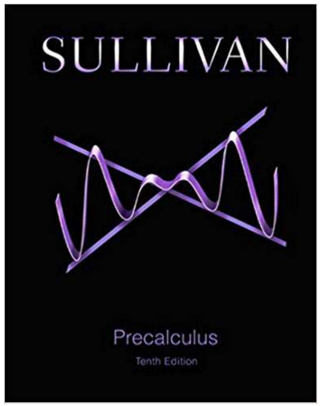
Precalculus 1 (Algebra)

Chapter 3. Linear and Quadratic Functions

3.1. Properties of Linear Functions and Linear Models—Exercises, Examples, Proofs



Page 126 Number 18(a)(b)

Page 126 Number 18(a)(b). Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and y-intercept h. **(b)** Use the slope and y-intercept to graph linear function h.

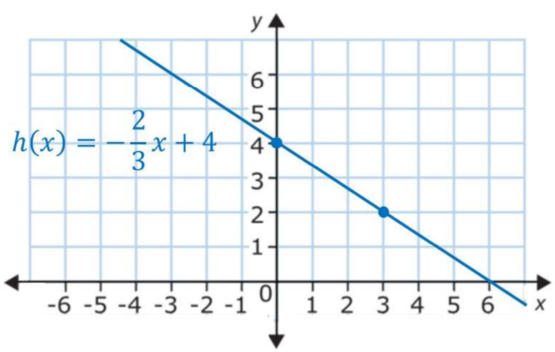
Solution. (a) Since $h(x) = mx + b = -\frac{2}{3}x + 4$, then the slope is $\boxed{m = -2/3}$. □

(b) The y-intercept is $b = 4$, so the point $(x, y) = (0, 4)$ is on the graph. Based on this point, if we take $m = \Delta y/\Delta x = -2/3$, $\Delta x = 3$, and $\Delta y = -2$, then another point on the graph of the function is $(x + \Delta x, y + \Delta y) = (0 + 3, 4 + (-2)) = (3, 2)$. This determines the graph of the linear function...

Page 126 Number 18(a)(b) (continued)

Page 126 Number 18(a)(b). Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and y-intercept of each function. **(b)** Use the slope and y-intercept to graph the linear function.

Solution (continued).



□

Page 126 Number 18(c)

Page 126 Number 18(c). Consider again $h(x) = -\frac{2}{3}x + 4$. Determine the average rate of change of each function.

Solution. By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope, so by part (a) the $\boxed{\text{average rate of change is } m = -2/3}$. □

Page 126 Number 22

Page 126 Number 22. Determine whether the function values given in the table could be associated with a linear function or a nonlinear function. If it could be linear, determine the slope.

x	f(x)
−2	1/4
−1	1/2
0	1
1	2
2	4

Solution. By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope. So if these function values lie on the graph of a linear function, the average rate of change will be same when computed for any consecutive pair of points. We get the following average rates of change: . . .

Page 126 Number 22 (continued)

Solution (continued).

Points	Average Rate of Change
$(-2,1/4)$, $(-1,1/2)$	$\Delta y/\Delta x = ((1/2)-(1/4))/((-1)-(-2)) = 1/4$
$(-1,1/2)$, $(0,1)$	$\Delta y/\Delta x = ((1)-(1/2))/((0)-(-1)) = 1/2$
$(0,1)$, $(1,2)$	$\Delta y/\Delta x = ((2)-(1))/((1)-(0)) = 1$
$(1,2)$, $(2,4)$	$\Delta y/\Delta x = ((4)-(2))/((2)-(1)) = 2$

So these are not points on a linear function. □

Page 126 Number 18(d)

Page 126 Number 18(d). Consider again $h(x) = -\frac{2}{3}x + 4$. Determine whether the linear function is increasing, decreasing, or constant.

Solution. By part (a) the slope of this linear function is $m = -2/3$ so by Theorem 3.1.B, Increasing, Decreasing, and Constant Linear Functions, this is a decreasing function on its domain $(-\infty, \infty) = \mathbb{R}$. □

Page 128 Number 30

Page 128 Number 30. Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

Solution. (a) To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $x = -5/3$. □

(b) To solve $f(x) < 0$ we consider $3x + 5 < 0$ or $3x < -5$ or $x < -5/3$. □

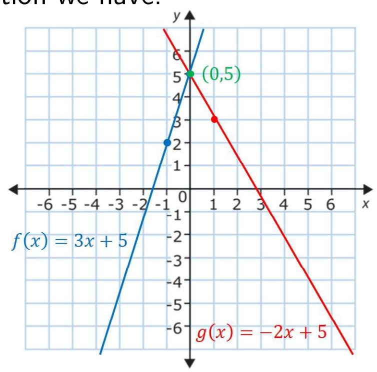
(c) To solve $f(x) = g(x)$ we consider $3x + 5 = -2x + 5$ or $5x = 0$ or $x = 0$. □

(d) So solve $f(x) \geq g(x)$ we consider $3x + 5 \geq -2x + 5$ or $5x \geq 0$ or $x \geq 0$. □

Page 128 Number 30 (continued)

Page 128 Number 30. Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

Solution (continued). The graph of $f(x) = 3x + 5$ has slope 3 and y-intercept 5. The graph of $g(x) = -2x + 5$ has slope -2 and y-intercept 5. With this information we have:



□

Page 128 Number 38

Page 128 Number 38. The monthly cost C, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where x is the number of minutes used.
(a) What is the cost if you talk on the phone for $x = 50$ minutes?
(b) Suppose that your monthly bill is \$21.64. How many minutes did you use the phone? **(c)** Suppose that you budget yourself \$50 per month for the phone. What is the maximum number of minutes that you can talk?
(d) What is the implied domain of C if there are 30 days in the month?

Solution. (a) For $x = 50$ we have $\boxed{C(50) = 0.26(50) + 5 = 18 \text{ dollars}}$. □

(b) With $C(x) = 21.64$ we solve $C(x) = 0.26x + 5 = 21.64$ or $0.26x = 16.64$ or $\boxed{x = 16.64/0.26 = 64 \text{ minutes}}$. □

(c) With $C(x) = 50$ we solve $C(x) = 0.26x + 5 = 50$ or $0.26x = 45$ or $\boxed{x = 45/0.26 \approx 173 \text{ minutes}}$. □

Page 128 Number 38 (continued)

Page 128 Number 38. The monthly cost C, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where x is the number of minutes used.
(d) What is the implied domain of C if there are 30 days in the month?

Solution (continued). (d) Since the set of x values is the number of minutes used, x must be at least 0. For a 30 day month, there are (30 days)(24 hours/day)(60 minutes/hour) = 43,200 minutes and so x can be no more than 43,200. The implied domain is $0 \le x \le 43{,}200$ or, in interval notation, $\boxed{[0,\ 43{,}200]}$. □

Page 129 Number 50

Page 129 Number 50. A cell phone company offers an international plan by charging \$30 for the first 80 minutes, plus \$0.50 for each minute over 80. **(a)** Write a linear model that relates the cost, in dollars, of talking x minutes, assuming $x \ge 80$. **(b)** What is the cost to talking 105 minutes? 120 minutes?

Solution. (a) Let $C(x)$ be the cost in dollars where x is the number of minutes and assume $x \ge 80$ (so the domain is the interval $[80, \infty)$). Then $\boxed{C(x) = 30 + 0.50(x - 80) \text{ dollars}}$. □

(b) When talking $x = 105$ minutes, we have the cost is $\boxed{C(105) = 30 + 0.50((105) - 80) = 30 + 0.50(25) = 42.50 \text{ dollars}}$. When talking $x = 120$ minutes, the cost is $\boxed{C(120) = 30 + 0.50((120) - 80) = 30 + 0.50(40) = 50 \text{ dollars}}$. □