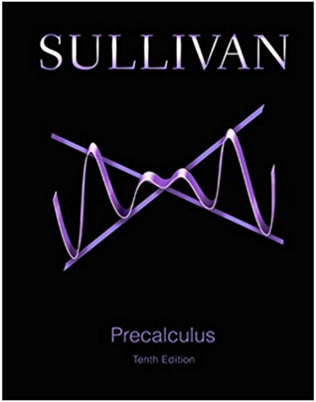
Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.3. Exponential Functions—Exercises, Examples, Proofs



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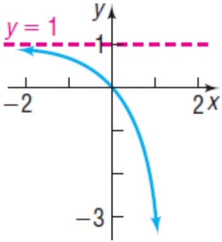
Page 281 Number 38. Match the graph with one of the following functions: **A.** $y = 3^x$, **B.** $y = 3^{2x}$, **C.** $y = 23^x$, **D.** $y = 23^{2x}$, **E.** $y = 3^x - 1$, **F.** $y = 3^{x-1}$, **G.** $y = 3^{1-x}$, **H.** $y = 1 - 3^x$.



Solution. Choices A, B (where $y = 3^{2x} = (3^2)^x = 9^x$), C, and D ((where $y = 23^{2x} = (23^2)^x = 529^x$) are simply exponential functions with base a where $a > 1$. Since such a function is increasing, then these do not match the graph.

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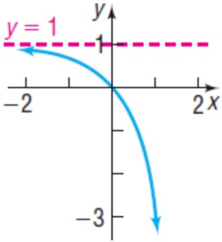

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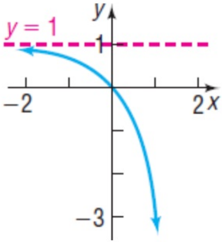

Solution (continued). Choice E is a vertical shift down by 1 unit of the graph of $y = 3^x$, which yields an increasing function and this does not match the graph. Choice F is a horizontal shift to the right by 1 unit of the graph of $y = 3^x$, which yields an increasing function and this does not match the graph.

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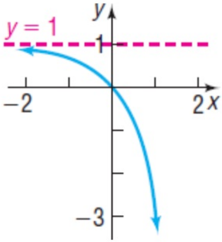

Solution (continued). Choice G is a function which is always positive, so this does not match the graph. **[Choice H must match the graph]**. □

Note. $y = 1 - 3^x$ results from the graph of $y = 3^x$ by (1) a reflection about the x axis (where $y = f(x)$ is replaced with $y = -f(x)$), and (2) a vertical shift up by 1 unit (where $y = g(x)$ is replaced with $y = g(x) + 1$).

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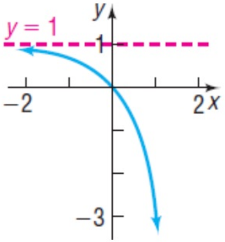

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Page 281 Number 52. Use transformations to graph the function $f(x) = 1 - 2^{x+3}$. Determine the domain, range, and horizontal asymptote.

Solution. We start with the graph of $y = 2^x$. First, we replace $y = f(x)$ with $y = -f(x)$ to get $y = -2^x$, which is a reflection about the x-axis of $y = 2^x$. Second, we replace x with $x + 3 = x - (-3)$ in $y = -2^x$ to get $y = -2^{x+3}$ which is a shift to the left by 3 units of $y = -2^x$. Third, we add 1 to -2^{x+3} to get $y = 1 - 2^{x+3}$ which is a vertical shift up by 1 unit of $y = -2^{x+3}$

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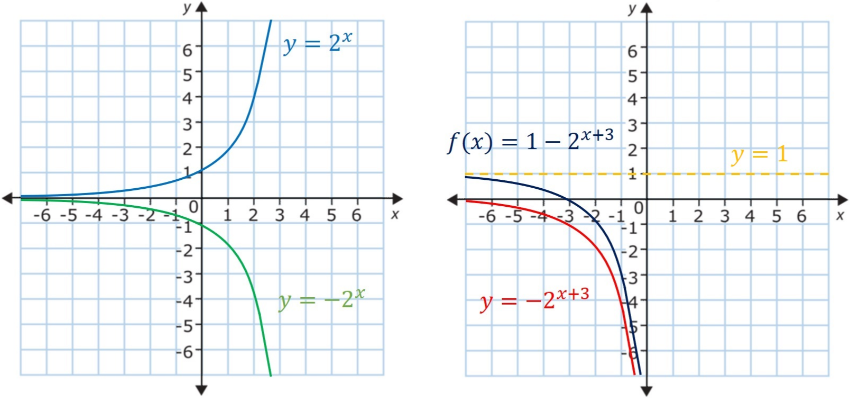

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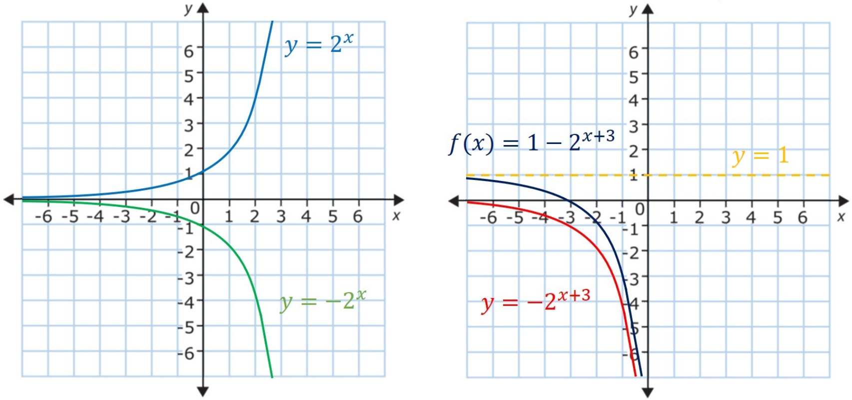

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Solution. Since $81 = 9^2 = (3^2)^2 = 3^4$, then the equation becomes $3^{-x} = 3^4$. By the previous note, since $f(x) = 3^x$ is one-to-one, then $-x = 4$ or $x = -4$. So the solution set is $\boxed{\{-4\}}$. □

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Page 282 Number 82

Page 282 Number 82. Solve the equation $(e^4)^x e^{x^2} = e^{12}$.

Solution. By the properties of exponents (given in Theorem 5.3.A), we have $(e^4)^x e^{x^2} = e^{4x} e^{x^2} = e^{4x+x^2}$, so the equation becomes $e^{4x+x^2} = e^{12}$. By the previous note, since $f(x) = e^x$ is one-to-one, then we need $4x + x^2 = 12$, or $x^2 + 4x - 12 = 0$, or $(x+6)(x-2) = 0$. So the solution set is $\boxed{\{-6, 2\}}$. □

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Solution. Since we are given $2^x = 3$, then $(2^x)^2 = 3^2$, or $2^{2x} = 9$, or $(2^2)^x = 9$, or $4^x = 9$. So $4^{-2x} = (4^x)^{-2} = (9)^{-2} = 1/9^2 = \boxed{1/81}$. □

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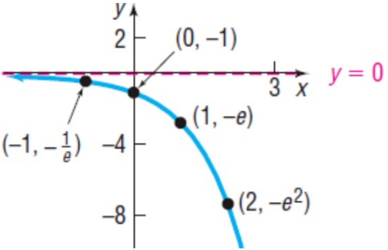

Solution. We look for a function of the form $f(x) = Ca^x$. Since the graph passes through the point $(0, -1)$ then we must have $f(0) = Ca^0 = (C)(1) = C = -1$, so that $C = -1$. Since the graph passes through the point $(1, -e)$ then we must have $f(1) = Ca^1 = Ca = (-1)a = -a = -e$, so that $a = e$. So the desired function must be $\boxed{f(x) = Ca^x = -e^x}$. □

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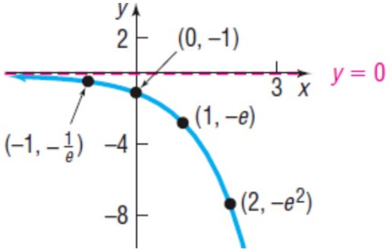

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Page 283 Number 106

Page 283 Number 106. The atmospheric pressure p on a balloon or airplane decreases with increasing height. This pressure, measured in millimeters of mercury, is related to the height h (in kilometers) above sea level by the function $p(h) = 760e^{-0.145h}$. **(a)** Find the atmospheric pressure at a height of 2 km (over a mile). **(b)** What is it at a height of 10 kilometers (over 30,000 feet)?

Solution. **(a)** When $h = 2$ km, we have the pressure $p(2) = 760e^{-0.145(2)} = \boxed{760e^{-0.290} \approx 568.680 \text{ mm}}$. □

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