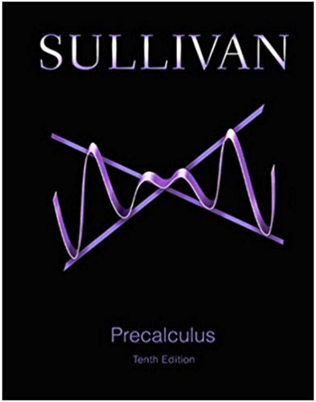
Precalculus 1 (Algebra)

Chapter 3. Linear and Quadratic Functions

3.5. Inequalities Involving Quadratic Functions—Exercises, Examples, Proofs



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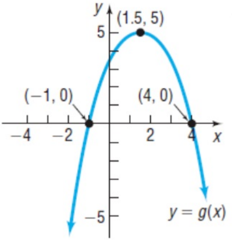
Page 160 Number 4. Use the figure to solve each inequality.



(a) $g(x) < 0$. **(b)** $g(x) \leq 0$.

Solution. **(a)** We see from the graph that $g(x) < 0$ when the graph of $y = g(x)$ is below the x-axis, so this is satisfied for the x values in $\boxed{(-\infty, -1) \cup (4, \infty)}$. □

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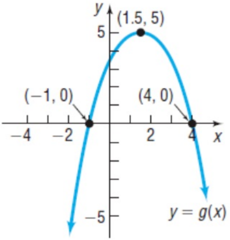

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(b) For the nonstrict inequality, we include the endpoints in part (a), and we have $g(x) \leq 0$ for x values in $\boxed{(-\infty, -1] \cup [4, \infty)}$.

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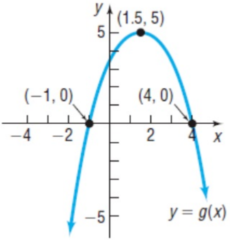

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Proof. The inequality $6x^2 < 6 + 5x$ is equivalent to the inequality $6x^2 - 5x - 6 < 0$. So we define the quadratic function $f(x) = 6x^2 - 5x - 6$. The graph of $y = f(x)$ opens up (since $a = 6 > 0$), so we look for the x-intercepts of f (if any) and then we must have f negative (strictly) between the intercepts.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(-6)}}{2(6)} = \frac{5 \pm \sqrt{25 + 144}}{12}$$

$$= \frac{5 \pm \sqrt{169}}{12} = \frac{5 \pm 13}{12},$$

so we have x-intercepts of $(5 - 13)/12 = -8/12 = -2/3$ and $(5 + 13)/12 = 18/12 = 3/2$. So $f(x) < 0$ and the original inequality is satisfied for x in $\boxed{(-2/3, 3/2)}$. □

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Page 160 Number 34. A ball is thrown vertically upward with an initial velocity of 96 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s(t) = 96t - 16t^2$. **(a)** At what time t will the ball strike the ground? **(b)** For what time t is the ball more than 128 feet above the ground?

Solution. **(a)** The ball strikes the ground when the distance from the ground s is 0, so we consider $s(t) = 96t - 16t^2 = 0$ or $16t(6 - t) = 0$. So we have the ball at ground level at $t = 0$ seconds (when it is first released) and at $\boxed{t = 6 \text{ seconds}}$.

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(b) We set the distance from the ground $s(t) = 96t - 16t^2$ greater than 128 to get $96t - 16t^2 > 128$ or $-16t^2 + 96t - 128 > 0$. The function $-16t^2 + 96t - 128$ is a quadratic function and its graph opens down (since $a = -16 < 0$). So this function is positive (that is, greater than 0) between its t-intercepts.

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Solution. So we set this function equal to 0 and consider $-16t^2 + 96t - 128 = 0$ or $-16(t^2 - 6t + 8) = 0$ or $-16(t - 2)(t - 4) = 0$. So this function is 0 when $t = 2$ seconds and $t = 4$ seconds. Hence, $-16t^2 + 96t - 128 > 0$ and the ball is more than 128 feet from the ground for time t in the interval $\boxed{(2, 4)}$. □