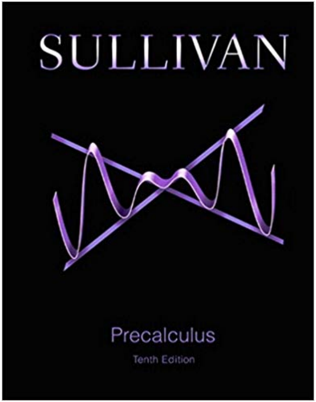
Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.4. Logarithmic Functions—Exercises, Examples, Proofs



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Page 294 Numbers 12 and 18

Page 294 Numbers 12 and 18. Change each exponential statement to an equivalent statement involving a logarithm: **(12)** $16 = 4^2$ and **(18)** $e^{2.2} = M$.

Solution. **(12)** Since $y = \log_a x$ if and only if $x = a^y$, then with $x = 16$, $a = 4$, and $y = 2$ we have that $16 = 4^2$ is equivalent to $\boxed{2 = \log_4 16}$. □

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(18) Since $y = \log_a x$ if and only if $x = a^y$, then with $x = M$, $a = e$, and $y = 2.2$ we have that $e^{2.2} = M$ is equivalent to $\boxed{2.2 = \log_e M}$. □

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Page 294 Number 22

Page 294 Number 22. Change the logarithmic statement $\log_b 4 = 2$ to an equivalent statement involving an exponent.

Solution. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = b$, $x = 4$, and $y = 2$ we have that $\log_b 4 = 2$ is equivalent to $\boxed{b^2 = 4}$. □

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Page 295 Numbers 30 and 36

Page 295 Numbers 30 and 36. Find the exact value without using a calculator: **(30)** $\log_3(1/9)$ and **(36)** $\log_{\sqrt{3}} 9$.

Solution. **(30)** We rewrite the logarithmic equation as an equivalent exponential equation. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = 3$, $x = 1/9$, and $y = \log_3(1/9)$ we have that $\log_3(1/9) = y$ is equivalent to $3^y = 1/9$. Since $3^y = 1/9 = 1/3^2 = 3^{-2}$, then (because exponential functions are one-to-one) we have $\boxed{y = \log_3(1/9) = -2}$. □

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(36) We rewrite the logarithmic equation as an equivalent exponential equation. Since $y = \log_a x$ if and only if $x = a^y$, then with $a = \sqrt{3}$, $x = 9$, and $y = \log_{\sqrt{3}} 9$ we have that $\log_{\sqrt{3}} 9 = y$ is equivalent to $\sqrt{3}^y = 9$. Since $\sqrt{3}^y = 9 = 3^2 = (\sqrt{3}^2)^2 = \sqrt{3}^4$, then (because exponential functions are one-to-one) we have $\boxed{y = \log_{\sqrt{3}} 9 = 4}$. □

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Page 295 Number 48

Page 295 Number 48. Find the domain of function $h(x) = \log_3 \left(\dfrac{x}{x-1} \right)$.

Solution. Since the domain of a logarithm function is $(0, \infty)$ then we need $\dfrac{x}{x-1} > 0$. As in 4.3. The Graph of a Rational Function, we divide the real number line into intervals using points where the numerator or denominator of the rational function $R(x) = \dfrac{x}{x-1}$ is 0. So we remove the points $x = 0$ and $x = 1$ to get the intervals $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$.

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Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Test Value c	-1	$1/2$	2
Value of $R(c)$	$(-1)/(-2)$	$(1/2)/(-1/2)$	$(2)/(1)$
Conclusion	R is positive	R negative	R positive

So $\dfrac{x}{x-1} > 0$ for $\boxed{(-\infty, 0) \cup (1, \infty)}$ and this is the domain of h. □

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So $\frac{x}{x-1} > 0$ for $\boxed{(-\infty, 0) \cup (1, \infty)}$ and this is the domain of h. □

Page 295 Number 62

Page 295 Number 62. Graph $f(x) = 4^x$ and $f^{-1}(x) = \log_4 x$ on the same set of axes.

Solution. We know the shape of exponential and logarithmic functions, so we simply plot a couple of special points on each and use the asymptotes. Notice that $f(0) = 4^0 = 1$ and $f(1) = 4^1 = 4$, so the points $(0, 1)$ and $(1, 4)$ are on the graph of $y = f(x)$; hence the points $(1, 0)$ and $(4, 1)$ are on the graph of $y = f^{-1}(x)$. $f(x)$ has $y = 0$ as a horizontal asymptote and $f^{-1}(x)$ has a vertical asymptote of $x = 0$.

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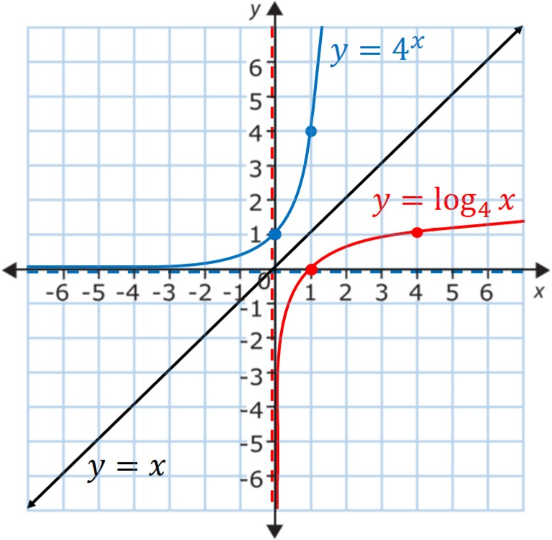
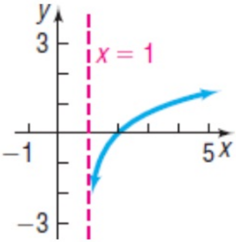
Page 295 Number 62 (continued)

Solution (continued). The graphs are:



Page 295 Number 66

Page 295 Number 66. Match the graph with one of the following functions: **A.** $y = \log_3 x$, **B.** $y = \log_3(-x)$, **C.** $y = 2\log_3(-x)$, **D.** $y = \log_3(-x)$, **E.** $y = \log_3(x) - 1$, **F.** $y = \log_3(x-1)$, **G.** $y = \log_3(1-x)$, **H.** $y = 1 - \log_3 x$.



Solution. We know that $\log_a x$ has a vertical asymptote of $x = 0$, so a logarithm function will have a vertical asymptote where the argument in the logarithm functions is 0. Hence choices A, B, C, D, E, and H have vertical asymptotes of $x = 0$ and these do not match the given graph.

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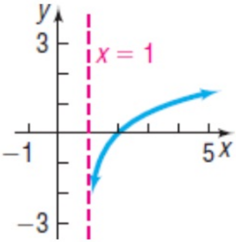

Solution. We know that $\log_a x$ has a vertical asymptote of $x = 0$, so a logarithm function will have a vertical asymptote where the argument in the logarithm functions is 0. Hence choices A, B, C, D, E, and H have vertical asymptotes of $x = 0$ and these do not match the given graph.

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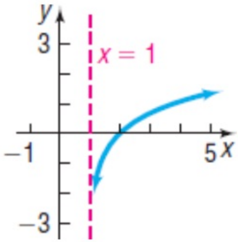

Solution (continued). Choices F and G both have a vertical asymptote of $x = 1$. The domain of F is $(1, \infty)$ and the domain of G is $(-\infty, 1)$, so the given graph must be for $\boxed{y = \log_3(x-1)}$. □

Page 295 Number 70

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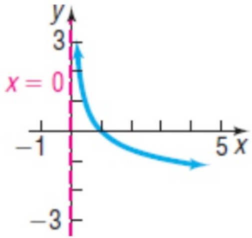

Solution. The domain of choices B, C, and D are each $(-\infty, 0)$, so these do not match the graph. The domain of F is $(1, \infty)$ and the domain of G is $(-\infty, 1)$, so these do not match the graph. Choices A and E are increasing functions, so these do not match the graph. So the given graph must be for $\boxed{y = 1 - \log_3 x}$. □

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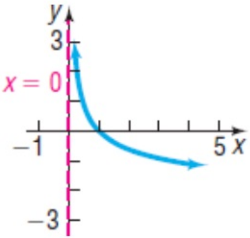

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Page 295 Number 74

Page 295 Number 74. Consider $f(x) = \ln(x-3)$. **(a)** Find the domain of f. **(b)** Graph f. **(c)** From the graph, determine the range and any asymptotes of f. **(d)** Find f^{-1}, the inverse of f. **(e)** Find the domain and the range of f^{-1}. **(f)** Graph f^{-1}.

Solution. (a) Logarithmic functions have domains $(0,\infty)$ so we need $x-3>0$ or $x>3$; the $\boxed{\text{domain of } f \text{ is } (3,\infty)}$. □

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(b) We consider $y = \ln x$ and replace x with $x-h = x-3$, which gives $f(x) = \ln(x-3)$ as a horizontal shift to the right (since $h = 3 > 0$) by 3 units of $y = \ln x$. Notice $y = \ln x$ contains points $(1,0)$ and $(e,1)$.

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Solution. **(a)** Logarithmic functions have domains $(0, \infty)$ so we need $x - 3 > 0$ or $x > 3$; the $\boxed{\text{domain of } f \text{ is } (3, \infty)}$. □

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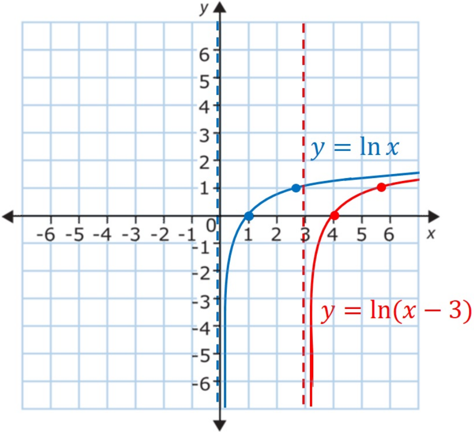
Page 295 Number 74 (continued 1)

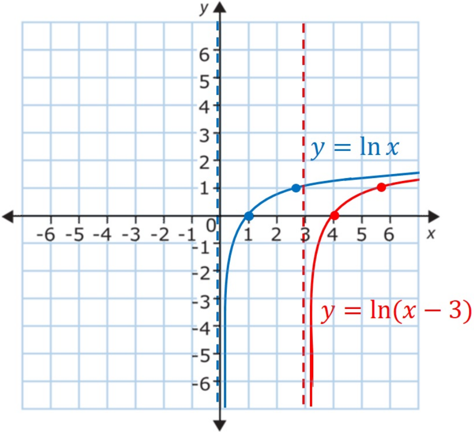
Solution (continued). We have:



(c) We see from the graph that the range of f is $\mathbb{R} = (-\infty, \infty)$ and the vertical asymptote is $x = 3$. □

Page 295 Number 74 (continued 1)

Solution (continued). We have:



(c) We see from the graph that the $\boxed{\text{range of } f \text{ is } \mathbb{R} = (-\infty, \infty)}$ and the $\boxed{\text{vertical asymptote is } x = 3}$. □

Page 295 Number 74 (continued 2)

Page 295 Number 74. Consider $f(x) = \ln(x-3)$. **(a)** Find the domain of f. **(b)** Graph f. **(c)** From the graph, determine the range and any asymptotes of f. **(d)** Find f^{-1}, the inverse of f. **(e)** Find the domain and the range of f^{-1}. **(f)** Graph f^{-1}.

Solution (continued). (d) Since $f(x) = \ln(x-3)$, we write $y = \ln(x-3)$, interchange x and y to get $x = \ln(y-3) = \log_e(y-3)$. We know that $x = \log_e(y-3)$ means $e^x = y-3$ so that $y = 3 + e^x$ and hence $\boxed{f^{-1}(x) = 3 + e^x}$. □

(e) The domain of f^{-1} is the same as the range of f and so the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R} = (-\infty, \infty)}$. The range of f^{-1} is the domain of f and so the $\boxed{\text{range of } f^{-1} \text{ is } (3, \infty)}$. □

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Solution (continued). **(d)** Since $f(x) = \ln(x-3)$, we write $y = \ln(x-3)$, interchange x and y to get $x = \ln(y-3) = \log_e(y-3)$. We know that $x = \log_e(y-3)$ means $e^x = y-3$ so that $y = 3 + e^x$ and hence $\boxed{f^{-1}(x) = 3 + e^x}$. □

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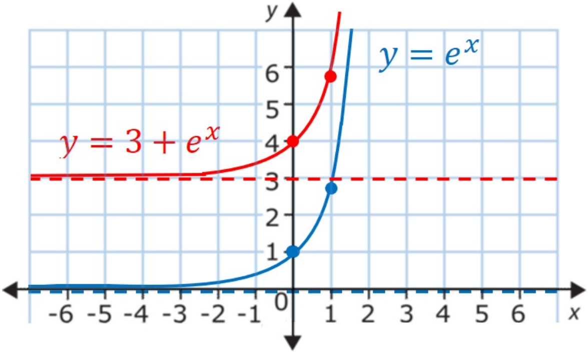
Page 295 Number 74 (continued 3)

Solution (continued). (f) We consider $y = e^x$ and add 3 to e^x, which gives $f^{-1}(x) = 3 + e^x$ as a shift up by 3 units of $y = e^x$. Notice $y = e^x$ contains points $(0, 1)$ and $(1, e)$:



Page 295 Number 80

Page 295 Number 80. Consider $f(x) = \frac{1}{2}\log(x) - 5$. **(a)** Find the domain of f. **(b)** Graph f. **(c)** From the graph, determine the range and any asymptotes of f. **(d)** Find f^{-1}, the inverse of f. **(e)** Find the domain and the range of f^{-1}. **(f)** Graph f^{-1}.

Solution. (a) Logarithmic functions have domains $(0, \infty)$ so the domain of f is $(0, \infty)$. □

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Solution. **(a)** Logarithmic functions have domains $(0, \infty)$ so the $\boxed{\text{domain of } f \text{ is } (0, \infty)}$. □

(b) We consider $y = \log x$ and first multiply $\log x$ by $1/2$ which gives a vertical compression by a factor of $1/2$ of $y = \log x$. Second we subtract 5 from $(1/2)\log x$ resulting in a vertical shift down by 5 units. So $f(x) = \frac{1}{2}\log(x) - 5$ results from $y = \log x$ by (1) a vertical compression by a factor of $1/2$, and (2) a vertical shift down by 5 units. Notice $y = \log x$ contains points $(1, 0)$ and $(10, 1)$.

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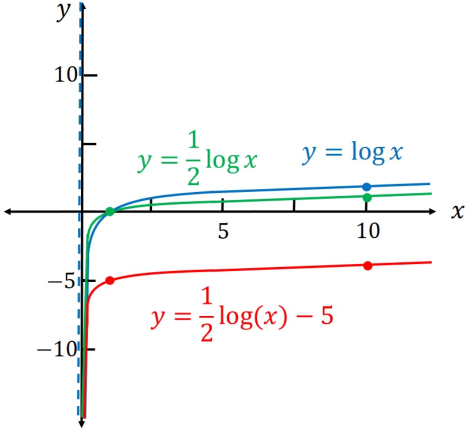
Page 295 Number 80 (continued 1)

Solution (continued). We have:



(c) We see from the graph that the range of f is $\mathbb{R} = (-\infty, \infty)$ and the vertical asymptote is $x = 0$. □

Page 295 Number 80 (continued 1)

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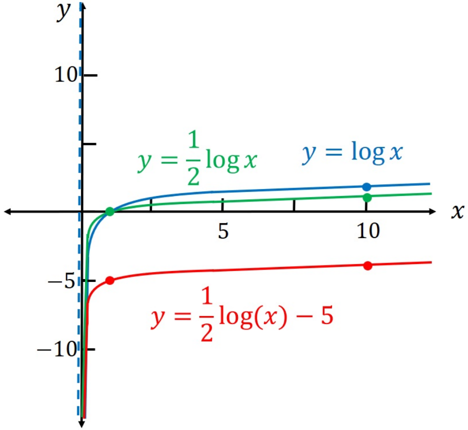

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Solution (continued). (d) Since $f(x) = \frac{1}{2}\log(x) - 5$, we write $y = \frac{1}{2}\log(x) - 5$, interchange x and y to get $x = \frac{1}{2}\log(y) - 5 = \frac{1}{2}\log_{10}(y) - 5$, or $x + 5 = \frac{1}{2}\log_{10} y$, or $2(x+5) = \log_{10} y$. We know that $2(x+5) = \log_{10} y$ means $10^{2(x+5)} = y$ and hence $\boxed{f^{-1}(x) = 10^{2x+10}}$. □

(e) The domain of f^{-1} is the same as the range of f and so the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R} = (-\infty, \infty)}$. The range of f^{-1} is the domain of f and so the $\boxed{\text{range of } f^{-1} \text{ is } (0, \infty)}$. □

Page 295 Number 80 (continued 2)

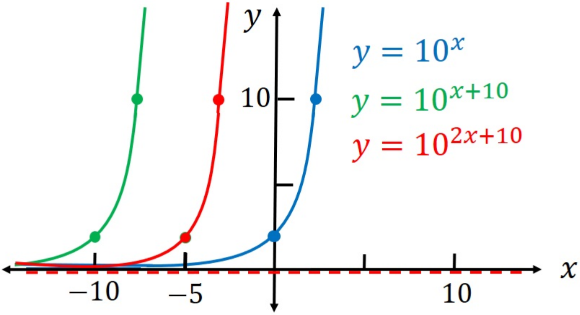
Page 295 Number 80. Consider $f(x) = \frac{1}{2}\log(x) - 5$. **(a)** Find the domain of f. **(b)** Graph f. **(c)** From the graph, determine the range and any asymptotes of f. **(d)** Find f^{-1}, the inverse of f. **(e)** Find the domain and the range of f^{-1}. **(f)** Graph f^{-1}.

Solution (continued). (d) Since $f(x) = \frac{1}{2}\log(x) - 5$, we write $y = \frac{1}{2}\log(x) - 5$, interchange x and y to get $x = \frac{1}{2}\log(y) - 5 = \frac{1}{2}\log_{10}(y) - 5$, or $x + 5 = \frac{1}{2}\log_{10} y$, or $2(x+5) = \log_{10} y$. We know that $2(x+5) = \log_{10} y$ means $10^{2(x+5)} = y$ and hence $\boxed{f^{-1}(x) = 10^{2x+10}}$. □

(e) The domain of f^{-1} is the same as the range of f and so the $\boxed{\text{domain of } f^{-1} \text{ is } \mathbb{R} = (-\infty, \infty)}$. The range of f^{-1} is the domain of f and so the $\boxed{\text{range of } f^{-1} \text{ is } (0, \infty)}$. □

Page 295 Number 80 (continued 3)

Solution (continued). (f) First, we consider $y = 10^x$ and replace x with $x + 10$, which gives $y = 10^{x+10}$ as a horizontal shift to the left by 10 units of $y = 10^x$. Second, we replace x by $2x$ in $y = 10^{x+10}$, which gives $f(x) = 10^{2x+10}$ which is a horizontal compression by a factor of 2 of $y = 10^{x+10}$. Notice $y = 10^x$ contains points $(0, 1)$ and $(1, 10)$:



Page 296 Numbers 90, 96, 102, and 112

Page 296 Numbers 90, 96, 102, and 112. Solve the equations: **(90)** $\log_5 x = 3$, **(96)** $\ln e^{-2x} = 8$, **(102)** $e^{-2x} = 1/3$, **(112)** $4e^{x+1} = 5$.

Solution. Recall that $y = \log_a x$ means $a^y = x$.

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Page 297 Number 124

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Solution. (a) We solve $F(t) = 1 - e^{-0.15t} = 0.50 = 1/2$ for t. So we need $e^{-0.15t} = 1/2$ which means $\log_e(1/2) = -0.15t$, or $\ln(1/2) = -0.15t$, or $\boxed{t = -(1/0.15)\ln(1/2) \approx 4.621 \text{ minutes.}}$ □

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