -1)$ and $(2, 2)$ is $m_3 = \frac{2-(-1)}{2-1} = \frac{3}{1} = 3$. The slope of the line segment joining $(4, 1)$ and $(5, 4)$ is $m_4 = \frac{4-1}{5-4} = \frac{3}{1} = 3$.

Page 32 Number 114 (continued)

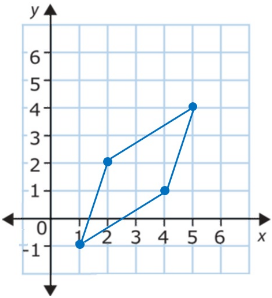
Solution (continued).



Since $m_1 = m_2$ then by Theorem 1.3.E, "Criterion for Parallel Lines," then the upper and lower edges of the quadrilateral are parallel. Since $m_3 = m_4$ then by Theorem 1.3.E, "Criterion for Parallel Lines," then the left and right edges of the quadrilateral are parallel. That is, the quadrilateral is a parallelogram. □

Page 32 Number 114 (continued)

Solution (continued).



Since $m_1 = m_2$ then by Theorem 1.3.E, "Criterion for Parallel Lines," then the upper and lower edges of the quadrilateral are parallel. Since $m_3 = m_4$ then by Theorem 1.3.E, "Criterion for Parallel Lines," then the left and right edges of the quadrilateral are parallel. That is, the quadrilateral is a parallelogram. □

Page 33 Number 126

Page 32 Number 126. A report in the Child Trends DataBase indicated that in 2000, 20.6% of twelfth grade students reported daily use of cigarettes. In 2012, 9.3% of twelfth grade students reported daily use of cigarettes.

(a) Write a linear equation that relates the percent y of twelfth grade students who smoke cigarettes daily to the number x of years after 2000.

(b) Find the intercepts of the graph of your equation.

(c) Do these intercepts have meaningful interpretation?

(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. **(a)** We are given that the points $(x_1, y_1) = (0, 20.6)$ and $(x_2, y_2) = (12, 9.3)$ are on the desired line, so the slope of the desired line is $m = (y_2 - y_1)/(x_2 - x_1) = (9.3 - 20.6)/(12 - 0) = (-11.3)/12 \approx -0.942$. Since the point $(0, 20.6)$ is on the desired line, then from the point-slope equation for a line we have $y - y_1 = m(x - x_1)$ or $y - 20.6 = -0.942(x - 0)$ or in point-slope form $\boxed{y = -0.942x + 20.6}$. □

Page 33 Number 126

Page 32 Number 126. A report in the Child Trends DataBase indicated that in 2000, 20.6% of twelfth grade students reported daily use of cigarettes. In 2012, 9.3% of twelfth grade students reported daily use of cigarettes.

(a) Write a linear equation that relates the percent y of twelfth grade students who smoke cigarettes daily to the number x of years after 2000.

(b) Find the intercepts of the graph of your equation.

(c) Do these intercepts have meaningful interpretation?

(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. (a) We are given that the points $(x_1, y_1) = (0, 20.6)$ and $(x_2, y_2) = (12, 9.3)$ are on the desired line, so the slope of the desired line is $m = (y_2 - y_1)/(x_2 - x_1) = (9.3 - 20.6)/(12 - 0) = (-11.3)/12 \approx -0.942$. Since the point $(0, 20.6)$ is on the desired line, then from the point-slope equation for a line we have $y - y_1 = m(x - x_1)$ or $y - 20.6 = -0.942(x - 0)$ or in point-slope form $\boxed{y = -0.942x + 20.6}$. □

Page 33 Number 126 (continued)

Page 33 Number 126.
(b) Find the intercepts of the graph of your equation.
(c) Do these intercepts have meaningful interpretation?
(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. (b) From part (a) we have $y = -0.942x + 20.6$. So with $y = 0$ we have $x = -20.6/(-0.942) \approx 21.868$ and $\boxed{\text{the } x\text{-intercept is } (21.868, 0)}$. With $x = 0$ we have $y = 20.6$ and $\boxed{\text{the } y\text{-intercept is } (0, 20.6)}$. □

(c) The meaning of the y-intercept $(0, 20.6)$ is that in the year 2000 (when $x = 0$) 20.6% (i.e., $y = 20.6$) of twelfth grade students reported daily use of cigarettes. The meaning of the x-intercept $(21.868, 0)$ is that in the year 2021.868 (when $x = 21.868$) 0% (i.e., $y = 0$) of twelfth grade students reported daily use of cigarettes. □

Page 33 Number 126 (continued)

Page 33 Number 126.
(b) Find the intercepts of the graph of your equation.
(c) Do these intercepts have meaningful interpretation?
(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. **(b)** From part (a) we have $y = -0.942x + 20.6$. So with $y = 0$ we have $x = -20.6/(-0.942) \approx 21.868$ and $\boxed{\text{the } x\text{-intercept is } (21.868, 0)}$. With $x = 0$ we have $y = 20.6$ and $\boxed{\text{the } y\text{-intercept is } (0, 20.6)}$. □

(c) The meaning of the y-intercept $(0, 20.6)$ is that in the year 2000 (when $x = 0$) 20.6% (i.e., $y = 20.6$) of twelfth grade students reported daily use of cigarettes. The meaning of the x-intercept $(21.868, 0)$ is that in the year 2021.868 (when $x = 21.868$) 0% (i.e., $y = 0$) of twelfth grade students reported daily use of cigarettes. □

Page 33 Number 126 (continued)

Page 33 Number 126.
(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. (d) From part (a) we have $y = -0.942x + 20.6$. In the year 2025 we have $x = 2025 - 2000 = 25$, so the percent for the year 2025 is the y value corresponding to $x = 25$. This gives $y = -0.942(25) + 20.6 = -2.95$. This would be interpreted as claiming that, in the year 2025, -2.95% of twelfth grade students would report daily use of cigarettes. But a negative percentage is not meaningful and this is not reasonable. $\square$

Page 33 Number 126 (continued)

Page 33 Number 126.
(d) Use your equation to predict the percent for the year 2025. Is this result reasonable?

Solution. (d) From part (a) we have $y = -0.942x + 20.6$. In the year 2025 we have $x = 2025 - 2000 = 25$, so the percent for the year 2025 is the y value corresponding to $x = 25$. This gives
$y = -0.942(25) + 20.6 = -2.95$. This would be interpreted as claiming that, in the year 2025, -2.95% of twelfth grade students would report daily use of cigarettes. But a negative percentage is not meaningful and this is not reasonable. $\square$