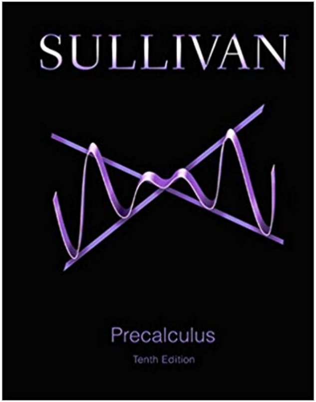
Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.2. One-to-One Functions; Inverse Functions—Exercises, Examples, Proofs



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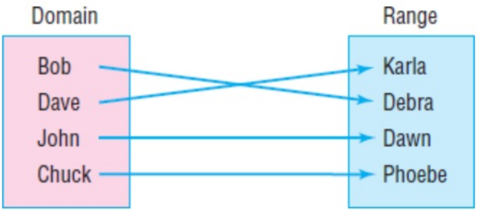
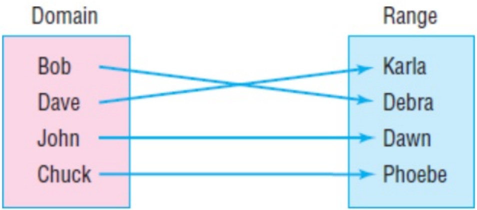
Page 265 Number 14. Determine whether the function is one-to-one:



Solution. A function is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. The arrows indicate function outputs, and since no two arrows converge on the same element of the range, then the function is one to one. □

Page 265 Number 14

Page 265 Number 14. Determine whether the function is one-to-one:



Solution. A function is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. The arrows indicate function outputs, and since no two arrows converge on the same element of the range, then the $\boxed{\text{function is one to one}}$. □

page 266 Number 16

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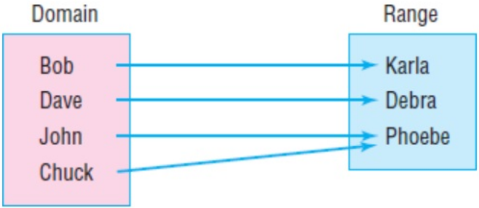

Solution. A function is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. The arrows indicate function outputs, and since two arrows converge on the same element of the range, namely the function maps both John and Chuck to Phoebe, then the **function is not one to one**. □

page 266 Number 16

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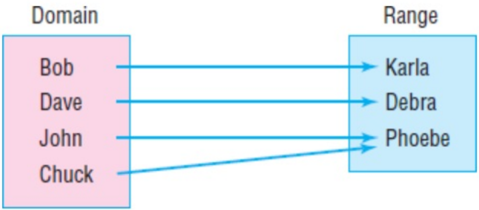
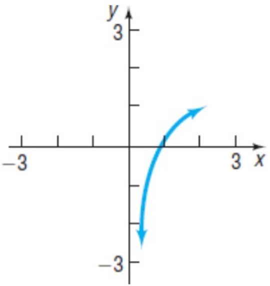


Solution. A function is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. The arrows indicate function outputs, and since two arrows converge on the same element of the range, namely the function maps both John and Chuck to Phoebe, then the **function is not one to one**. □

Page 266 Number 22

Page 266 Number 22. Consider the given graph of a function f. Use the horizontal-line test to determine whether f is one-to-one.

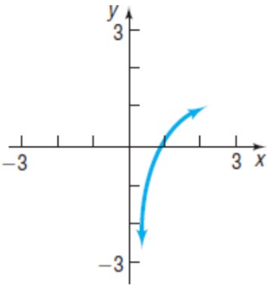


Solution. A horizontal line will intersect the graph (assuming it is extended according to the pattern we see) in exactly one point. So by Theorem 5.2.A, Horizontal Line Test,

the graph is that of a one-to-one function.

Page 266 Number 22

Page 266 Number 22. Consider the given graph of a function f. Use the horizontal-line test to determine whether f is one-to-one.



Solution. A horizontal line will intersect the graph (assuming it is extended according to the pattern we see) in exactly one point. So by Theorem 5.2.A, Horizontal Line Test,

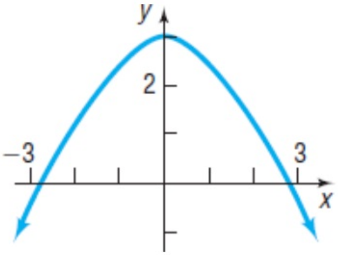
the graph is that of a one-to-one function

.

□

Page 266 Number 24

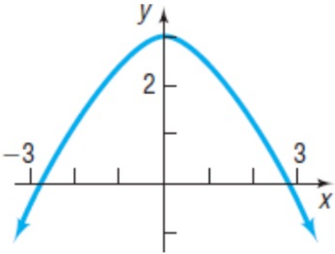
Page 266 Number 24. Consider the given graph of a function f. Use the horizontal-line test to determine whether f is one-to-one.



Solution. A horizontal line will intersect the graph in two points, provided the line is of the form $y = k$ where $k < 3$ (assuming it is extended according to the pattern we see). So by Theorem 5.2.A, Horizontal Line Test, the graph is that of a function that is NOT one-to-one. □

Page 266 Number 24

Page 266 Number 24. Consider the given graph of a function f. Use the horizontal-line test to determine whether f is one-to-one.



Solution. A horizontal line will intersect the graph in two points, provided the line is of the form $y = k$ where $k < 3$ (assuming it is extended according to the pattern we see). So by Theorem 5.2.A, Horizontal Line Test, $\boxed{\text{the graph is that of a function that is NOT one-to-one}}$. □

Page 266 Number 30

Page 266 Number 30. Find the inverse of the one-to-one function. State the domain and the range of the inverse function.

Source: United States Bureau of Labor Statistics, March 2014

Solution. Notice that the function represented here has the domain $\{\text{Virgina}, \text{Nevada}, \text{Tennessee}, \text{Texas}\}$ and range $\{5.0\%, 5.5\%, 6.7\%, 8.5\%\}$.

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Page 266 Number 30 (continued)

Source: United States Bureau of Labor Statistics, March 2014

Solution (continued). So the inverse function has

domain $\{5.0\%, 5.5\%, 6.7\%, 8.5\%\}$ and

range $\{\text{Virginia}, \text{Nevada}, \text{Tennessee}, \text{Texas}\}$. The inverse function is given by the set of ordered pairs

$\{(5.0\%, \text{Virginia}), (5.5\%, \text{Texas}), (6.7\%, \text{Tennessee}), (8.5\%, \text{Nevada})\}$. □

Page 266 Number 30 (continued)

Source: United States Bureau of Labor Statistics, March 2014

Solution (continued). So the inverse function has $\boxed{\text{domain } \{5.0\%, 5.5\%, 6.7\%, 8.5\%\}}$ and $\boxed{\text{range } \{\text{Virginia}, \text{Nevada}, \text{Tennessee}, \text{Texas}\}}$. The inverse function is given by the set of ordered pairs

$$\boxed{\{(5.0\%, \text{Virginia}), (5.5\%, \text{Texas}), (6.7\%, \text{Tennessee}), (8.5\%, \text{Nevada})\}}. \quad \square$$

Page 266 Number 32

Page 266 Number 32. Find the inverse of the one-to-one function:

$$\{(-2,2),(-1,6),(0,8),(1,-3),(2,9)\}.$$

State the domain and the range of the inverse function.

Solution. Notice that the function represented here has the domain $\{-2,-1,0,1,2\}$ and range $\{-3,2,6,8,9\}$. So the inverse function has $\boxed{\text{domain } \{-3,2,6,8,9\}}$ and $\boxed{\text{range } \{-2,-1,0,1,2\}}$.

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Page 266 Number 40

Page 266 Number 40. Consider $f(x) = (x-2)^2$ for $x \geq 2$ and $g(x) = \sqrt{x} + 2$. Verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g.

Solution. We have $f(g(x)) = f(\sqrt{x} + 2)$ where $x \geq 0$, or $((\sqrt{x}+2)-2)^2 = (\sqrt{x})^2 = x$ where $x \geq 0$. Also, $g(f(x)) = g((x-2)^2)$ for $x \geq 2$, or $\sqrt{(x-2)^2} + 2 = |x-2| + 2 = (x-2) + 2$ since $x \geq 2$, or $g(f(x)) = x$ for $x \geq 2$.

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Solution. We have $f(g(x)) = f(\sqrt{x} + 2)$ where $x \geq 0$, or $((\sqrt{x} + 2) - 2)^2 = (\sqrt{x})^2 = x$ where $x \geq 0$. Also, $g(f(x)) = g((x-2)^2)$ for $x \geq 2$, or $\sqrt{(x-2)^2} + 2 = |x-2| + 2 = (x-2) + 2$ since $x \geq 2$, or $g(f(x)) = x$ for $x \geq 2$. That is, $f(g(x)) = x$ for all $x \geq 0$ (i.e., for all x in the domain of g) and $g(f(x)) = x$ for all $x \geq 2$ (i.e., for all x in the domain of f). So f and g are inverses of each other, as claimed. □

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Solution. We have $f(g(x)) = f(\sqrt{x}+2)$ where $x \geq 0$, or $((\sqrt{x}+2)-2)^2 = (\sqrt{x})^2 = x$ where $x \geq 0$. Also, $g(f(x)) = g((x-2)^2)$ for $x \geq 2$, or $\sqrt{(x-2)^2} + 2 = |x-2| + 2 = (x-2) + 2$ since $x \geq 2$, or $g(f(x)) = x$ for $x \geq 2$. That is, $f(g(x)) = x$ for all $x \geq 0$ (i.e., for all x in the domain of g) and $g(f(x)) = x$ for all $x \geq 2$ (i.e., for all x in the domain of f). So f and g are inverses of each other, as claimed. □

Page 267 Number 44

Page 267 Number 44. Consider $f(x) = (x-5)/(2x+3)$ and $g(x) = (3x+5)/(1-2x)$. Verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g.

Solution. First, notice that the domain of f is $(-\infty, -3/2) \cup (-3/2, \infty)$ and the domain of g is $(-\infty, 1/2) \cup (1/2, \infty)$.

Page 267 Number 44

Page 267 Number 44. Consider $f(x) = (x-5)/(2x+3)$ and $g(x) = (3x+5)/(1-2x)$. Verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g.

Solution. First, notice that the domain of f is $(-\infty, -3/2) \cup (-3/2, \infty)$ and the domain of g is $(-\infty, 1/2) \cup (1/2, \infty)$. We have

$$f(g(x)) = f((3x+5)/(1-2x)) = \frac{(3x+5)/(1-2x)-5}{2(3x+5)/(1-2x)+3}$$

$$= \frac{(3x+5)/(1-2x)-5}{2(3x+5)/(1-2x)+3}\left(\frac{1-2x}{1-2x}\right) = \frac{(3x+5)-5(1-2x)}{2(3x+5)+3(1-2x)} \text{ for } x \neq 1/2$$

$$= \frac{3x+5-5+10x}{6x+10+3-6x} \text{ for } x \neq 1/2$$

$$= \frac{13x}{13} \text{ for } x \neq 1/2$$

$$= x \text{ for } x \neq 1/2.$$

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Solution. First, notice that the domain of f is $(-\infty, -3/2) \cup (-3/2, \infty)$ and the domain of g is $(-\infty, 1/2) \cup (1/2, \infty)$. We have

$$f(g(x)) = f((3x + 5)/(1 - 2x)) = \frac{(3x + 5)/(1 - 2x) - 5}{2(3x + 5)/(1 - 2x) + 3}$$

$$= \frac{(3x + 5)/(1 - 2x) - 5}{2(3x + 5)/(1 - 2x) + 3}\left(\frac{1 - 2x}{1 - 2x}\right) = \frac{(3x + 5) - 5(1 - 2x)}{2(3x + 5) + 3(1 - 2x)} \text{ for } x \neq 1/2$$

$$= \frac{3x + 5 - 5 + 10x}{6x + 10 + 3 - 6x} \text{ for } x \neq 1/2$$

$$= \frac{13x}{13} \text{ for } x \neq 1/2$$

$$= x \text{ for } x \neq 1/2.$$

Page 267 Number 44 (continued)

Solution (continued). We have

$$g(f(x)) = g((x-5)/(2x+3)) = \frac{3(x-5)/(2x+3)+5}{1-2(x-5)/(2x+3)}$$

$$= \frac{3(x-5)/(2x+3)+5}{1-2(x-5)/(2x+3)}\left(\frac{2x+3}{2x+3}\right) = \frac{3(x-5)+5(2x+3)}{(2x+3)-2(x-5)} \text{ for } x \neq -3/2$$

$$= \frac{3x-15+10x+15}{2x+3-2x+10} \text{ for } x \neq -3/2$$

$$= \frac{13x}{13} \text{ for } x \neq -3/2$$

$$= x \text{ for } x \neq -3/2.$$

That is, $f(g(x)) = x$ for all $x \neq 1/2$ (i.e., for all x in the domain of g) and $g(f(x)) = x$ for all $x \neq -3/2$ (i.e., for all x in the domain of f). So f and g are inverses of each other, as claimed. □

Page 267 Number 44 (continued)

Solution (continued). We have

$$g(f(x)) = g((x-5)/(2x+3)) = \frac{3(x-5)/(2x+3)+5}{1-2(x-5)/(2x+3)}$$

$$= \frac{3(x-5)/(2x+3)+5}{1-2(x-5)/(2x+3)}\left(\frac{2x+3}{2x+3}\right) = \frac{3(x-5)+5(2x+3)}{(2x+3)-2(x-5)} \text{ for } x \neq -3/2$$

$$= \frac{3x-15+10x+15}{2x+3-2x+10} \text{ for } x \neq -3/2$$

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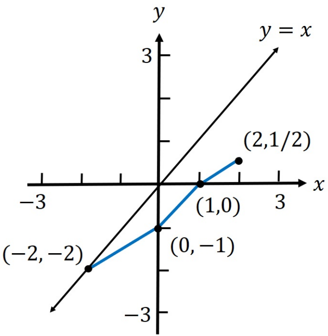
$$= x \text{ for } x \neq -3/2.$$

That is, $f(g(x)) = x$ for all $x \neq 1/2$ (i.e., for all x in the domain of g) and $g(f(x)) = x$ for all $x \neq -3/2$ (i.e., for all x in the domain of f). So f and g are inverses of each other, as claimed. □

Page 267 Number 46

Page 267 Number 46. Consider the graph of the one-to-one function f given below. Draw the graph of the inverse function f^{-1}.



Solution. To get points on the graph of f^{-1}, we simply take the given points and interchange the x- and y-coordinates. So f^{-1} includes the points $(-2,-2)$, $(-1,0)$, $(0,1)$, and $(1/2,2)$.

Page 267 Number 46

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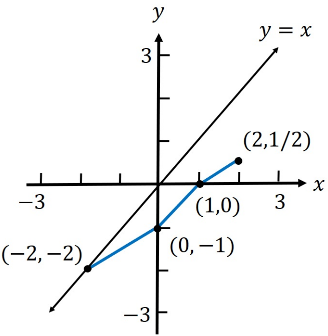
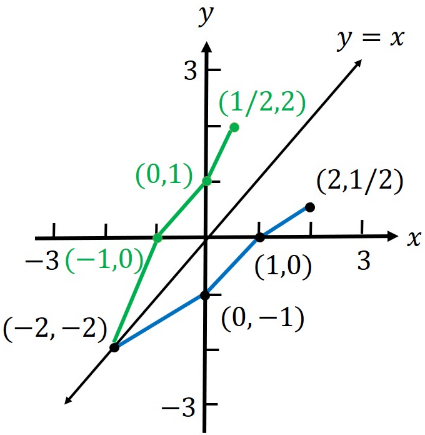


Solution. To get points on the graph of f^{-1}, we simply take the given points and interchange the x- and y-coordinates. So f^{-1} includes the points $(-2,-2)$, $(-1,0)$, $(0,1)$, and $(1/2,2)$.

Page 267 Number 46 (continued)

Solution (continued). So the graph of f^{-1} as as given here in green:



Page 267 Number 56

Page 267 Number 56. Consider the one-to-one function $f(x) = x^3 + 1$. **(a)** Find its inverse function f^{-1} and check your answer. **(b)** Find the domain and the range of f and f^{-1}. **(c)** Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

Solution. **(a)** We follow the 3 steps. Step 1 first gives $y = x^3 + 1$, and then gives (interchanging x and y) $x = y^3 + 1$. In Step 2, we solve for y to get: $y^3 = x - 1$, or $\sqrt[3]{y^3} = \sqrt[3]{x-1}$, or $y = \sqrt[3]{x-1}$. Hence $\boxed{f^{-1}(x) = \sqrt[3]{x-1}}$. Notice that the domains of both f and f^{-1} are $\mathbb{R} = (-\infty, \infty)$.

Page 267 Number 56

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Solution. (a) We follow the 3 steps. Step 1 first gives $y = x^3 + 1$, and then gives (interchanging x and y) $x = y^3 + 1$. In Step 2, we solve for y to get: $y^3 = x - 1$, or $\sqrt[3]{y^3} = \sqrt[3]{x-1}$, or $y = \sqrt[3]{x-1}$. Hence $\boxed{f^{-1}(x) = \sqrt[3]{x-1}}$. Notice that the domains of both f and f^{-1} are $\mathbb{R} = (-\infty, \infty)$. To check, consider $f(f^{-1}(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = (x-1) + 1 = x$ for all x in $\mathbb{R}$. Next, consider $f^{-1}(f(x)) = f^{-1}(x^3+1) = \sqrt[3]{(x^3+1) - 1} = \sqrt[3]{x^3} = x$ for all x in $\mathbb{R}$. So f and f^{-1} actually are inverse functions.

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Page 267 Number 56. Consider the one-to-one function $f(x) = x^3 + 1$. **(a)** Find its inverse function f^{-1} and check your answer. **(b)** Find the domain and the range of f and f^{-1}. **(c)** Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

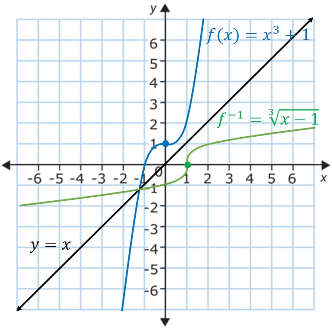
Solution. **(a)** We follow the 3 steps. Step 1 first gives $y = x^3 + 1$, and then gives (interchanging x and y) $x = y^3 + 1$. In Step 2, we solve for y to get: $y^3 = x - 1$, or $\sqrt[3]{y^3} = \sqrt[3]{x-1}$, or $y = \sqrt[3]{x-1}$. Hence $\boxed{f^{-1}(x) = \sqrt[3]{x-1}}$. Notice that the domains of both f and f^{-1} are $\mathbb{R} = (-\infty, \infty)$. To check, consider $f(f^{-1}(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = (x-1) + 1 = x$ for all x in $\mathbb{R}$. Next, consider $f^{-1}(f(x)) = f^{-1}(x^3 + 1) = \sqrt[3]{(x^3+1) - 1} = \sqrt[3]{x^3} = x$ for all x in $\mathbb{R}$. So f and f^{-1} actually are inverse functions.

Page 267 Number 56 (continued)

Solution (continued). (b) We have $f(x) = x^3 + 1$ and $f^{-1}(x) = \sqrt[3]{x-1}$. Since the $\boxed{\text{domain of } f \text{ is } \mathbb{R}}$ then the $\boxed{\text{range of } f^{-1} \text{ is } \mathbb{R}}$. Since the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R}}$, then the $\boxed{\text{range of } f \text{ is } \mathbb{R}}$. **(c)** The graphs are related to translations of the library of functions members $y = x^3$ and $y = \sqrt[3]{x}$:

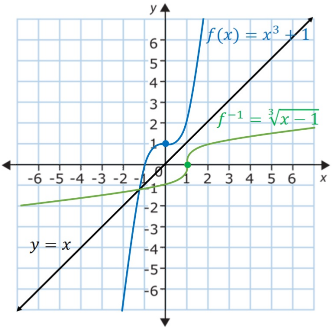
Page 267 Number 56 (continued)

Solution (continued). (b) We have $f(x) = x^3 + 1$ and $f^{-1}(x) = \sqrt[3]{x-1}$. Since the $\boxed{\text{domain of } f \text{ is } \mathbb{R}}$ then the $\boxed{\text{range of } f^{-1} \text{ is } \mathbb{R}}$. Since the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R}}$, then the $\boxed{\text{range of } f \text{ is } \mathbb{R}}$. **(c)** The graphs are related to translations of the library of functions members $y = x^3$ and $y = \sqrt[3]{x}$:



Page 267 Number 56 (continued)

Solution (continued). (b) We have $f(x) = x^3 + 1$ and $f^{-1}(x) = \sqrt[3]{x-1}$. Since the $\boxed{\text{domain of } f \text{ is } \mathbb{R}}$ then the $\boxed{\text{range of } f^{-1} \text{ is } \mathbb{R}}$. Since the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R}}$, then the $\boxed{\text{range of } f \text{ is } \mathbb{R}}$. **(c)** The graphs are related to translations of the library of functions members $y = x^3$ and $y = \sqrt[3]{x}$:



□

Page 267 Number 58

Page 267 Number 58. Consider the one-to-one function $f(x) = x^2 + 9$ where $x \geq 0$. **(a)** Find its inverse function f^{-1} and check your answer. **(b)** Find the domain and the range of f and f^{-1}. **(c)** Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

Solution. (a) We follow the 3 steps. Step 1 first gives $y = x^2 + 9$ where $x \geq 0$, and then gives (interchanging x and y) $x = y^2 + 9$ where $y \geq 0$. In Step 2, we solve for y to get: $y^2 = x - 9$ where $y \geq 0$, or $\sqrt{y^2} = \sqrt{x - 9}$ where $y \geq 0$, or $|y| = \sqrt{x - 9}$ where $y \geq 0$, or $y = \sqrt{x - 9}$ (since $y \geq 0$). Hence $\boxed{f^{-1}(x) = \sqrt{x - 9}}$. Notice that the domain of f^{-1} is $x \geq 9$ or x in $[9, \infty)$.

Page 267 Number 58

Page 267 Number 58. Consider the one-to-one function $f(x) = x^2 + 9$ where $x \geq 0$. **(a)** Find its inverse function f^{-1} and check your answer. **(b)** Find the domain and the range of f and f^{-1}. **(c)** Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

Solution. **(a)** We follow the 3 steps. Step 1 first gives $y = x^2 + 9$ where $x \geq 0$, and then gives (interchanging x and y) $x = y^2 + 9$ where $y \geq 0$. In Step 2, we solve for y to get: $y^2 = x - 9$ where $y \geq 0$, or $\sqrt{y^2} = \sqrt{x-9}$ where $y \geq 0$, or $|y| = \sqrt{x-9}$ where $y \geq 0$, or $y = \sqrt{x-9}$ (since $y \geq 0$). Hence $\boxed{f^{-1}(x) = \sqrt{x-9}}$. Notice that the domain of f^{-1} is $x \geq 9$ or x in $[9, \infty)$. To check, consider
$f(f^{-1}(x)) = f(\sqrt{x-9}) = (\sqrt{x-9})^2 + 9 = (x-9) + 9 = x$ for all $x \geq 9$ (i.e., for all x in the domain of f^{-1}). Next, consider
$f^{-1}(f(x)) = f^{-1}(x^2+9) = \sqrt{(x^2+9) - 9} = \sqrt{x^2} = |x| = x$ for all $x \geq 0$ (i.e., for all x in the domain of f). So f and f^{-1} actually are inverse functions.

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Solution (continued). (b) We have $f(x) = x^2 + 9$ where $x \geq 0$ and $f^{-1}(x) = \sqrt{x - 9}$. Since the $\boxed{\text{domain of } f \text{ is } [0, \infty)}$ (as given) then the $\boxed{\text{range of } f^{-1} \text{ is } [0, \infty)}$. Since the $\boxed{\text{domain of } f^{-1} \text{ is } [9, \infty)}$ (as observed above), then the $\boxed{\text{range of } f \text{ is } [9, \infty)}$. **(c)** The graphs are related to translations of the library of functions members $y = x^2$ and $y = \sqrt{x}$:

Page 267 Number 58 (continued)

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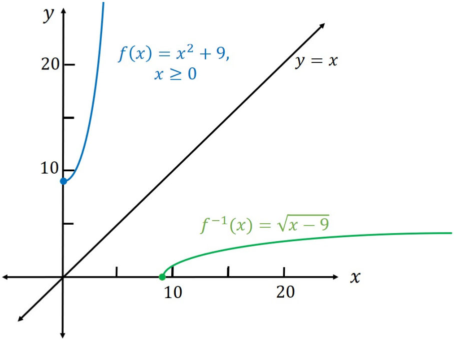

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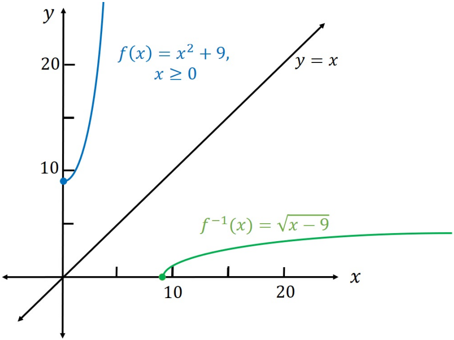

□

Page 268 Number 94

Page 268 Number 94. The function $F(C) = \frac{9}{5}C + 32$ converts a temperature from C degrees Celsius to F degrees Fahrenheit. **(a)** Express the temperature in degrees Celsius C as a function of the temperature in degrees F. **(b)** Verify that $C = C(F)$ is the inverse of $F = F(C)$ by showing that $C(F(C)) = C$ and $F(C(F)) = F$. **(c)** What is the temperature in degrees Celsius if it is 70 degrees Fahrenheit?

Solution. **(a)** We solve for C in the equation $F = \frac{9}{5}C + 32$. We have $\frac{9}{5}C = F - 32$, or $C = \frac{5}{9}(F - 32)$, or $\boxed{C(F) = \frac{5}{9}(F - 32)}$. □

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(b) Notice that the domains of both $F(C)$ and $C(F)$ are $\mathbb{R}$ (since there is an "absolute zero," not all of the domain makes physical sense, but it makes algebraic sense). We have
$C(F(C)) = C\left(\frac{9}{5}C + 32\right) = \frac{5}{9}\left(\left(\frac{9}{5}C + 32\right) - 32\right) = \frac{5}{9}\left(\frac{9}{5}C\right) = C$ for all C in $\mathbb{R}$, and
$F(C(F)) = F\left(\frac{5}{9}(F - 32)\right) = \frac{9}{5}\left(\frac{5}{9}(F - 32)\right) + 32 = (F - 32) + 32 = F$ for all F in $\mathbb{R}$. So $F(C)$ and $C(F)$ actually are inverse functions. □

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Solution. **(a)** We solve for C in the equation $F = \frac{9}{5}C + 32$. We have $\frac{9}{5}C = F - 32$, or $C = \frac{5}{9}(F - 32)$, or $\boxed{C(F) = \frac{5}{9}(F - 32)}$. □

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$C(F(C)) = C\left(\frac{9}{5}C + 32\right) = \frac{5}{9}\left(\left(\frac{9}{5}C + 32\right) - 32\right) = \frac{5}{9}\left(\frac{9}{5}C\right) = C$ for all C in $\mathbb{R}$, and
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Page 268 Number 94 (continued)

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Solution (continued). **(c)** If $F = 70$ then

$C(70) = \frac{5}{9}((70) - 32) = \frac{5}{9}(38) = \frac{190}{9} = \boxed{21\frac{1}{9} \text{ degrees Celsius}}$. □

NOTE. Since $C(F)$ and $F(C)$ are linear functions, then (since their slopes are different) they intersect at one point. We can set $C = F(C)$ to get $C = \frac{9}{5}C + 32$, or $\frac{9}{5}C - C = -32$, or $\frac{4}{5}C = -32$ or $C = \frac{5}{4}(-32) = -40$. So -40 degrees Fahrenheit is the same as -40 degrees Celsius. □

Page 268 Number 94 (continued)

Page 268 Number 94. The function $F(C) = \frac{9}{5}C + 32$ converts a temperature from C degrees Celsius to F degrees Fahrenheit. **(a)** Express the temperature in degrees Celsius C as a function of the temperature in degrees F. **(b)** Verify that $C = C(F)$ is the inverse of $F = F(C)$ by showing that $C(F(C)) = C$ and $F(C(F)) = F$. **(c)** What is the temperature in degrees Celsius if it is 70 degrees Fahrenheit?

Solution (continued). **(c)** If $F = 70$ then $C(70) = \frac{5}{9}((70) - 32) = \frac{5}{9}(38) = \frac{190}{9} = \boxed{21\frac{1}{9} \text{ degrees Celsius}}$. □

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