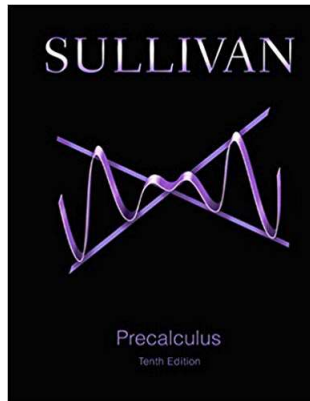


Page 294 Numbers 12 and 18

Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.4. Logarithmic Functions—Exercises, Examples, Proofs



()

Page 294 Number 22

Page 294 Number 22. Change the logarithmic statement $\log_b 4 = 2$ to an equivalent statement involving an exponent.

Solution. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = b$, $x = 4$, and $y = 2$ we have that $\log_b 4 = 2$ is equivalent to $b^2 = 4$. \square

()

Page 294 Numbers 12 and 18

Page 294 Numbers 12 and 18. Change each exponential statement to an equivalent statement involving a logarithm: **(12)** $16 = 4^2$ and **(18)** $e^{2.2} = M$.

Solution. (12) Since $y = \log_a x$ if and only if $x = a^y$, then with $x = 16$, $a = 4$, and $y = 2$ we have that $16 = 4^2$ is equivalent to $2 = \log_4 16$. \square

(18) Since $y = \log_a x$ if and only if $x = a^y$, then with $x = M$, $a = e$, and $y = 2.2$ we have that $e^{2.2} = M$ is equivalent to $2.2 = \log_e M$. \square

()

Page 295 Numbers 30 and 36

Page 295 Numbers 30 and 36. Find the exact value without using a calculator: **(30)** $\log_3(1/9)$ and **(36)** $\log_{\sqrt{3}} 9$.

Solution. (30) We rewrite the logarithmic equation as an equivalent exponential equation. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = 3$, $x = 1/9$, and $y = \log_3(1/9)$ we have that $\log_3(1/9) = y$ is equivalent to $3^y = 1/9$. Since $3^y = 1/9 = 1/3^2 = 3^{-2}$, then (because exponential functions are one-to-one) we have $y = \log_3(1/9) = -2$. \square

(36) We rewrite the logarithmic equation as an equivalent exponential equation. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = \sqrt{3}$, $x = 9$, and $y = \log_{\sqrt{3}} 9$ we have that $\log_{\sqrt{3}} 9 = y$ is equivalent to $\sqrt{3}^y = 9$. Since $\sqrt{3}^y = 9 = 3^2 = (\sqrt{3}^2)^2 = \sqrt{3}^4$, then (because exponential functions are one-to-one) we have $y = \log_{\sqrt{3}} 9 = 4$. \square

()

Page 295 Number 48

Page 295 Number 48. Find the domain of function

$$h(x) = \log_3 \left(\frac{x}{x-1} \right).$$

Solution. Since the domain of a logarithm function is $(0, \infty)$ then we need $\frac{x}{x-1} > 0$. As in 4.3. **The Graph of a Rational Function**, we divide the real number line into intervals using points where the numerator or denominator of the rational function $R(x) = \frac{x}{x-1}$ is 0. So we remove the points $x = 0$ and $x = 1$ to get the intervals $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$. Consider

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Test Value c	-1	1/2	2
Value of $R(c)$	$(-1)/(-2)$	$(1/2)/(-1/2)$	$(2)/(1)$
Conclusion	R is positive	R negative	R positive

So $\frac{x}{x-1} > 0$ for $(-\infty, 0) \cup (1, \infty)$ and this is the domain of h . \square

()

Page 295 Number 62

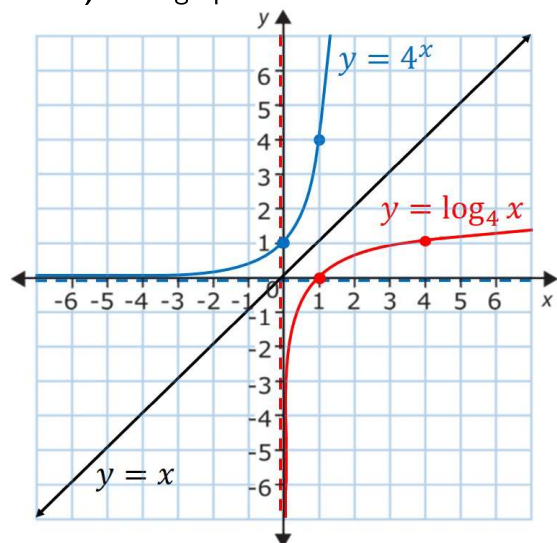
Page 295 Number 62. Graph $f(x) = 4^x$ and $f^{-1}(x) = \log_4 x$ on the same set of axes.

Solution. We know the shape of exponential and logarithmic functions, so we simply plot a couple of special points on each and use the asymptotes. Notice that $f(0) = 4^0 = 1$ and $f(1) = 4^1 = 4$, so the points $(0, 1)$ and $(1, 4)$ are on the graph of $y = f(x)$; hence the points $(1, 0)$ and $(4, 1)$ are on the graph of $y = f^{-1}(x)$. $f(x)$ has $y = 0$ as a horizontal asymptote and $f^{-1}(x)$ has a vertical asymptote of $x = 0$.

()

Page 295 Number 62 (continued)

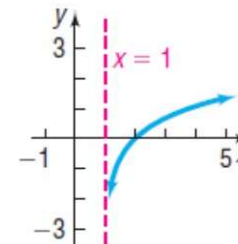
Solution (continued). The graphs are:

 \square

()

Page 295 Number 66

Page 295 Number 66. Match the graph with one of the following functions: **A.** $y = \log_3 x$, **B.** $y = \log_3(-x)$, **C.** $y = 2 \log_3(-x)$, **D.** $y = \log_3(-x)$, **E.** $y = \log_3(x) - 1$, **F.** $y = \log_3(x - 1)$, **G.** $y = \log_3(1 - x)$, **H.** $y = 1 - \log_3 x$.

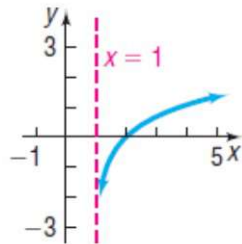


Solution. We know that $\log_a x$ has a vertical asymptote of $x = 0$, so a logarithm function will have a vertical asymptote where the argument in the logarithm functions is 0. Hence choices A, B, C, D, E, and H have vertical asymptotes of $x = 0$ and these do not match the given graph.

()

Page 295 Number 66 (continued)

Page 295 Number 66. Match the graph with one of the following functions: **A.** $y = \log_3 x$, **B.** $y = \log_3(-x)$, **C.** $y = 2 \log_3(-x)$, **D.** $y = \log_3(-x)$, **E.** $y = \log_3(x) - 1$, **F.** $y = \log_3(x - 1)$, **G.** $y = \log_3(1 - x)$, **H.** $y = 1 - \log_3 x$.

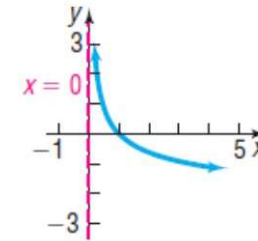


Solution (continued). Choices F and G both have a vertical asymptote of $x = 1$. The domain of F is $(1, \infty)$ and the domain of G is $(-\infty, 1)$, so the given graph must be for $y = \log_3(x - 1)$.

()

Page 295 Number 70

Page 295 Number 70. Match the graph with one of the following functions: **A.** $y = \log_3 x$, **B.** $y = \log_3(-x)$, **C.** $y = 2 \log_3(-x)$, **D.** $y = \log_3(-x)$, **E.** $y = \log_3(x) - 1$, **F.** $y = \log_3(x - 1)$, **G.** $y = \log_3(1 - x)$, **H.** $y = 1 - \log_3 x$.



Solution. The domain of choices B, C, and D are each $(-\infty, 0)$, so these do not match the graph. The domain of F is $(1, \infty)$ and the domain of G is $(-\infty, 1)$, so these do not match the graph. Choices A and E are increasing functions, so these do not match the graph. So the given graph must be for $y = 1 - \log_3 x$.

()

Page 295 Number 74

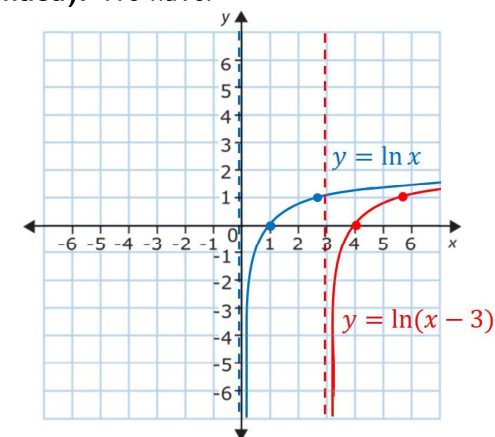
Page 295 Number 74. Consider $f(x) = \ln(x - 3)$. **(a)** Find the domain of f . **(b)** Graph f . **(c)** From the graph, determine the range and any asymptotes of f . **(d)** Find f^{-1} , the inverse of f . **(e)** Find the domain and the range of f^{-1} . **(f)** Graph f^{-1} .

Solution. **(a)** Logarithmic functions have domains $(0, \infty)$ so we need $x - 3 > 0$ or $x > 3$; the domain of f is $(3, \infty)$.

(b) We consider $y = \ln x$ and replace x with $x - h = x - 3$, which gives $f(x) = \ln(x - 3)$ as a horizontal shift to the right (since $h = 3 > 0$) by 3 units of $y = \ln x$. Notice $y = \ln x$ contains points $(1, 0)$ and $(e, 1)$.

Page 295 Number 74 (continued 1)

Solution (continued). We have:



(c) We see from the graph that the range of f is $\mathbb{R} = (-\infty, \infty)$ and the vertical asymptote is $x = 3$.

()

()

Page 295 Number 74 (continued 2)

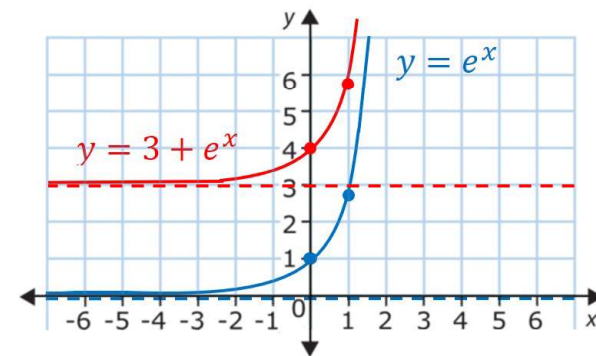
Page 295 Number 74. Consider $f(x) = \ln(x - 3)$. **(a)** Find the domain of f . **(b)** Graph f . **(c)** From the graph, determine the range and any asymptotes of f . **(d)** Find f^{-1} , the inverse of f . **(e)** Find the domain and the range of f^{-1} . **(f)** Graph f^{-1} .

Solution (continued). **(d)** Since $f(x) = \ln(x - 3)$, we write $y = \ln(x - 3)$, interchange x and y to get $x = \ln(y - 3) = \log_e(y - 3)$. We know that $x = \log_e(y - 3)$ means $e^x = y - 3$ so that $y = 3 + e^x$ and hence $f^{-1}(x) = 3 + e^x$. \square

(e) The domain of f^{-1} is the same as the range of f and so the domain of f^{-1} is $\mathbb{R} = (-\infty, \infty)$. The range of f^{-1} is the domain of f and so the range of f^{-1} is $(3, \infty)$. \square

Page 295 Number 74 (continued 3)

Solution (continued). **(f)** We consider $y = e^x$ and add 3 to e^x , which gives $f^{-1}(x) = 3 + e^x$ as a shift up by 3 units of $y = e^x$. Notice $y = e^x$ contains points $(0, 1)$ and $(1, e)$:



Page 295 Number 80

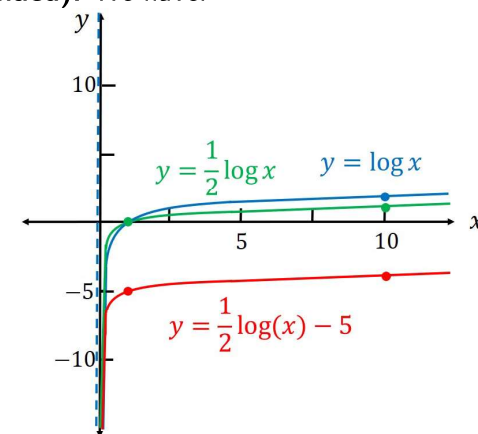
Page 295 Number 80. Consider $f(x) = \frac{1}{2} \log(x) - 5$. **(a)** Find the domain of f . **(b)** Graph f . **(c)** From the graph, determine the range and any asymptotes of f . **(d)** Find f^{-1} , the inverse of f . **(e)** Find the domain and the range of f^{-1} . **(f)** Graph f^{-1} .

Solution. **(a)** Logarithmic functions have domains $(0, \infty)$ so the domain of f is $(0, \infty)$. \square

(b) We consider $y = \log x$ and first multiply $\log x$ by $1/2$ which gives a vertical compression by a factor of $1/2$ of $y = \log x$. Second we subtract 5 from $(1/2) \log x$ resulting in a vertical shift down by 5 units. So $f(x) = \frac{1}{2} \log(x) - 5$ results from $y = \log x$ by (1) a vertical compression by a factor of $1/2$, and (2) a vertical shift down by 5 units. Notice $y = \log x$ contains points $(1, 0)$ and $(10, 1)$.

Page 295 Number 80 (continued 1)

Solution (continued). We have:



(c) We see from the graph that the range of f is $\mathbb{R} = (-\infty, \infty)$ and the vertical asymptote is $x = 0$. \square

Page 295 Number 80 (continued 2)

Page 295 Number 80. Consider $f(x) = \frac{1}{2} \log(x) - 5$. **(a)** Find the domain of f . **(b)** Graph f . **(c)** From the graph, determine the range and any asymptotes of f . **(d)** Find f^{-1} , the inverse of f . **(e)** Find the domain and the range of f^{-1} . **(f)** Graph f^{-1} .

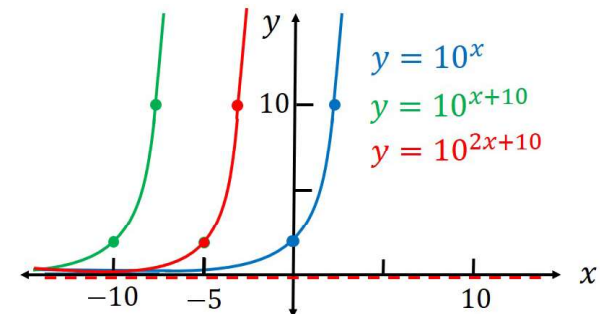
Solution (continued). **(d)** Since $f(x) = \frac{1}{2} \log(x) - 5$, we write $y = \frac{1}{2} \log(x) - 5$, interchange x and y to get $x = \frac{1}{2} \log(y) - 5 = \frac{1}{2} \log_{10}(y) - 5$, or $x + 5 = \frac{1}{2} \log_{10} y$, or $2(x + 5) = \log_{10} y$. We know that $2(x + 5) = \log_{10} y$ means $10^{2(x+5)} = y$ and hence $f^{-1}(x) = 10^{2x+10}$. □

(e) The domain of f^{-1} is the same as the range of f and so the domain of f^{-1} is $\mathbb{R} = (-\infty, \infty)$. The range of f^{-1} is the domain of f and so the range of f^{-1} is $(0, \infty)$. □

()

Page 295 Number 80 (continued 3)

Solution (continued). **(f)** First, we consider $y = 10^x$ and replace x with $x + 10$, which gives $y = 10^{x+10}$ as a horizontal shift to the left by 10 units of $y = 10^x$. Second, we replace x by $2x$ in $y = 10^{x+10}$, which gives $f(x) = 10^{2x+10}$ which is a horizontal compression by a factor of 2 of $y = 10^{x+10}$. Notice $y = 10^x$ contains points $(0, 1)$ and $(1, 10)$:



□

()

Page 296 Numbers 90, 96, 102, and 112

Page 296 Numbers 90, 96, 102, and 112. Solve the equations: **(90)** $\log_5 x = 3$, **(96)** $\ln e^{-2x} = 8$, **(102)** $e^{-2x} = 1/3$, **(112)** $4e^{x+1} = 5$.

Solution. Recall that $y = \log_a x$ means $a^y = x$.

(90) $\log_5 x = 3$ means $5^3 = x$ so that $x = 5^3 = 125$. □

(96) The natural log function \ln can be written \log_e so that $\ln e^{-2x} = 8$ is equivalent to $\log_e e^{-2x} = 8$ which means $e^8 = e^{-2x}$ so that (since exponential functions are one-to-one) $8 = -2x$ or $x = -4$. □

(102) $e^{-2x} = 1/3$ means $\log_e(1/3) = -2x$ so that $x = -(1/2) \log_e(1/3) = -(1/2) \ln(1/3)$. □

(112) We rewrite $4e^{x+1} = 5$ as $e^{x+1} = 5/4$ which means $\log_e(5/4) = x + 1$ or $x = \log_e(5/4) - 1 = \ln(5/4) - 1$. □

()

Page 297 Number 124

Page 297 Number 124. Between 5:00 pm and 6:00 pm, cars arrive at Jiffy Lube at the rate of 9 cars per hour (0.15 car per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 5:00 pm: $F(t) = 1 - e^{-0.15t}$.

(a) Determine how many minutes are needed for the probability to reach 50%. **(b)** Determine how many minutes are needed for the probability to reach 80%.

Solution. **(a)** We solve $F(t) = 1 - e^{-0.15t} = 0.50 = 1/2$ for t . So we need $e^{-0.15t} = 1/2$ which means $\log_e(1/2) = -0.15t$, or $\ln(1/2) = -0.15t$, or $t = -(1/0.15) \ln(1/2) \approx 4.621$ minutes. □

(b) We solve $F(t) = 1 - e^{-0.15t} = 0.80 = 4/5$ for t . So we need $e^{-0.15t} = 1/5$ which means $\log_e(1/5) = -0.15t$, or $\ln(1/5) = -0.15t$, or $t = -(1/0.15) \ln(1/5) \approx 10.730$ minutes. □

()