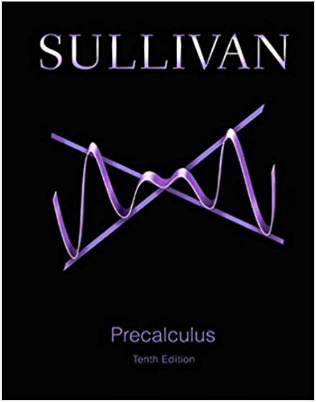
# Precalculus 1 (Algebra)

**Chapter 3. Linear and Quadratic Functions**

3.1. Properties of Linear Functions and Linear Models—Exercises, Examples, Proofs



# Table of contents

# Page 126 Number 18(a)(b)

**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept $h$. **(b)** Use the slope and $y$-intercept to graph linear function $h$.

**Solution.** **(a)** Since $h(x) = mx + b = -\frac{2}{3}x + 4$, then the slope is $\boxed{m = -2/3}$. □

# Page 126 Number 18(a)(b)

**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept $h$. **(b)** Use the slope and $y$-intercept to graph linear function $h$.

**Solution. (a)** Since $h(x) = mx + b = -\frac{2}{3}x + 4$, then the slope is $\boxed{m = -2/3}$. □

**(b)** The $y$-intercept is $b = 4$, so the point $(x, y) = (0, 4)$ is on the graph. Based on this point, if we take $m = \Delta y/\Delta x = -2/3$, $\Delta x = 3$, and $\Delta y = -2$, then another point on the graph of the function is $(x + \Delta x, y + \Delta y) = (0 + 3, 4 + (-2)) = (3, 2)$. This determines the graph of the linear function...

# Page 126 Number 18(a)(b)

**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept $h$. **(b)** Use the slope and $y$-intercept to graph linear function $h$.

**Solution.** **(a)** Since $h(x) = mx + b = -\frac{2}{3}x + 4$, then the slope is $\boxed{m = -2/3}$. □

**(b)** The $y$-intercept is $b = 4$, so the point $(x, y) = (0, 4)$ is on the graph. Based on this point, if we take $m = \Delta y/\Delta x = -2/3$, $\Delta x = 3$, and $\Delta y = -2$, then another point on the graph of the function is $(x + \Delta x, y + \Delta y) = (0 + 3, 4 + (-2)) = (3, 2)$. This determines the graph of the linear function...

# Page 126 Number 18(a)(b) (continued)

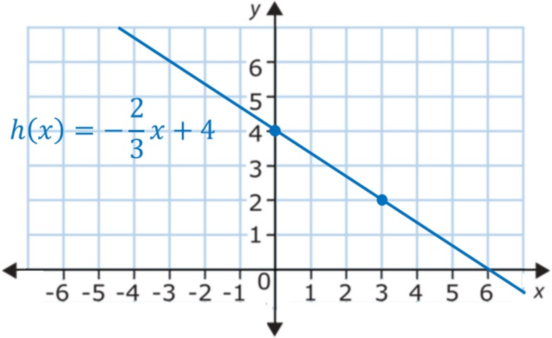
**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept of each function. **(b)** Use the slope and $y$-intercept to graph the linear function.

**Solution (continued).**

# Page 126 Number 18(a)(b) (continued)

**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept of each function. **(b)** Use the slope and $y$-intercept to graph the linear function.

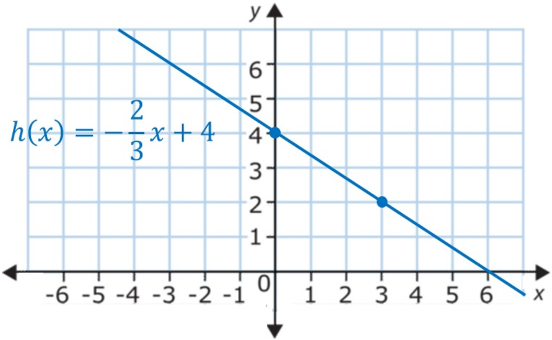
**Solution (continued).**



# Page 126 Number 18(a)(b) (continued)

**Page 126 Number 18(a)(b).** Consider $h(x) = -\frac{2}{3}x + 4$. **(a)** Determine the slope and $y$-intercept of each function. **(b)** Use the slope and $y$-intercept to graph the linear function.

**Solution (continued).**



# Page 126 Number 18(c)

**Page 126 Number 18(c).** Consider again $h(x) = -\frac{2}{3}x + 4$. Determine the average rate of change of each function.

**Solution.** By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope, so by part (a) the $\boxed{\text{average rate of change is } m = -2/3}$. □

# Page 126 Number 18(c)

**Page 126 Number 18(c).** Consider again $h(x) = -\frac{2}{3}x + 4$. Determine the average rate of change of each function.

**Solution.** By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope, so by part (a) the $\boxed{\text{average rate of change is } m = -2/3}$. □

# Page 126 Number 22

**Page 126 Number 22.** Determine whether the function values given in the table could be associated with a linear function or a nonlinear function. If it could be linear, determine the slope.

| **x** | **f(x)** |
|---|---|
| $-2$ | $1/4$ |
| $-1$ | $1/2$ |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

**Solution.** By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope. So if these function values lie on the graph of a linear function, the average rate of change will be same when computed for any consecutive pair of points. We get the following average rates of change: ...

# Page 126 Number 22

**Page 126 Number 22.** Determine whether the function values given in the table could be associated with a linear function or a nonlinear function. If it could be linear, determine the slope.

| **x** | **f(x)** |
|---|---|
| $-2$ | $1/4$ |
| $-1$ | $1/2$ |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

**Solution.** By Theorem 3.1.A, Average Rate of Change of a Linear Function, the average rate of change of a linear function is its slope. So if these function values lie on the graph of a linear function, the average rate of change will be same when computed for any consecutive pair of points. We get the following average rates of change: ...

# Page 126 Number 22 (continued)

**Solution (continued).**

| Points | Average Rate of Change |
|---|---|
| $(-2, 1/4)$, $(-1, 1/2)$ | $\Delta y/\Delta x = ((1/2) - (1/4))/((-1) - (-2)) = 1/4$ |
| $(-1, 1/2)$, $(0, 1)$ | $\Delta y/\Delta x = ((1) - (1/2))/((0) - (-1)) = 1/2$ |
| $(0, 1)$, $(1, 2)$ | $\Delta y/\Delta x = ((2) - (1))/((1) - (0)) = 1$ |
| $(1, 2)$, $(2, 4)$ | $\Delta y/\Delta x = ((4) - (2))/((2) - (1)) = 2$ |

So these $\boxed{\text{are not points on a linear function}}$. □

# Page 126 Number 18(d)

**Page 126 Number 18(d).** Consider again $h(x) = -\frac{2}{3}x + 4$. Determine whether the linear function is increasing, decreasing, or constant.

**Solution.** By part (a) the slope of this linear function is $m = -2/3$ so by Theorem 3.1.B, Increasing, Decreasing, and Constant Linear Functions, this is a $\boxed{\text{decreasing function on its domain } (-\infty, \infty) = \mathbb{R}}$. □

# Page 126 Number 18(d)

**Page 126 Number 18(d).** Consider again $h(x) = -\frac{2}{3}x + 4$. Determine whether the linear function is increasing, decreasing, or constant.

**Solution.** By part (a) the slope of this linear function is $m = -2/3$ so by Theorem 3.1.B, Increasing, Decreasing, and Constant Linear Functions, this is a $\boxed{\text{decreasing function on its domain } (-\infty, \infty) = \mathbb{R}}$. □

# Page 128 Number 30

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution.** **(a)** To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $\boxed{x = -5/3}$. □

# Page 128 Number 30

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution.** **(a)** To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $\boxed{x = -5/3}$. □

**(b)** To solve $f(x) < 0$ we consider $3x + 5 < 0$ or $3x < -5$ or $\boxed{x < -5/3}$. □

# Page 128 Number 30

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution.** **(a)** To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $\boxed{x = -5/3}$. □

**(b)** To solve $f(x) < 0$ we consider $3x + 5 < 0$ or $3x < -5$ or $\boxed{x < -5/3}$. □

**(c)** To solve $f(x) = g(x)$ we consider $3x + 5 = -2x + 5$ or $5x = 0$ or $\boxed{x = 0}$. □

# Page 128 Number 30

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution. (a)** To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $\boxed{x = -5/3}$. □

**(b)** To solve $f(x) < 0$ we consider $3x + 5 < 0$ or $3x < -5$ or $\boxed{x < -5/3}$. □

**(c)** To solve $f(x) = g(x)$ we consider $3x + 5 = -2x + 5$ or $5x = 0$ or $\boxed{x = 0}$. □

**(d)** So solve $f(x) \geq g(x)$ we consider $3x + 5 \geq -2x + 5$ or $5x \geq 0$ or $\boxed{x \geq 0}$. □

# Page 128 Number 30

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(a)** Solve $f(x) = 0$. **(b)** Solve $f(x) < 0$. **(c)** Solve $f(x) = g(x)$. **(d)** Solve $f(x) \geq g(x)$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution.** **(a)** To solve $f(x) = 0$ we consider $3x + 5 = 0$ or $3x = -5$ or $\boxed{x = -5/3}$. □

**(b)** To solve $f(x) < 0$ we consider $3x + 5 < 0$ or $3x < -5$ or $\boxed{x < -5/3}$. □

**(c)** To solve $f(x) = g(x)$ we consider $3x + 5 = -2x + 5$ or $5x = 0$ or $\boxed{x = 0}$. □

**(d)** So solve $f(x) \geq g(x)$ we consider $3x + 5 \geq -2x + 5$ or $5x \geq 0$ or $\boxed{x \geq 0}$. □

# Page 128 Number 30 (continued)

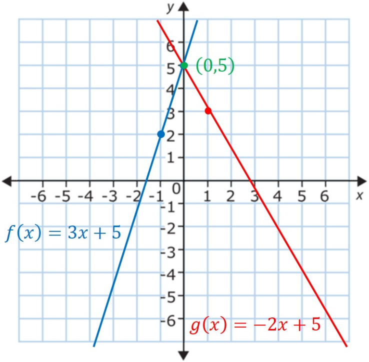
**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution (continued).** The graph of $f(x) = 3x + 5$ has slope 3 and $y$-intercept 5. The graph of $g(x) = -2x + 5$ has slope $-2$ and $y$-intercept 5. With this information we have:

# Page 128 Number 30 (continued)

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

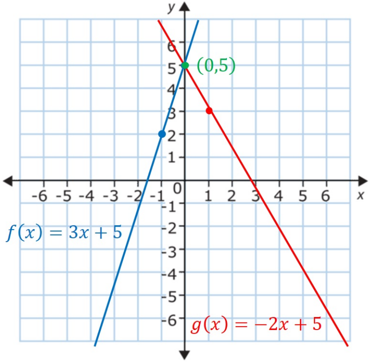
**Solution (continued).** The graph of $f(x) = 3x + 5$ has slope 3 and $y$-intercept 5. The graph of $g(x) = -2x + 5$ has slope $-2$ and $y$-intercept 5. With this information we have:



# Page 128 Number 30 (continued)

**Page 128 Number 30.** Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. **(e)** Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

**Solution (continued).** The graph of $f(x) = 3x + 5$ has slope 3 and $y$-intercept 5. The graph of $g(x) = -2x + 5$ has slope $-2$ and $y$-intercept 5. With this information we have:




□

# Page 128 Number 38

**Page 128 Number 38.** The monthly cost $C$, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where $x$ is the number of minutes used.
**(a)** What is the cost if you talk on the phone for $x = 50$ minutes?
**(b)** Suppose that your monthly bill is \$21.64. How many minutes did you use the phone? **(c)** Suppose that you budget yourself \$50 per month for the phone. What is the maximum number of minutes that you can talk?
**(d)** What is the implied domain of $C$ if there are 30 days in the month?

**Solution. (a)** For $x = 50$ we have
$\boxed{C(50) = 0.26(50) + 5 = 18 \text{ dollars}}$. □

# Page 128 Number 38

**Page 128 Number 38.** The monthly cost $C$, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where $x$ is the number of minutes used.
**(a)** What is the cost if you talk on the phone for $x = 50$ minutes?
**(b)** Suppose that your monthly bill is \$21.64. How many minutes did you use the phone? **(c)** Suppose that you budget yourself \$50 per month for the phone. What is the maximum number of minutes that you can talk?
**(d)** What is the implied domain of $C$ if there are 30 days in the month?

**Solution. (a)** For $x = 50$ we have
$\boxed{C(50) = 0.26(50) + 5 = 18 \text{ dollars}}$. □

**(b)** With $C(x) = 21.64$ we solve $C(x) = 0.26x + 5 = 21.64$ or $0.26x = 16.64$ or $\boxed{x = 16.64/0.26 = 64 \text{ minutes}}$. □

# Page 128 Number 38

**Page 128 Number 38.** The monthly cost $C$, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where $x$ is the number of minutes used.
**(a)** What is the cost if you talk on the phone for $x = 50$ minutes?
**(b)** Suppose that your monthly bill is \$21.64. How many minutes did you use the phone? **(c)** Suppose that you budget yourself \$50 per month for the phone. What is the maximum number of minutes that you can talk?
**(d)** What is the implied domain of $C$ if there are 30 days in the month?

**Solution. (a)** For $x = 50$ we have
$\boxed{C(50) = 0.26(50) + 5 = 18 \text{ dollars}}$. □

**(b)** With $C(x) = 21.64$ we solve $C(x) = 0.26x + 5 = 21.64$ or $0.26x = 16.64$ or $\boxed{x = 16.64/0.26 = 64 \text{ minutes}}$. □

**(c)** With $C(x) = 50$ we solve $C(x) = 0.26x + 5 = 50$ or $0.26x = 45$ or $\boxed{x = 45/0.26 \approx 173 \text{ minutes}}$. □

# Page 128 Number 38

**Page 128 Number 38.** The monthly cost $C$, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where $x$ is the number of minutes used.
**(a)** What is the cost if you talk on the phone for $x = 50$ minutes?
**(b)** Suppose that your monthly bill is \$21.64. How many minutes did you use the phone? **(c)** Suppose that you budget yourself \$50 per month for the phone. What is the maximum number of minutes that you can talk?
**(d)** What is the implied domain of $C$ if there are 30 days in the month?

**Solution. (a)** For $x = 50$ we have
$\boxed{C(50) = 0.26(50) + 5 = 18 \text{ dollars}}$. □

**(b)** With $C(x) = 21.64$ we solve $C(x) = 0.26x + 5 = 21.64$ or $0.26x = 16.64$ or $\boxed{x = 16.64/0.26 = 64 \text{ minutes}}$. □

**(c)** With $C(x) = 50$ we solve $C(x) = 0.26x + 5 = 50$ or $0.26x = 45$ or $\boxed{x = 45/0.26 \approx 173 \text{ minutes}}$. □

# Page 128 Number 38 (continued)

**Page 128 Number 38.** The monthly cost $C$, in dollars, for calls from the United States to Germany on a certain phone plan is modeled by the function $C(x) = 0.26x + 5$, where $x$ is the number of minutes used.
**(d)** What is the implied domain of $C$ if there are 30 days in the month?

**Solution (continued). (d)** Since the set of $x$ values is the number of minutes used, $x$ must be at least 0. For a 30 day month, there are (30 days)(24 hours/day)(60 minutes/hour) = 43,200 minutes and so $x$ can be no more than 43,200. The implied domain is $0 \le x \le 43{,}200$ or, in interval notation, $\boxed{[0, 43{,}200]}$. □

# Page 129 Number 50

**Page 129 Number 50.** A cell phone company offers an international plan by charging \$30 for the first 80 minutes, plus \$0.50 for each minute over 80. **(a)** Write a linear model that relates the cost, in dollars, of talking $x$ minutes, assuming $x \geq 80$. **(b)** What is the cost to talking 105 minutes? 120 minutes?

**Solution.** **(a)** Let $C(x)$ be the cost in dollars where $x$ is the number of minutes and assume $x \geq 80$ (so the domain is the interval $[80, \infty)$). Then $\boxed{C(x) = 30 + 0.50(x - 80) \text{ dollars}}$. □

# Page 129 Number 50

**Page 129 Number 50.** A cell phone company offers an international plan by charging \$30 for the first 80 minutes, plus \$0.50 for each minute over 80. **(a)** Write a linear model that relates the cost, in dollars, of talking $x$ minutes, assuming $x \geq 80$. **(b)** What is the cost to talking 105 minutes? 120 minutes?

**Solution. (a)** Let $C(x)$ be the cost in dollars where $x$ is the number of minutes and assume $x \geq 80$ (so the domain is the interval $[80, \infty)$). Then $\boxed{C(x) = 30 + 0.50(x - 80) \text{ dollars}}$. □

**(b)** When talking $x = 105$ minutes, we have the cost is $\boxed{C(105) = 30 + 0.50((105) - 80) = 30 + 0.50(25) = 42.50 \text{ dollars}}$. When talking $x = 120$ minutes, the cost is $\boxed{C(120) = 30 + 0.50((120) - 80) = 30 + 0.50(40) = 50 \text{ dollars}}$. □

# Page 129 Number 50

**Page 129 Number 50.** A cell phone company offers an international plan by charging \$30 for the first 80 minutes, plus \$0.50 for each minute over 80. **(a)** Write a linear model that relates the cost, in dollars, of talking $x$ minutes, assuming $x \geq 80$. **(b)** What is the cost to talking 105 minutes? 120 minutes?

**Solution. (a)** Let $C(x)$ be the cost in dollars where $x$ is the number of minutes and assume $x \geq 80$ (so the domain is the interval $[80, \infty)$). Then $\boxed{C(x) = 30 + 0.50(x - 80) \text{ dollars}}$. □

**(b)** When talking $x = 105$ minutes, we have the cost is $\boxed{C(105) = 30 + 0.50((105) - 80) = 30 + 0.50(25) = 42.50 \text{ dollars}}$. When talking $x = 120$ minutes, the cost is $\boxed{C(120) = 30 + 0.50((120) - 80) = 30 + 0.50(40) = 50 \text{ dollars}}$. □