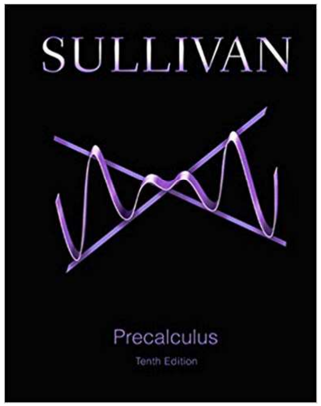
Precalculus 1 (Algebra)

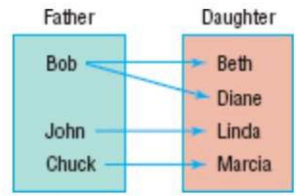
Chapter 2. Functions and Their Graphs

2.1. Functions—Exercises, Examples, Proofs



Page 57 Number 20

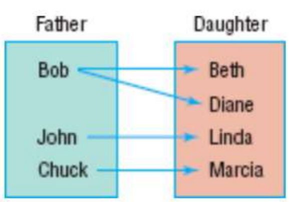
Page 57 Number 20. State the domain and range for the relation pictured here. Determine whether the relation represents a function.



Solution. We are using an arrow $\longrightarrow$ to represent the correspondence in the relation. So the domain is the set "Father" since an arrow emanates from each of the listed fathers Bob, John, and Chuck. The range is the set "Daughter" since an arrow terminates at each of the listed daughters Beth, Diane, Linda, and Marcia.

Page 57 Number 20 (continued)

Page 57 Number 20. State the domain and range for the relation pictured here. Determine whether the relation represents a function.



Solution. By definition, a function associates with each element of the domain exactly one element of the range. But here we see that Bob (in the domain) is associated with the two elements Beth and Diane (in the range), so that this relation is not a function. □

Page 57 Number 30

Page 57 Number 30. Consider the relation

$$\{(-2,16),(-1,4),(0,3),(1,4)\}.$$

State the domain and range for this relation. Determine whether the relation represents a function.

Solution. For (x,y) in the relation we say x corresponds to y, so the domain X is the set of all first entries in the list of ordered pair correspondences. That is, the $\boxed{\text{domain is } X=\{-2,-1,0,1\}}$. The range Y is the set of all second entries in the list of ordered pair correspondences. That is, the $\boxed{\text{range is } X=\{3,4,16\}}$. By definition, a function associates with each element of the domain exactly one element of the range. Since all the first entries in the list of ordered pair correspondences are different, then the relation is a function. □

Page 57 Number 34

Page 57 Number 34. Determine whether the equation $y = |x|$ defines y as a function of x.

Solution. As commented above, for a function you can take any good input (that is, an element of the domain) and it gives you a single output (that is, element of the range). Here, we can let x be any real number (so the domain is all of $\mathbb{R}$) and this will produce a single output, namely the absolute value of x. So $\boxed{y \text{ is a function of } x}$. □

Page 57 Number 36

Page 57 Number 36. Determine whether the equation $y = \pm\sqrt{1-2x}$ defines y as a function of x.

Solution. As commented above, for a function you can take any good input (that is, an element of the domain) and it gives you a single output (that is, element of the range). Here, we cannot let x be just any real number since we cannot take the square root of a negative number (for example, 2 is not in the domain since $x = 2$ implies that $y = \pm\sqrt{1-2(2)} = \pm\sqrt{-3}$ and $\sqrt{-3}$ is undefined). But this does not affect whether the equation determines a function or not (it just determines the domain). The problem is the presence of the $\pm$. If $x = 0$ then $y = \pm\sqrt{1-2(0)} = \pm\sqrt{1} = \pm 1$. So the input value $x = 0$ produces two output values (namely, -1 and $+1$) and hence $\boxed{\text{the equation does not define a function}}$. □

Page 57 Number 48

Page 57 Number 48. Consider the function $f(x) = \sqrt{x^2+x}$. Find each of the following: **(a)** $f(0)$, **(b)** $f(1)$, **(c)** $f(-1)$, **(d)** $f(-x)$, **(e)** $-f(x)$, **(f)** $f(x+1)$, **(g)** $f(2x)$, **(h)** $f(x+h)$, **(i)** $f(\text{BOB})$.

Solution. We simply substitute for the independent value x whatever input is given.
(a) $f(0) = \sqrt{(0)^2+(0)} = \sqrt{0} = 0$. **(b)** $f(1) = \sqrt{(1)^2+(1)} = \sqrt{2}$.
(c) $f(-1) = \sqrt{(-1)^2+(-1)} = \sqrt{1-1} = \sqrt{0} = 0$.
(d) $f(-x) = \sqrt{(-x)^2+(-x)} = \sqrt{x^2-x}$. **(e)** $-f(x) = -\sqrt{x^2+x}$.
(f) $f(x+1) = \sqrt{(x+1)^2+(x+1)} = \sqrt{(x^2+2x+1)+(x+1)} = \sqrt{x^2+3x+2}$.
(g) $f(2x) = \sqrt{(2x)^2+(2x)} = \sqrt{4x^2+2x}$.
(h) $f(x+h) = \sqrt{(x+h)^2+(x+h)} = \sqrt{(x^2+2xh+h^2)+(x+h)} = \sqrt{x^2+2xh+h^2+x+h}$.
(i) $f(\text{BOB}) = \sqrt{(\text{BOB})^2+(\text{BOB})}$. □

Page 57 Number 58

Page 57 Number 58. Find the domain of the function $G(x) = \dfrac{x+4}{x^3-4x}$. Express you answer in interval notation (see Appendix A9. Algebra Essentials).

Solution. By the previous comment, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. There are no square roots in the definition of G but there is division. So we factor the denominator of G:

$$G(x) = \frac{x+4}{x^3-4x} = \frac{x+4}{x(x^2-4)} = \frac{x+4}{x(x-2)(x+2)}.$$

So x is in the domain of G *unless* $x(x-2)(x+2) = 0$. We have $x(x-2)(x+2) = 0$ when either $x = 0$, $x = 2$, or $x = -2$. Hence the domain of G is all real numbers *except* $-2, 0, 2$; that is (in interval notation) the $\boxed{\text{domain of } G \text{ is } (-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)}$. □

Page 57 Number 60

Page 57 Number 60. Find the domain of the function $G(x) = \sqrt{1-x}$. Express you answer in interval notation (see Appendix A9. Algebra Essentials).

Solution. As before, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. There is no division in the definition of G but there is a square root. We need the expression under the square root radical to be nonnegative (that is, greater than or equal to 0): $1 - x \geq 0$. This is equivalent to $1 \geq x$ or $x \leq 1$. Hence the domain of G is all real numbers x such that $x \leq 1$; that is (in interval notation) the $\boxed{\text{domain of } G \text{ is } (-\infty, 1]}$. □

Page 58 Number 80

Page 58 Number 80. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $h \neq 0$, and simplify for $f(x) = -3x+1$.

Solution. We have

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(-3(x+h)+1)-(-3x+1)}{h} \text{ since } f(x) = -3x+1\\ &= \frac{(-3x-3h+1)+3x-1}{h} = \frac{-3h}{h} \text{ simplifying}\\ &= -3 \text{ since } h \neq 0.\end{aligned}$$

So the difference quotient (where $h \neq 0$) is $\boxed{-3}$. □

Page 58 Number 90

Page 58 Number 90. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $h \neq 0$, and simplify for $f(x) = \sqrt{x+1}$.

Solution. We have

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{(x+h)+1}-\sqrt{x+1}}{h} \text{ since } f(x) = \sqrt{x+1}\\ &= \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\left(\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\right)\\ &\quad \text{multiplying by a form of 1 to rationalize numerator}\\ &= \frac{(\sqrt{x+h+1})^2-(\sqrt{x+1})^2}{h(\sqrt{x+h+1}+\sqrt{x+1})} \text{ multiplying out}\\ &= \frac{(x+h+1)-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})}\end{aligned}$$

Page 58 Number 90 (continued)

Page 58 Number 90. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $h \neq 0$, and simplify for $f(x) = \sqrt{x+1}$.

Solution (continued). We have

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h+1)-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})}\\ &= \frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})} \text{ simplifying}\\ &= \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} \text{ since } h \neq 0.\end{aligned}$$

So the difference quotient is $\boxed{\dfrac{1}{\sqrt{x+h+1}+\sqrt{x+1}}}$ (where $h \neq 0$). □

Page 58 Number 76

Page 58 Number 76. For $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{2}{x}$ find the formula for and domain of (in interval notation): **(a)** $(f+g)(x)$, **(b)** $(f-g)(x)$, **(c)** $(f \cdot g)(x)$, **(d)** $\left(\dfrac{f}{g}\right)(x)$.

Solution. (a) We have

$$(f+g)(x) = f(x)+g(x) = \sqrt{x+1}+\frac{2}{x} = \sqrt{x+1}\frac{x}{x}+\frac{2}{x} = \boxed{\frac{x\sqrt{x+1}+2}{x}}.$$

As before, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. Since $f+g$ has a denominator of x then we must have $x \neq 0$. Since $f+g$ involves $\sqrt{x+1}$ then we must have $x+1 \geq 0$ or $x \geq -1$. Combining these two pieces of information we see that the $\boxed{\text{domain is } [-1,0) \cup (0,\infty)}$. □

Page 58 Number 76 (continued 1)

Page 58 Number 76. For $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{2}{x}$ find the formula for and domain of (in interval notation): **(a)** $(f+g)(x)$, **(b)** $(f-g)(x)$, **(c)** $(f \cdot g)(x)$, **(d)** $\left(\dfrac{f}{g}\right)(x)$.

Solution.
(b) We have

$$(f-g)(x) = f(x)-g(x) = \sqrt{x+1}-\frac{2}{x} = \sqrt{x+1}\frac{x}{x}-\frac{2}{x} = \boxed{\frac{x\sqrt{x+1}-2}{x}}.$$

As before, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. Since $f-g$ has a denominator of x then we must have $x \neq 0$. Since $f-g$ involves $\sqrt{x+1}$ then we must have $x+1 \geq 0$ or $x \geq -1$. Combining these two pieces of information we see that the $\boxed{\text{domain is } [-1,0) \cup (0,\infty)}$. □

Page 58 Number 76 (continued 2)

Page 58 Number 76. For $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{2}{x}$ find the formula for and domain of (in interval notation): **(a)** $(f+g)(x)$, **(b)** $(f-g)(x)$, **(c)** $(f \cdot g)(x)$, **(d)** $\left(\dfrac{f}{g}\right)(x)$.

Solution.

(c) We have $(f \cdot g)(x) = f(x)g(x) = \sqrt{x+1}\dfrac{2}{x} = \boxed{\dfrac{2\sqrt{x+1}}{x}}$. As before, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. Since $f \cdot g$ has a denominator of x then we must have $x \neq 0$. Since $f \cdot g$ involves $\sqrt{x+1}$ then we must have $x+1 \geq 0$ or $x \geq -1$. Combining these two pieces of information we see that the $\boxed{\text{domain is } [-1,0) \cup (0,\infty)}$. □

Page 58 Number 76 (continued 3)

Page 58 Number 76. For $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{2}{x}$ find the formula for and domain of (in interval notation): **(a)** $(f+g)(x)$, **(b)** $(f-g)(x)$, **(c)** $(f \cdot g)(x)$, **(d)** $\left(\dfrac{f}{g}\right)(x)$.

Solution.

(d) We have $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x+1}}{2/x} = \boxed{\dfrac{x\sqrt{x+1}}{2} \text{ if } x \neq 0}$. As before, we look for potential incidents of (1) division by 0, and (2) square roots of negatives. Since $x = 0$ is not in the domain of g then $x = 0$ is not in the domain of f/g. Since f/g involves $\sqrt{x+1}$ then we must have $x+1 \geq 0$ or $x \geq -1$. So the $\boxed{\text{domain is } [-1,0) \cup (0,\infty)}$. □

Page 58 Number 104

Page 58 Number 104. If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x) = 20 - 13x^2$.

(a) What is the height of the rock when $x = 1$ second? When $x = 1.1$ seconds? When $x = 1.2$ seconds?
(b) When is the height of the rock 15 meters? When is the height 10 meters? When is it 5 meters?
(c) When does the rock strike the ground? (That is, when is the height 0? Ironically, as a gas giant planet Jupiter doesn't actually have a "ground"!)

Page 58 Number 104

Page 58 Number 104. If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x) = 20 - 13x^2$.

(a) What is the height of the rock when $x = 1$ second? When $x = 1.1$ seconds? When $x = 1.2$ seconds?
(b) When is the height of the rock 15 meters? when is the height 10 meters? When is it 5 meters?
(c) When does the rock strike the ground? (That is, when is the height 0? Ironically, as a gas giant planet Jupiter doesn't actually have a "ground"!)

Solution. (a) When $x = 1$ second we have a height of $H(1) = 20 - 13(1)^2 = 20 - 13 = \boxed{7}$ meters. When $x = 1.1$ seconds we have a height of $H(1.1) = 20 - 13(1.1)^2 = 20 - 15.73 = \boxed{4.27}$ meters. When $x = 1.2$ seconds we have a height of $H(1.1) = 20 - 13(1.2)^2 = 20 - 18.72 = \boxed{1.28}$ meters.

Page 58 Number 104 (continued 1)

Page 58 Number 104. If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x) = 20 - 13x^2$. **(b)** When is the height of the rock 15 meters? When is the height 10 meters? When is it 5 meters?

Solution. (b) When the height $H = 15$ meters we have $H(x) = 20 - 13x^2 = 15$ or $-13x^2 = 15 - 20 = -5$ or $x^2 = -5/(-13) = 5/13$ or $x = \pm\sqrt{5/13}$. Presumably we take x positive and so $\boxed{x = \sqrt{5/13} \approx 0.620 \text{ seconds}}$. When the height $H = 10$ meters we have $H(x) = 20 - 13x^2 = 10$ or $-13x^2 = 10 - 20 = -10$ or $x^2 = -10/(-13) = 10/13$ or $x = \pm\sqrt{10/13}$. Presumably we take x positive and so $\boxed{x = \sqrt{10/13} \approx 0.877 \text{ seconds}}$. When the height $H = 5$ meters we have $H(x) = 20 - 13x^2 = 5$ or $-13x^2 = 5 - 20 = -15$ or $x^2 = -15/(-13) = 15/13$ or $x = \pm\sqrt{15/13}$. Presumably we take x positive and so $\boxed{x = \sqrt{15/13} \approx 1.074 \text{ seconds}}$. □

Page 58 Number 104 (continued 2)

Page 58 Number 104. If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately $H(x) = 20 - 13x^2$. **(c)** When does the rock strike the ground? (That is, when is the height 0? Ironically, as a gas giant planet Jupiter doesn't actually have a "ground"!)

Solution. (c) When the height $H = 0$ meters we have $H(x) = 20 - 13x^2 = 0$ or $-13x^2 = -20$ or $x^2 = -20/(-13) = 20/13$ or $x = \pm\sqrt{20/13}$. Presumably we take x positive and so $\boxed{x = \sqrt{20/13} \approx 1.240 \text{ seconds}}$. □

Page 59 Number 112

Page 59 Number 112. Suppose that the revenue R, in dollars, from selling x clocks is $R(x) = 30x$. The cost C, in dollars, of selling x clocks is $C(x) = 0.1x^2 + 7x + 400$.

(a) Find the profit function, $P(x) = R(x) - C(x)$.

(b) Find the profit if $x = 30$ clocks are sold.

(c) Interpret $P(30)$.

Solution. (a) Since the profit function is $P(x) = R(x) - C(x)$ then we have $\boxed{P(x) = (30x) - (0.1x^2 + 7x + 400) = -0.1x^2 + 23x - 400}$.

(b) If $x = 30$ clocks are sold then the profit, by part (a), is $\boxed{P(30) = -0.1(30)^2 + 23(30) - 400 = 200 \text{ dollars}}$.

(c) We interpret $P(30)$ as $\boxed{\text{if 30 clocks are sold then the profit is \$200}}$. □