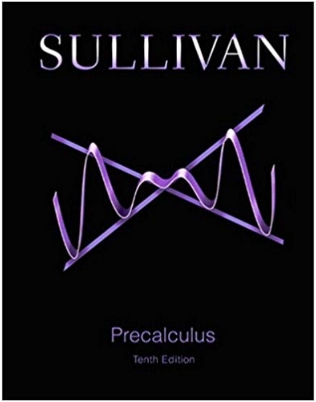
Precalculus 1 (Algebra)

Chapter 2. Functions and Their Graphs

2.4. Library of Functions; Piecewise-defined Functions—Exercises, Examples, Proofs



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Page 90 Number 28. Consider $f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ 2x^2 + 1 & \text{if } x > -1. \end{cases}$
Find **(a)** $f(-2)$, **(b)** $f(0)$, **(c)** $f(1)$, and **(d)** $f(3)$.

Solution. **(a)** To find $f(-2)$, we see that $x = -2$ satisfies $x < -1$ so we use the piece of f defined as $-3x$. Hence $f(-2) = -3(-2) = \boxed{6}$. □

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(c) To find $f(1)$, we see that $x = 1$ satisfies $x > -1$ so we use the piece of f defined as $2x^2 + 1$. Hence $f(1) = 2(1)^2 + 1 = \boxed{3}$. □

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(d) To find $f(3)$, we see that $x = 3$ satisfies $x > -1$ so we use the piece of f defined as $2x^2 + 1$. Hence $f(3) = 2(3)^2 + 1 = \boxed{19}$. □

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(a) Find the domain of f. **(b)** Locate any intercepts. **(c)** Graph f.
(d) Based on the graph, find the range. **(e)** Is f continuous on its domain?

Solution. **(a)** We see that f is defined for all real number (and there is no division or square roots in any piece of the definition of f), so the

$\boxed{\text{domain of } f \text{ is all real numbers } \mathbb{R} = (-\infty, \infty)}$. □

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(b) For the y-intercept, we let $x = 0$. We have $f(0) = 2(0)^2 + 1 = 1$, so $\boxed{\text{the } y\text{-intercept is } 1}$.

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(b) For the y-intercept, we let $x = 0$. We have $f(0) = 2(0)^2 + 1 = 1$, so $\boxed{\text{the } y\text{-intercept is } 1}$. For the x-intercept, we set $y = f(x) = 0$. The first piece of f, $-3x$, is 0 when $x = 0$ but we do not use this piece of f when $x = 0$ and so the first piece of f has no x-intercept. The second piece of f, 0, *is* 0 and so gives an x-intercept of -1 (since this is the set of x values for which we use the second piece). The third piece of f, $2x^2 + 1$, is never 0 so this gives no x-intercept. So $\boxed{\text{the } x\text{-intercept of } f \text{ is } -1}$. □

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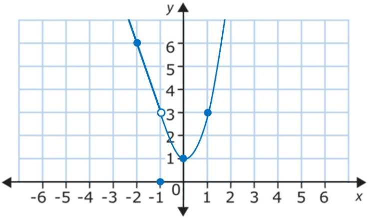

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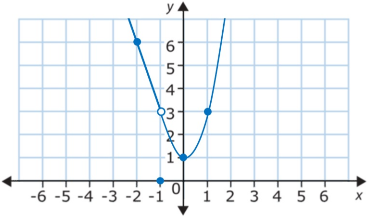

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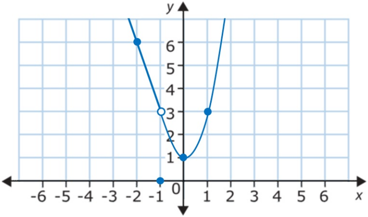

(d) Based on the graph, find the range. **(e)** Is f continuous on its domain?

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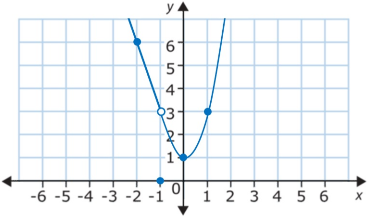

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Solution (continued). (d) The range is the set of all y-values for which there is a corresponding point on the graph of f, so we see from the graph that the range includes all $y \geq 1$ along with $y = 0$. That is, the range is $\{0\} \cup [1, \infty)$.

(e) f is not continuous on its domain because it has a discontinuity at $x = -1$ where there is a hole in the graph.

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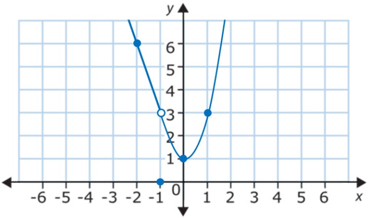

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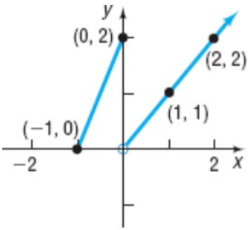

Solution. The left piece of function f is a line segment containing the two points $(x_1, y_1) = (-1, 0)$ and $(x_2, y_2) = (0, 2)$, so we find a formula for this line. The slope of the line is
$m = (y_2 - y_1)/(x_2 - x_1) = ((2) - (0))/((0) - (-1)) = 2$, so from the point-slope formula for a line we have $y - y_1 = m(x - x_1)$ or
$y - (0) = 2(x - (-1))$ or $y = 2x + 2$. We see from the graph that we use this piece for x in the interval $[-1, 0]$ (or $-1 \leq x \leq 0$).

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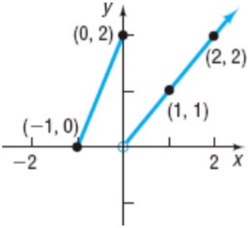

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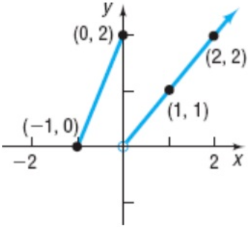

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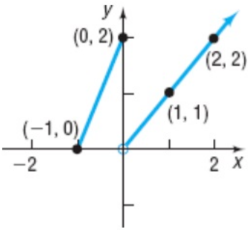

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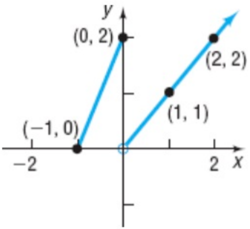

Solution (continued). So the definition of f is

$$f(x) = \begin{cases} 2x+2 & \text{if } -1 \leq x \leq 0 \\ x & \text{if } x > 0. \end{cases}$$

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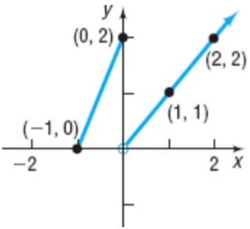

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Page 92 Number 54

Page 92 Number 54. Two 2014 Tax Rate Schedules are given in the accompanying table. If x equals taxable income and y equals the tax due, construct a function $y = f(x)$ for Schedule Y-1.

2014 Tax Rate Schedules											
Schedule X—Single						Schedule Y-1—Married Filing Jointly or Qualified Widow(er)					
If Taxable Income is Over	But Not Over	The Tax is This Amount		Plus This %	Of the Excess Over	If Taxable Income is Over	But Not Over	The Tax is This Amount		Plus This %	Of the Excess Over
$0	$9,075	$0	+	10%	$0	$0	$18,150	$0	+	10%	$0
9,075	36,900	907.50	+	15%	9,075	18,150	73,800	1,815	+	15%	18,150
36,900	89,350	5,081.25	+	25%	36,900	73,800	148,850	10,162.50	+	25%	73,800
89,350	186,350	18,193.75	+	28%	89,350	148,850	226,850	28,925.00	+	28%	148,850
186,350	405,100	45,353.75	+	33%	186,350	226,850	405,100	50,765.00	+	33%	226,850
405,100	406,750	117,541.25	+	35%	405,100	405,100	457,600	109,587.50	+	35%	405,100
406,750	–	118,188.75	+	39.6%	406,750	457,600	–	127,962.50	+	39.6%	457,600

Solution. With x as taxable income, we first see that for $0 < x \leq 18{,}150$ the tax due is 0 plus 10% of x, or $0 + 0.10x$. For $18{,}150 < x \leq 73{,}800$ the tax due is 1,815 plus 15% of the excess of x over 18,150, or $1{,}815 + 0.15(x - 18{,}150)$.

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9,075	36,900	907.50	+	15%	9,075	18,150	73,800	1,815	+	15%	18,150
36,900	89,350	5,081.25	+	25%	36,900	73,800	148,850	10,162.50	+	25%	73,800
89,350	186,350	18,193.75	+	28%	89,350	148,850	226,850	28,925.00	+	28%	148,850
186,350	405,100	45,353.75	+	33%	186,350	226,850	405,100	50,765.00	+	33%	226,850
405,100	406,750	117,541.25	+	35%	405,100	405,100	457,600	109,587.50	+	35%	405,100
406,750	–	118,188.75	+	39.6%	406,750	457,600	–	127,962.50	+	39.6%	457,600

Solution. With x as taxable income, we first see that for $0 < x \leq 18{,}150$ the tax due is 0 plus 10% of x, or $0 + 0.10x$. For $18{,}150 < x \leq 73{,}800$ the tax due is 1,815 plus 15% of the excess of x over 18,150, or $1{,}815 + 0.15(x - 18{,}150)$.

Page 92 Number 54 (continued)

Solution (continued). Similarly for $73{,}800 < x \le 148{,}850$ the tax due is $10{,}162.50 + 0.25(x - 73{,}800)$.
For $148{,}850 < x \le 226{,}850$ the tax due is $28{,}925 + 0.28(x - 148{,}850)$.
For $226{,}850 < x \le 405{,}100$ the tax due is $50{,}765 + 0.33(x - 226{,}850)$.
For $405{,}100 < x \le 457{,}600$ the tax due is $109{,}587.50 + 0.35(x - 405{,}100)$.
For $457{,}600 < x$ the tax due is $127{,}962.50 + 0.396(x - 457{,}600)$. So

$$f(x) = \begin{cases} 0 + 0.10x & \text{if } 0 < x \le 18{,}150 \\ 1{,}815 + 0.15(x - 18{,}150) & \text{if } 18{,}150 < x \le 73{,}800 \\ 10{,}162.50 + 0.25(x - 73{,}800) & \text{if } 73{,}800 < x \le 148{,}850 \\ 28{,}925 + 0.28(x - 148{,}850) & \text{if } 148{,}850 < x \le 226{,}850 \\ 50{,}765 + 0.33(x - 226{,}850) & \text{if } 226{,}850 < x \le 405{,}100 \\ 109{,}587.50 + 0.35(x - 405{,}100) & \text{if } 405{,}100 < x \le 457{,}600 \\ 127{,}962.50 + 0.396(x - 457{,}600) & \text{if } 457{,}600 < x \end{cases}$$

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For $148{,}850 < x \le 226{,}850$ the tax due is $28{,}925 + 0.28(x - 148{,}850)$.
For $226{,}850 < x \le 405{,}100$ the tax due is $50{,}765 + 0.33(x - 226{,}850)$.
For $405{,}100 < x \le 457{,}600$ the tax due is $109{,}587.50 + 0.35(x - 405{,}100)$.
For $457{,}600 < x$ the tax due is $127{,}962.50 + 0.396(x - 457{,}600)$. So

$$f(x) = \begin{cases} 0 + 0.10x & \text{if } 0 < x \le 18{,}150 \\ 1{,}815 + 0.15(x - 18{,}150) & \text{if } 18{,}150 < x \le 73{,}800 \\ 10{,}162.50 + 0.25(x - 73{,}800) & \text{if } 73{,}800 < x \le 148{,}850 \\ 28{,}925 + 0.28(x - 148{,}850) & \text{if } 148{,}850 < x \le 226{,}850 \\ 50{,}765 + 0.33(x - 226{,}850) & \text{if } 226{,}850 < x \le 405{,}100 \\ 109{,}587.50 + 0.35(x - 405{,}100) & \text{if } 405{,}100 < x \le 457{,}600 \\ 127{,}962.50 + 0.396(x - 457{,}600) & \text{if } 457{,}600 < x \end{cases}$$

□