Precalculus 1 (Algebra)

Chapter 4. Polynomial and Rational Functions

4.1. Polynomial Functions and Models—Exercises, Examples, Proofs

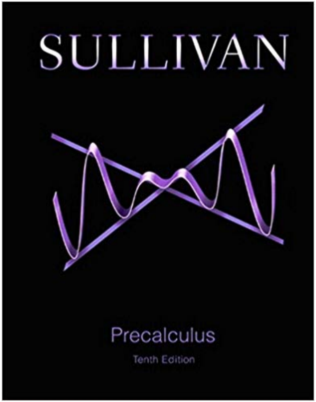

Table of contents

Page 184 Numbers 24 and 28

Page 184 Numbers 24 and 28. Determine which functions are polynomial functions. For those that are, state the degree. For those that are not, tell why not. Write each polynomial in standard form. Then identify the leading term and the constant term.
(24) $h(x) = \sqrt{x}(\sqrt{x} - 1)$, and **(28)** $G(x) = -3x^2(x + 2)^3$.

Solution. (24) We multiply out to get $h(x) = \sqrt{x}(\sqrt{x} - 1) = \sqrt{x}\sqrt{x} - \sqrt{x} = x - \sqrt{x}$. So h is $\boxed{\text{not a polynomial function}}$ since it is not of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ (the $\sqrt{x} = x^{1/2}$ term causes the problem). □

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Solution. We start with the graph of $y = x^4$ and replace x with $x - (-2) = x + 2$ to get $y = (x+2)^4$; this is a horizontal shift of $y = x^4$ to the left by 2 units. Next, we subtract 3 from the graph of $y = (x+2)^4 - 3$; this is a vertical shift down by 3 units of $y = (x+2)^4$. So the graph $y = (x+2)^4 - 3$ results from the graph of $y = x^4$ by (1) a horizontal shift to the left by 2 units, and then (2) a vertical shift down by 3 units. The graph is...

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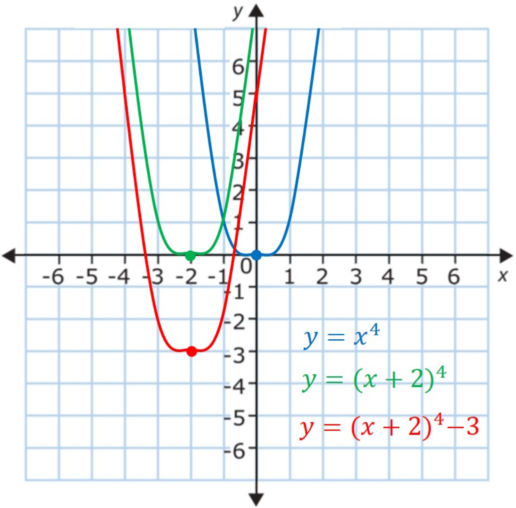

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Page 185 Number 46. Form a polynomial function whose real zeros are -4, 0, and 2, and whose degree is 3. Answers will vary depending on the choice of the leading coefficient.

Solution. Since we want polynomial function f to have roots -4, 0, and 2, then require that f has factors of $x - (-4) = x + 4$, $x - 0 = x$, and $x - 2$. So the product $(x + 4)x(x - 2) = x^3 + 2x^2 - 8x$ must be a factor of f. Since f is required to be degree 3, it can have no other factors involving x, and we can take f as $\boxed{f(x) = x^3 + 2x^2 - 8x}$. □

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Page 185 Number 52. Find the polynomial function of degree 3 with zeros -2, 0, and 2, and whose graph passes through the point $(-4, 16)$.

Solution. Since we want polynomial function f to have roots -2, 0, and 2, then we require that f has factors of $x - (-2) = x + 2$, $x - 0 = x$, and $x - 2$. So the product $(x + 2)x(x - 2) = x^3 - 4x$ must be a factor of f. Function f is required to be degree 3, so it can have no other factors involving x, and f must be a constant multiple of $x^3 - 4x$, say $f(x) = a(x^3 - 4x)$.

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Solution. Since we want polynomial function f to have roots -1 of multiplicity 2, 0, and 3 of multiplicity 2, then we require that f has factors of $(x-(-1))^2 = (x+1)^2$, $x - 0 = x$, and $(x-(3))^2 = (x-3)^2$. So the product $(x+1)^2 x(x-3)^2$ must be a factor of f. Function f is required to be degree 5, so it can have no other factors involving x, and f must be a constant multiple of $x(x+1)^2(x-3)^2$, say $f(x) = ax(x+1)^2(x-3)^2$.

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Example 4.1.A

Example 4.1.A. Consider $f(x) = 2x^2(x-4)$. **(a)** List each real zero and its multiplicity. **(b)** Determine whether the graph crosses or touches the x-axis at each x-intercept.

Solution. **(a)** Since f is already factored, we see that $\boxed{\text{0 is a zero of multiplicity 2}}$ (because of the factor $x^2 = (x-0)^2$) and $\boxed{\text{4 is a zero of multiplicity 1}}$ (because of the factor $(x-4)$).

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(b) To find the x-intercepts, we set $y = f(x) = 2x^2(x-4) = 0$ and find that $x = 0$ and $x = 4$ are the x-intercepts. We now perform a sign check of f by dividing the real line into intervals using the x intercepts. This results in the intervals $(-\infty, 0)$, $(0, 4)$, and $(4, \infty)$. We choose test values from the intervals and get the following:

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Solution (continued).

Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Test Value c	-1	1	5
Value of $f(c)$	$f(-1) = -10$	$f(1) = -6$	$f(5) = 50$
Location of Graph	Below x-axis	Below x-axis	Above x-axis
Point on Graph	$(-1, -10)$	$(1, -6)$	$(5, 50)$

Since the graph does not change signs at $x = 0$, then the $\boxed{\text{graph touches the } x\text{-axis at } x = 0}$. Since the graph does change sign at $x = 4$, then the $\boxed{\text{graph crosses the } x\text{-axis at } x = 4}$. □

Example 4.3.B

Example 4.3.B. Consider $f(x) = 2x^2(x-4)$. **(c)** Determine the maximum number of turning points on the graph. **(d)** Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

Solution. **(c)** Since $f(x) = 2x^2(x-4) = 2x^3 - 8x^2$ is a polynomial function of degree $n = 3$, then by Theorem 4.1.A, $\boxed{f \text{ has at most } n - 1 = 2 \text{ turning points}}$. □

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(d) Since $f(x) = 2x^3 - 8x^2$, then by Theorem 4.1.B, the end behavior of f is given by the graph of $\boxed{y = 2x^3}$. □

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(d) Since $f(x) = 2x^3 - 8x^2$, then by Theorem 4.1.B, the end behavior of f is given by the graph of $\boxed{y = 2x^3}$. □

Page 186 Number 74

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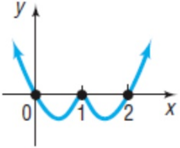

Solution. Since the function has x-intercepts, and hence zeros, at $x = 0$, $x = 1$, and $x = 2$, then the function f must have factors of $x - 0 = x$, $x - 1$, and $x - 2$. By Note 4.1.C, since the graph of f crosses the x-axis at $x = 0$ and $x = 2$, then the factors x and $x - 2$ must be of odd multiplicity. Similarly, since the graph of f touches the x-axis at $x = 1$, then the factor $x - 1$ must be of even multiplicity.

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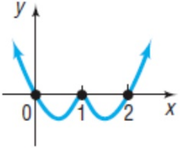

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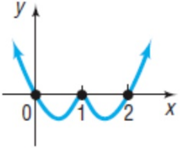

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Page 186 Number 74 (continued)

Solution (continued). NOTE: We have some liberty in choosing the exponents of each factor. For example, we could take $g(x) = x^3(x-1)^4(x-2)^3$ which has end behavior given by the graph of $y = x^{10}$ (also positive for $|x|$ large). We could also take any positive multiple of our f or g. So there are LOTS of possible answers!

We could have: $10x(x-1)^2(x-2)$, $10{,}000x(x-1)^2(x-2)$, $x^{15}(x-1)^{20}(x-2)^7$, $77x^{15}(x-1)^{20}(x-2)^7$, $10^{-5}x^{77}(x-1)^{2020}(x-2)^{755}$, $\ldots$

Page 186 Number 80

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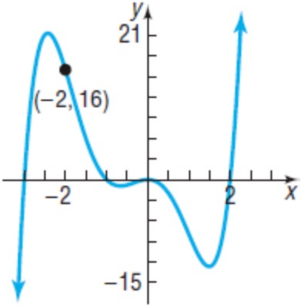

Solution. Since the function has x-intercepts, and hence zeros, at $x = -3$, $x = -1$, $x = 0$, and $x = 2$, then the function f must have factors of $x - (-3) = x + 3$, $x - (-1) = x + 1$, $x - 0 = x$, and $x - 2$.

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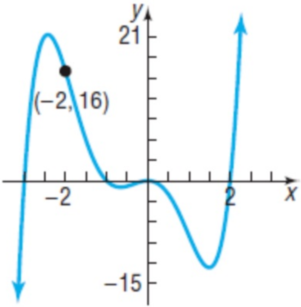

Solution. Since the function has x-intercepts, and hence zeros, at $x = -3$, $x = -1$, $x = 0$, and $x = 2$, then the function f must have factors of $x - (-3) = x + 3$, $x - (-1) = x + 1$, $x - 0 = x$, and $x - 2$.

Page 186 Number 80 (continued)

Solution (continued). Since the graph of f crosses the x-axis at $x = -3$, $x = -1$, and $x = 2$, then the factors $x + 3$, $x + 1$, and $x - 2$ must be of odd multiplicity by Note 4.1.C. Similarly, since the graph of f touches the x-axis at $x = 0$, then the factor x must be of even multiplicity. Consider $f(x) = a(x + 3)(x + 1)x^2(x - 2)$, where $a > 0$, which has end behavior given by the graph of $y = ax^5$, as desired (the given graph is negative for x large and negative, and positive for x large and positive).

Page 186 Number 80 (continued)

Solution (continued). Since the graph of f crosses the x-axis at $x = -3$, $x = -1$, and $x = 2$, then the factors $x + 3$, $x + 1$, and $x - 2$ must be of odd multiplicity by Note 4.1.C. Similarly, since the graph of f touches the x-axis at $x = 0$, then the factor x must be of even multiplicity. Consider $f(x) = a(x + 3)(x + 1)x^2(x - 2)$, where $a > 0$, which has end behavior given by the graph of $y = ax^5$, as desired (the given graph is negative for x large and negative, and positive for x large and positive). Since the graph of f passes through the point $(-2, 16)$, then we need $f(-2) = 16$ or $f(-2) = a((-2) + 3)((-2) + 1)(-2)^2((-2) - 2) = a(1)(-1)(4)(-4) = 16a = 16$, or $a = 1$. So we take $\boxed{f(x) = x^2(x + 3)(x + 1)(x - 2)}$. □

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NOTE: As in Exercise 74, we can find other solutions by varying the exponents of the factors (keeping the parity the same), but this will also require adjusting the leading coefficient. An example of function with a similar graph is $g(x) = (1/16)x^2(x + 3)^3(x + 1)^3(x - 2)^3$.

Page 186 Number 80 (continued)

Solution (continued). Since the graph of f crosses the x-axis at $x = -3$, $x = -1$, and $x = 2$, then the factors $x + 3$, $x + 1$, and $x - 2$ must be of odd multiplicity by Note 4.1.C. Similarly, since the graph of f touches the x-axis at $x = 0$, then the factor x must be of even multiplicity. Consider $f(x) = a(x + 3)(x + 1)x^2(x - 2)$, where $a > 0$, which has end behavior given by the graph of $y = ax^5$, as desired (the given graph is negative for x large and negative, and positive for x large and positive). Since the graph of f passes through the point $(-2, 16)$, then we need $f(-2) = 16$ or $f(-2) = a((-2) + 3)((-2) + 1)(-2)^2((-2) - 2) = a(1)(-1)(4)(-4) = 16a = 16$, or $a = 1$. So we take $\boxed{f(x) = x^2(x + 3)(x + 1)(x - 2)}$. □

NOTE: As in Exercise 74, we can find other solutions by varying the exponents of the factors (keeping the parity the same), but this will also require adjusting the leading coefficient. An example of function with a similar graph is $g(x) = (1/16)x^2(x + 3)^3(x + 1)^3(x - 2)^3$.

Page 186 Number 114

Page 186 Number 114. Analyze polynomial function $f(x) = -x^5 + 5x^4 + 4x^3 - 20x^2$ by following Steps 1 through 5.

Solution. Based on Note 4.1.D, we list the steps one at a time.
Step 1. Determine the end behavior of the graph of the function. By Theorem 4.1.B, the end behavior is given by the graph of $\boxed{y = -x^5}$. □

Page 186 Number 114

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Step 2. Find the x- and y-intercepts of the graph of the function. First, notice that f factors as
$f(x) = -x^5 + 5x^4 + 4x^3 - 20x^2 = -x^2(x^3 - 5x^2 - 4x + 20) = -x^2(x^2(x - 5) - 4(x - 5)) = -x^2(x^2 - 4)(x - 5) = -x^2(x - 2)(x + 2)(x - 5) = 0$.
For the x-intercepts, we set $y = f(x) = 0$ and consider $-x^2(x - 2)(x + 2)(x - 5) = 0$. So the $\boxed{x\text{-intercepts are } x = -2,\ x = 0,\ x = 2, \text{ and } x = 5}$. For the y-intercepts, we set $x = 0$ and consider $f(0) = -(0)^2((0) - 2)((0) + 2)((0) - 5) = 0$. So the $\boxed{y\text{-intercept is } 0}$. □

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Page 186 Number 114 (continued 1)

Solution (continued). Step 3. Determine the zeros of the function and their multiplicity. We have $f(x) = -x^2(x-2)(x+2)(x-5)$, so the zeros of f are $\boxed{\text{0 of multiplicity 2, and } -2\text{, 2, and 5 each of multiplicity 1}}$. □

Step 4. Determine the maximum number of turning points on the graph of the function. Since f is a polynomial function of degree $n = 5$, by Theorem 4.1.A $\boxed{f \text{ has at most } n-1=4 \text{ turning points}}$. □

Page 186 Number 114 (continued 1)

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Step 5. Use the information in Steps 1 through 4 to draw a complete graph of the function. We plot a few points to establish the y-axis scale. With $x = -3$ we have $f(-3) = -(9)(-5)(-1)(-8) = 360$, so that $(-3, 360)$ is on the graph of f. With $x = -1$ we have $f(-1) = -(1)(-3)(1)(-6) = -18$, so that $(-1, -18)$ is on the graph of f. With $x = 1$ we have $f(1) = -(1)(-1)(3)(-4) = -12$, so that $(1, -12)$ is on the graph of f. With $x = 3$ we have $f(3) = -(9)(1)(5)(-2) = 90$, so that $(3, 90)$ is on the graph of f.

Page 186 Number 114 (continued 1)

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Page 186 Number 114 (continued 2)

Solution (continued). Graphing the intercepts, known points, and using the facts that 0 is a zero of multiplicity 2 (so the graph touches the x-axis at $x = 0$) and that -2, 2, and 5 are zeros of multiplicity 1 (so the graph crosses the x-axis at $x = -2$, $x = 2$, and $x = 5$) we get: