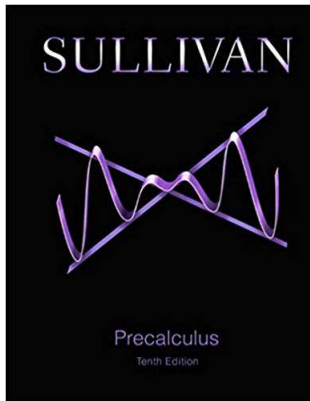


## Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions  
5.3. Exponential Functions—Exercises, Examples, Proofs

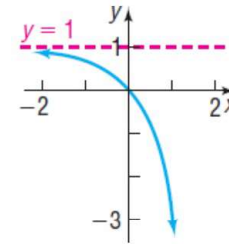
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## Page 281 Number 38

**Page 281 Number 38.** Match the graph with one of the following functions: **A.**  $y = 3^x$ , **B.**  $y = 3^{2x}$ , **C.**  $y = 23^x$ , **D.**  $y = 23^{2x}$ , **E.**  $y = 3^x - 1$ , **F.**  $y = 3^{x-1}$ , **G.**  $y = 3^{1-x}$ , **H.**  $y = 1 - 3^x$ .



**Solution.** Choices A, B (where  $y = 3^{2x} = (3^2)^x = 9^x$ ), C, and D (where  $y = 23^{2x} = (23^2)^x = 529^x$ ) are simply exponential functions with base  $a$  where  $a > 1$ . Since such a function is increasing, then these do not match the graph.

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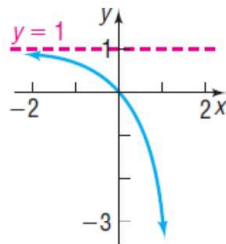
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Page 281 Number 38

## Page 281 Number 38 (continued 1)

**Page 281 Number 38.** Match the graph with one of the following functions: **A.**  $y = 3^x$ , **B.**  $y = 3^{2x}$ , **C.**  $y = 23^x$ , **D.**  $y = 23^{2x}$ , **E.**  $y = 3^x - 1$ , **F.**  $y = 3^{x-1}$ , **G.**  $y = 3^{1-x}$ , **H.**  $y = 1 - 3^x$ .



**Solution (continued).** Choice E is a vertical shift down by 1 unit of the graph of  $y = 3^x$ , which yields an increasing function and this does not match the graph. Choice F is a horizontal shift to the right by 1 unit of the graph of  $y = 3^x$ , which yields an increasing function and this does not match the graph.

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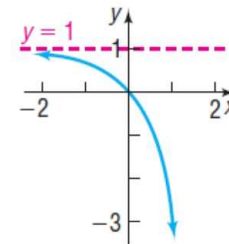
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Page 281 Number 38

## Page 281 Number 38 (continued 2)

**Page 281 Number 38.** Match the graph with one of the following functions: **A.**  $y = 3^x$ , **B.**  $y = 3^{2x}$ , **C.**  $y = 23^x$ , **D.**  $y = 23^{2x}$ , **E.**  $y = 3^x - 1$ , **F.**  $y = 3^{x-1}$ , **G.**  $y = 3^{1-x}$ , **H.**  $y = 1 - 3^x$ .



**Solution (continued).** Choice G is a function which is always positive, so this does not match the graph. Choice H must match the graph.

**Note.**  $y = 1 - 3^x$  results from the graph of  $y = 3^x$  by (1) a reflection about the  $x$  axis (where  $y = f(x)$  is replaced with  $y = -f(x)$ ), and (2) a vertical shift up by 1 unit (where  $y = g(x)$  is replaced with  $y = g(x) + 1$ ).

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## Page 281 Number 52

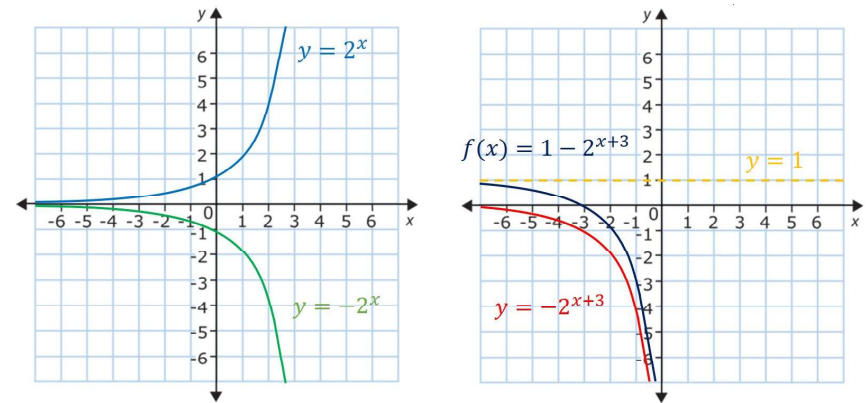
**Page 281 Number 52.** Use transformations to graph the function  $f(x) = 1 - 2^{x+3}$ . Determine the domain, range, and horizontal asymptote.

**Solution.** We start with the graph of  $y = 2^x$ . First, we replace  $y = f(x)$  with  $y = -f(x)$  to get  $y = -2^x$ , which is a reflection about the  $x$ -axis of  $y = 2^x$ . Second, we replace  $x$  with  $x + 3 = x - (-3)$  in  $y = -2^x$  to get  $y = -2^{x+3}$  which is a shift to the left by 3 units of  $y = -2^x$ . Third, we add 1 to  $-2^{x+3}$  to get  $y = 1 - 2^{x+3}$  which is a vertical shift up by 1 unit of  $y = -2^{x+3}$ . So we start with  $y = 2^x$  and (1) reflect about the  $x$ -axis, (2) shift to the left by 3 units, and (3) shift up 1 unit to produce the graph of  $f(x) = 1 - 2^{x+3}$ .

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## Page 281 Number 52 (continued)

**Solution.** We have the following graph:



We see from the graph that the domain of  $f$  is  $\mathbb{R}$ , the range is  $(-\infty, 1)$ , and  $y = 1$  is a horizontal asymptote. □

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## Page 282 Number 66

**Page 282 Number 66.** Solve the equation  $3^{-x} = 81$ .

**Solution.** Since  $81 = 9^2 = (3^2)^2 = 3^4$ , then the equation becomes  $3^{-x} = 3^4$ . By the previous note, since  $f(x) = 3^x$  is one-to-one, then  $-x = 4$  or  $x = -4$ . So the solution set is  $\{-4\}$ . □

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## Page 282 Number 82

**Page 282 Number 82.** Solve the equation  $(e^4)^x e^{x^2} = e^{12}$ .

**Solution.** By the properties of exponents (given in Theorem 5.3.A), we have  $(e^4)^x e^{x^2} = e^{4x} e^{x^2} = e^{4x+x^2}$ , so the equation becomes  $e^{4x+x^2} = e^{12}$ . By the previous note, since  $f(x) = e^x$  is one-to-one, then we need  $4x + x^2 = 12$ , or  $x^2 + 4x - 12 = 0$ , or  $(x + 6)(x - 2) = 0$ . So the solution set is  $\{-6, 2\}$ . □

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## Page 282 Number 84

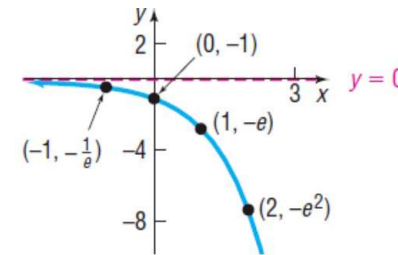
**Page 282 Number 84.** If  $2^x = 3$ , what does  $4^{-2x}$  equal?

**Solution.** Since we are given  $2^x = 3$ , then  $(2^x)^2 = 3^2$ , or  $2^{2x} = 9$ , or  $(2^2)^x = 9$ , or  $4^x = 9$ . So  $4^{-2x} = (4^x)^{-2} = (9)^{-2} = 1/9^2 = \boxed{1/81}$ .  $\square$

**Note.** We solved this problem without actually finding  $x$ ; we don't yet have a technique to solve for  $x$ , but soon will when we cover 5.5. Properties of Logarithms (see Change-of-Base Formula, Theorem 5.5.C).

## Page 282 Number 92

**Page 282 Number 92.** Determine the exponential function with this graph:



**Solution.** We look for a function of the form  $f(x) = Ca^x$ . Since the graph passes through the point  $(0, -1)$  then we must have  $f(0) = Ca^0 = (C)(1) = C = -1$ , so that  $C = -1$ . Since the graph passes through the point  $(1, -e)$  then we must have  $f(1) = Ca^1 = Ca = (-1)a = -a = -e$ , so that  $a = e$ . So the desired function must be  $\boxed{f(x) = Ca^x = -e^x}$ .  $\square$

## Page 283 Number 106

**Page 283 Number 106.** The atmospheric pressure  $p$  on a balloon or airplane decreases with increasing height. This pressure, measured in millimeters of mercury, is related to the height  $h$  (in kilometers) above sea level by the function  $p(h) = 760e^{-0.145h}$ . **(a)** Find the atmospheric pressure at a height of 2 km (over a mile). **(b)** What is it at a height of 10 kilometers (over 30,000 feet)?

**Solution. (a)** When  $h = 2$  km, we have the pressure  $p(2) = 760e^{-0.145(2)} = \boxed{760e^{-0.290} \approx 568.680 \text{ mm}}$ .  $\square$

**(b)** When  $h = 10$  km, we have the pressure  $p(10) = 760e^{-0.145(10)} = \boxed{760e^{-1.450} \approx 178.273 \text{ mm}}$ .  $\square$