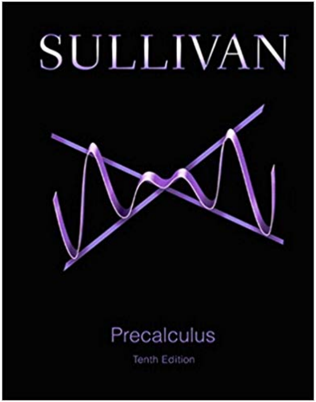
Precalculus 1 (Algebra)

Chapter 2. Functions and Their Graphs

2.2. The Graph of a Function—Exercises, Examples, Proofs



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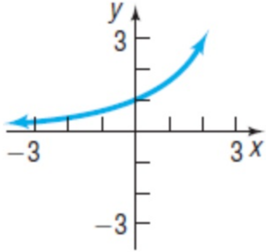
Page 65 Number 14. Determine whether the graph is that of a function by using the vertical line test. If it is, use the graph to find (a) the domain and range, (b) the intercepts (if any), (c) any symmetry with respect to the x-axis, y-axis, or the origin.



Solution. A vertical line will intersect the graph (assuming it is extended according to the pattern we see) in exactly one point. So by Theorem 2.2.A, Vertical Line Test, the graph is that of a function.

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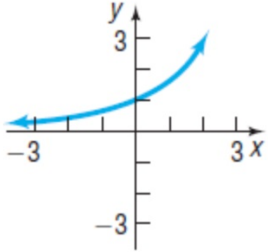

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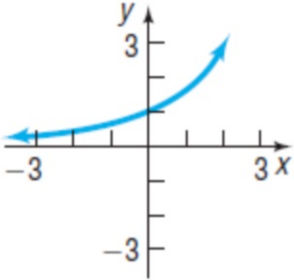
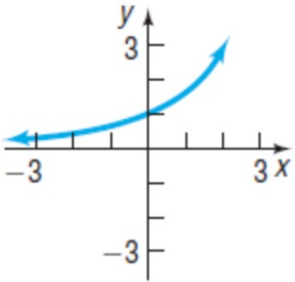
Page 65 Number 14 (continued 1)



Solution (continued). (a) The domain is all possible x values where the function is defined (and so the set of all x values for which the point $(x, f(x))$ is on the graph). So the graph (assuming it is extended according to the pattern we see) indicates that the $\boxed{\text{domain is all real numbers, } \mathbb{R}}$. (This is due to the fact that *every* vertical line intersects the graph of the function in exactly one point.)

Page 65 Number 14 (continued 1)



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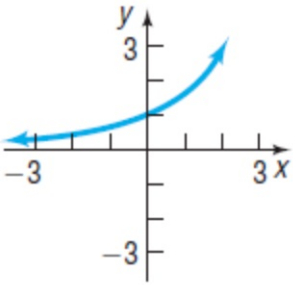

Solution (continued). The range is all possible y values where the function is defined (and so the set of all y values for which the point $(x, f(x)) = (x, y)$ is on the graph). So the graph (assuming it is extended according to the pattern we see) indicates that the

range is all positive real numbers, $(0, \infty)$. (This is due to the fact that the horizontal lines $y = b$, where $b > 0$, intersect the graph of the function.) □

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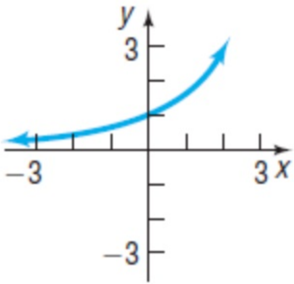

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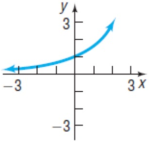
Page 65 Number 14 (continued 3)



Solution (continued). (b) The graph (assuming it is extended according to the pattern we see) indicates that there is

no x-intercept and that the y-intercept is $(0, 1)$.

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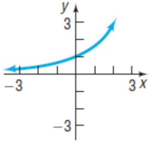

Solution (continued). (b) The graph (assuming it is extended according to the pattern we see) indicates that there is

no x-intercept and that the y-intercept is $(0, 1)$.

(c) The graph is not symmetric with respect to the x-axis (since there are no points in the third and fourth quadrants, but there are points in the first and second quadrants). The graph is not symmetric with respect to the y-axis (since the points in the first quadrant do not correspond to the points in the second quadrant). The graph is not symmetric with respect to the origin (since there are not points in the third and fourth quadrant, but there are points in the first and second quadrants).

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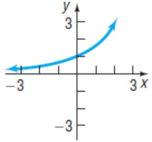

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Page 65 Number 18

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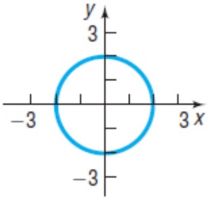

Solution. The vertical line $x = 0$ (that is, the y-axis) intersects the graph in the two points $(0, 2)$ and $(0, -2)$. So by Theorem 2.2.A, Vertical Line Test, the graph is not that of a function. □

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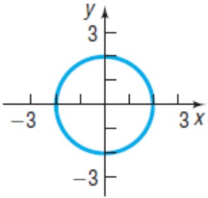

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Page 65 Number 26

Page 65 Number 26. Consider $f(x) = -3x^2 + 5x$.

(a) Is the point $(-1, 2)$ on the graph of f?

(b) If $x = -2$, what is $f(x)$? Based on this information, what point is on the graph of f?

(c) if $f(x) = -2$, what is x? Based on this information, what point(s) are on the graph of f?

(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

(f) List the y-intercept, if there is one, of the graph of f.

Solution. (a) With $x = -1$ we have $f(x) = f(-1) = -3(-1)^2 + 5(-1) = -3 - 5 = -8$ so the point $(-1, -8)$ is on the graph, but the point $\boxed{(-1, 2) \text{ is not on the graph}}$.

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Solution. (b) If $x = -2$ then $f(x) = f(-2) = -3(-2)^2 + 5(-2) = -3(4) - 10 = \boxed{-22}$. So $\boxed{\text{the point } (x, f(x)) = (-2, -22) \text{ is on the graph of } f}$. □

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(c) If $f(x) = -2$ the we must have $f(x) = -3x^2 + 5x = -2$ or $3x^2 - 5x - 2 = 0$. We have by the quadratic equation that
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-2)}}{2(3)} = \frac{5\pm\sqrt{25+24}}{6} = \frac{5\pm\sqrt{49}}{6} = \frac{5\pm7}{6}$.
So $\boxed{\text{either } x = -2/6 = -1/3 \text{ or } x = 12/6 = 2}$. So
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(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

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Solution. (d) We look for potential incidents of (1) division by 0, and (2) square roots of negatives. Since f has no division nor square roots, then the domain of f is $\boxed{\text{all real numbers, } \mathbb{R}}$. □

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(e) For x-intercepts, we set $y = f(x) = 0$ or $-3x^2 + 5x = 0$ or $x(-3x + 5) = 0$. So the $\boxed{x\text{-intercepts are } (0,0) \text{ and } (5/3,0)}$. □

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(f) For the y-intercepts, we set $x = 0$ and get $f(x) = f(0) = -3(0)^2 + 5(0) = 0$. So the $\boxed{y\text{-intercept is } (0,0)}$. □

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Page 66 Number 32. Granny Shots

Page 66 Number 32. If a player in the NBA shoots an underhand foul shot, releasing the ball at a 70-degree angle from a position 3.5 feet above the floor, then the path of the ball can be modeled by the function $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5$, where h is the height of the ball above the floor, x is the forward distance of the ball in front of the foul line, and v is the initial velocity with which the ball is shot in feet per second.

(a) The center of the hoop is 10 feet above the floor and 15 feet in front of the foul line. Determine the initial velocity with which the ball must be shot in order for the ball to go through the hoop.

(b) Write the function for the path of the ball using the velocity found in part (a).

(c) Determine the height of the ball after it has traveled 9 feet in front of the foul line.

(d) Find additional points and graph the path of the basketball.

Page 66 Number 32

Page 66 Number 32. $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5.$

(a) The center of the hoop is 10 feet above the floor and 15 feet in front of the foul line. Determine the initial velocity with which the ball must be shot in order for the ball to go through the hoop.

Solution. (a) The center of the hoop is, in terms of the variables we have, $x = 15$ feet and $h = 10$ feet. So we want the point $(x, h) = (15, 10)$ on the graph of $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5$. So we set $x = 15$ and $h = 10$ to get the equation $10 = -\frac{136(15)^2}{v^2} + 2.7(15) + 3.5$ or $10 = -\frac{30{,}600}{v^2} + 40.5 + 3.5 = -\frac{30{,}600}{v^2} + 44$ or $\frac{30{,}600}{v^2} = 44 - 10 = 34$ or $v^2 = 30{,}600/34 = 900$ or $v = \pm\sqrt{900} = \pm 30$. Presumably (based on the physics) we take the velocity as positive, so we take the $\boxed{\text{velocity as } v = 30 \text{ feet/second}}$.

Page 66 Number 32

Page 66 Number 32. $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5.$

(a) The center of the hoop is 10 feet above the floor and 15 feet in front of the foul line. Determine the initial velocity with which the ball must be shot in order for the ball to go through the hoop.

Solution. **(a)** The center of the hoop is, in terms of the variables we have, $x = 15$ feet and $h = 10$ feet. So we want the point $(x, h) = (15, 10)$ on the graph of $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5$. So we set $x = 15$ and $h = 10$ to get the equation $10 = -\frac{136(15)^2}{v^2} + 2.7(15) + 3.5$ or $10 = -\frac{30{,}600}{v^2} + 40.5 + 3.5 = -\frac{30{,}600}{v^2} + 44$ or $\frac{30{,}600}{v^2} = 44 - 10 = 34$ or $v^2 = 30{,}600/34 = 900$ or $v = \pm\sqrt{900} = \pm 30$. Presumably (based on the physics) we take the velocity as positive, so we take the $\boxed{\text{velocity as } v = 30 \text{ feet/second}}$.

Page 66 Number 32 (continued 1)

Page 66 Number 32. $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5.$

(b) Write the function for the path of the ball using the velocity found in part (a).

(c) Determine the height of the ball after it has traveled 9 feet in front of the foul line.

(d) Find additional points and graph the path of the basketball.

Solution. **(b)** We found in part (a) that $v = 30$ feet/second and so $v^2 = 900$ feet^2/second^2. So the function for the path of the ball is

$$\boxed{h(x) = -\frac{136x^2}{900} + 2.7x + 3.5 = -\frac{34x^2}{225} + 2.7x + 3.5} \text{ feet.}$$

Page 66 Number 32 (continued 1)

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Page 66 Number 32 (continued 2)

Page 66 Number 32. $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5.$

(c) Determine the height of the ball after it has traveled 9 feet in front of the foul line.

(d) Find additional points and graph the path of the basketball.

Solution. (c) The question is $h(x) =?$ when $x = 9$ feet. So we consider

$$h(9) = -\frac{34(9)^2}{225} + 2.7(9) + 3.5 = \boxed{15.56 \text{ feet}}.$$

Page 66 Number 32 (continued 2)

Page 66 Number 32. $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5.$

(c) Determine the height of the ball after it has traveled 9 feet in front of the foul line.

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Solution. **(c)** The question is $h(x) =?$ when $x = 9$ feet. So we consider

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Page 66 Number 32 (continued 3)

Page 66 Number 32. $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5.$

(d) Find additional points and graph the path of the basketball.

Solution. **(d)** From part (c), $h(x) = -\frac{34x^2}{225} + 2.7x + 3.5 = 0$. So some additional points are (approximately):

x	$h(x)$	x	$h(x)$	x	$h(x)$
0	3.5	6	14.26	12	14.14
1	6.05	7	15.00	13	13.06
2	8.30	8	15.43	14	11.68
3	10.24	9	15.56	15	10.00
4	11.88	10	15.39		
5	13.22	11	14.92		

Page 66 Number 32 (continued 3)

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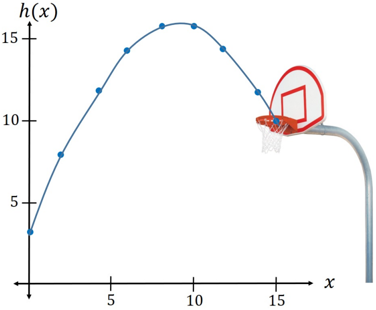

Page 66 Number 32 (continued 3)

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x	$h(x)$	x	$h(x)$	x	$h(x)$
0	3.5	6	14.26	12	14.14
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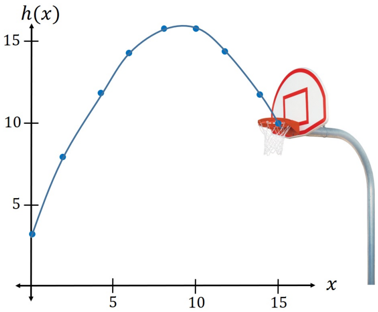
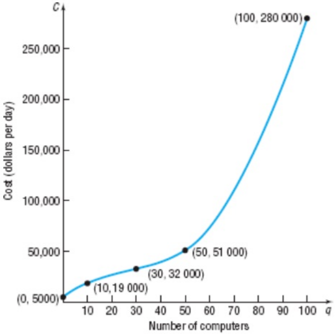
4	11.88	10	15.39		
5	13.22	11	14.92		



□

Page 67 Number 38

Page 67 Number 38. Let C be the function whose graph is given. This graph represents the cost C of manufacturing q computers in a day.



(a) Determine $C(0)$. Interpret the value.

(b) Determine $C(10)$. Interpret the value.

(c) Determine $C(50)$. Interpret the value.

(d) What is the domain of C? What does this domain imply in terms of daily production?

(e) Describe the shape of the graph.

(f) The point $(30, 32{,}000)$ is called an *inflection point* Describe the behavior of the graph around the inflection point.

Page 67 Number 38

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(a) Determine $C(0)$. Interpret the value.
(b) Determine $C(10)$. Interpret the value.
(c) Determine $C(50)$. Interpret the value.

Solution. (a) The point (0, 5000) is on the graph of C, so $\boxed{C(0) = 5000}$. This means that if no computers are manufactured, then the cost is \$5,000 (this is called *fixed cost* and represents the cost of having employees, keeping a factory open, paying the electricity bill, and so forth). □

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(b) The point (10, 19,000) is on the graph of C, so $\boxed{C(10) = 19{,}000}$. This means that if 10 computers are manufactured, then the cost is \$19,000. □

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(c) The point (50, 51,000) is on the graph of C, so $\boxed{C(50) = 51{,}000}$. This means that if 50 computers are manufactured, then the cost is \$51,000. □

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(d) What is the domain of C? What does this domain imply in terms of daily production?

(e) Describe the shape of the graph.

Solution (continued). (d) The domain of C is the set of all q values for which C is defined. We see from the values on the q-axis that C is defined for $0 \leq q \leq 100$ or, in interval notation, the $\boxed{\text{domain is } [0, 100]}$. So daily production can be as little as 0 computers but cannot exceed 100 computers. □

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(e) The graph increases slowly up to $q = 30$ when it starts to increase more rapidly; it starts to increase very rapidly around $q = 50$. We interpret this as the cost increases rapidly when the manufacturer produces computers at a level of 50 or more. □

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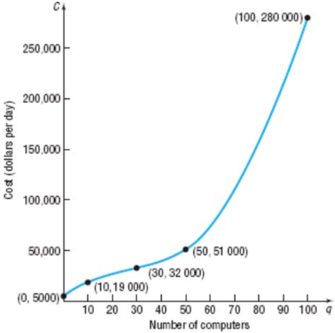
(f) The point (30, 32,000) is called an *inflection point* Describe the behavior of the graph around the inflection point.



Solution (continued). (f) As observed in part (e), this is where the rate of increase goes from growing slowly to growing rapidly. In economics, this is called the *point of diminishing returns*. □

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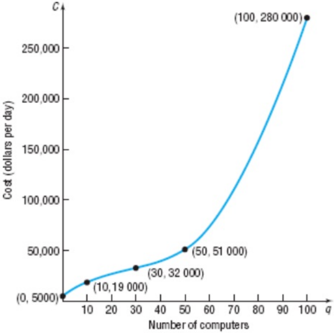

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