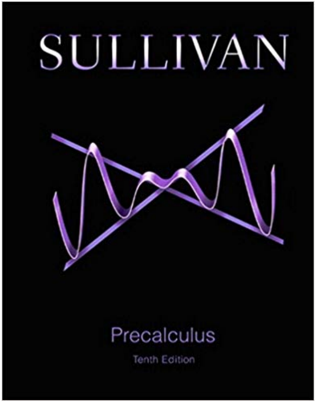
Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.5. Properties of Logarithms—Exercises, Examples, Proofs



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Theorem 5.5.A. Properties of Logarithms

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

1. $a^{\log_a M} = M$.
2. $\log_a a^r = r$.
3. The Log of a Product Equals the Sum of the Logs:
$$\log_a(MN) = \log_a M + \log_a N.$$
4. The Log of a Quotient Equals the Difference of the Logs:
$$\log_a(M/N) = \log_a M - \log_a N.$$
5. The Log of a Power Equals the Product of the Power and the Log:
$$\log_a M^r = r \log_a M \text{ and } a^r = e^{r \ln a}.$$

Theorem 5.5.A. Properties of Logarithms (continued 1)

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

1. $a^{\log_a M} = M$.
2. $\log_a a^r = r$.

Proof. (1) With $f(x) = a^x$, we have by definition that $f^{-1}(x) = \log_a x$. For any inverse functions, we have $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}. Therefore

$$f(f^{-1}(x)) = a^{\log_a x} = x \text{ for all } x > 0.$$

In particular, with $x = M > 0$ we have $a^{\log_a M} = M$, as claimed.

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In particular, with $x = M > 0$ we have $a^{\log_a M} = M$, as claimed.

(2) For any inverse functions, we also have $f^{-1}(f(x)) = x$ for all x in the domain of f. Therefore

$$f^{-1}(f(x)) = \log_a a^x = x \text{ for all real } x.$$

In particular, with $x = r$ we have $\log_a a^r = r$, as claimed.

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Theorem 5.5.A. Properties of Logarithms (continued 2)

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

3. The Log of a Product Equals the Sum of the Logs:

$$\log_a(MN) = \log_a M + \log_a N.$$

Proof (continued). (3) Let $A = \log_a M$ and let $B = \log_a N$. Translating these logarithmic expressions into exponential expressions gives $a^A = M$ and $a^B = N$. Then

$$\begin{aligned} \log_a(MN) &= \log_a(a^A a^B) = \log_a a^{A+B} \text{ by Theorem 5.3.A} \\ &= A + B \text{ by Property 2 of this theorem} \\ &= \log_a M + \log_a N, \end{aligned}$$

as claimed.

Theorem 5.5.A. Properties of Logarithms (continued 3)

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

4. The Log of a Quotient Equals the Difference of the Logs:

$$\log_a(M/N) = \log_a M - \log_a N.$$

Proof (continued). (4) Let $A = \log_a M$ and let $B = \log_a N$. Translating these logarithmic expressions into exponential expressions gives $a^A = M$ and $a^B = N$. Then

$$\begin{aligned} \log_a(M/N) &= \log_a(a^A/a^B) = \log_a a^{A-B} \text{ by Theorem 5.3.A} \\ &= A - B \text{ by Property 2 of this theorem} \\ &= \log_a M - \log_a N, \end{aligned}$$

as claimed. (This also appears as Exercise 5.5.109.)

Theorem 5.5.A. Properties of Logarithms (continued 4)

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

5. The Log of a Power Equals the Product of the Power and the Log:

$$\log_a M^r = r \log_a M \text{ and } a^r = e^{r \ln a}.$$

Proof (continued). (5) Let $A = \log_a M$. Translating this logarithmic expression into an exponential expression gives $a^A = M$. Then

$$\begin{aligned} \log_a M^r &= \log_a (a^A)^r = \log_a a^{rA} \text{ by Theorem 5.3.A} \\ &= rA \text{ by Property 2 of this theorem} \\ &= r \log_a M, \end{aligned}$$

as claimed.

Theorem 5.5.A. Properties of Logarithms (continued 5)

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

5. The Log of a Power Equals the Product of the Power and the Log:

$$\log_a M^r = r \log_a M \text{ and } a^r = e^{r \ln a}.$$

Proof (continued). Next, by (1) of this theorem with $a = e$ (and so $\log_a$ is $\log_e = \ln$), we have $e^{\ln M} = M$. Then

$$\begin{aligned} e^{\ln a^r} &= e^{r \ln a} \text{ by this theorem, the first part of (5)} \\ &= (e^{\ln a})^r \text{ by Theorem 5.3.A} \\ &= a^r \text{ by this theorem, part (1), as described above,} \end{aligned}$$

as claimed. □

Page 305 Number 20

Page 305 Number 20. Use properties of logarithms to find the exact value of $\log_6 9 + \log_6 4$.

Solution. Since the log of a product equals the sum of the logs (Theorem 5.5.A(3)), then $\log_6 9 + \log_6 4 = \log_6((9)(4)) = \log_6(36)$ and by Theorem 5.5.A(3) $\log_6(36) = \log_6(6^2) = \boxed{2}$ by Theorem 5.5.A(2). □

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Page 305 Number 36

Page 305 Number 36. Suppose that $\ln 2 = a$ and $\ln 3 = b$. Use properties of logarithms to write $\ln \sqrt[4]{2/3}$ in terms of a and b.

Solution. First, $\ln \sqrt[4]{2/3} = \ln\left((2/3)^{1/4}\right)$, and by Theorem 5.5.A(5), the log of a power equals the product of the power and the log, $\ln\left((2/3)^{1/4}\right) = \frac{1}{4}\ln(2/3)$. Since the log of a quotient equals the difference of the logs, Theorem 5.5.A(4), then $\frac{1}{4}\ln(2/3) = \frac{1}{4}(\ln 2 - \ln 3)$.

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$$\ln \sqrt[4]{2/3} = \ln\left((2/3)^{1/4}\right) = \frac{1}{4}\ln(2/3) = \frac{1}{4}(\ln 2 - \ln 3).$$

With $a = \ln 2$ and $b = \ln 3$ we then have $\ln \sqrt[4]{2/3} = \frac{1}{4}(\ln 2 - \ln 3) = \boxed{\frac{1}{4}(a - b)}$. □

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With $a = \ln 2$ and $b = \ln 3$ we then have
$\ln \sqrt[4]{2/3} = \frac{1}{4}(\ln 2 - \ln 3) = \boxed{\frac{1}{4}(a-b)}$. □

Page 305 Number 48

Page 305 Number 48. Write $\ln\left(x\sqrt{1+x^2}\right)$ as a sum and/or difference of logarithms. Express powers as factors.

Solution. Since the log of a product equals the sum of the logs (Theorem 5.5.A(3)), then $\ln\left(x\sqrt{1+x^2}\right) = \ln x + \ln\sqrt{1+x^2}$. By Theorem 5.5.A(5), the log of a power equals the product of the power and the log, so

$$\ln x + \ln\sqrt{1+x^2} = \ln x + \ln(1+x^2)^{1/2} = \ln x + \frac{1}{2}\ln(1+x^2).$$

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$$\ln x + \ln\sqrt{1+x^2} = \ln x + \ln(1+x^2)^{1/2} = \ln x + \frac{1}{2}\ln(1+x^2).$$

Hence $\ln\left(x\sqrt{1+x^2}\right) = \boxed{\ln x + \frac{1}{2}\ln(1+x^2)}$. □

Page 305 Number 56

Page 305 Number 56. Write $\ln\left(\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right)$, where $0 < x < 1$, as a sum and/or difference of logarithms. Express powers as factors.

Solution. Applying the parts of Theorem 5.5.A, we have

$$\begin{aligned}\ln\left(\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right) &= \ln(5x^2\sqrt[3]{1-x}) - \ln(4(x+1)^2) \text{ by Theorem 5.5.A(4)}\\ &= \ln 5 + \ln x^2 + \ln\sqrt[3]{1-x} - (\ln 4 + \ln(x+1)^2)\\ &\quad \text{by Theorem 5.5.A(3)}\\ &= \ln 5 + \ln x^2 + \ln(1-x)^{1/3} - (\ln 4 + \ln(x+1)^2)\\ &= \boxed{\ln 5 + 2\ln x + \tfrac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)}\\ &\quad \text{by Theorem 5.5.A(5).}\end{aligned}$$

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$$\begin{aligned}
\ln\left(\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right) &= \ln(5x^2\sqrt[3]{1-x}) - \ln(4(x+1)^2) \text{ by Theorem 5.5.A(4)} \\
&= \ln 5 + \ln x^2 + \ln\sqrt[3]{1-x} - (\ln 4 + \ln(x+1)^2) \\
&\quad \text{by Theorem 5.5.A(3)} \\
&= \ln 5 + \ln x^2 + \ln(1-x)^{1/3} - (\ln 4 + \ln(x+1)^2) \\
&= \boxed{\ln 5 + 2\ln x + \tfrac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)} \\
&\quad \text{by Theorem 5.5.A(5).}
\end{aligned}$$

□

Page 305 Numbers 62 and 70

Page 305 Numbers 62 and 70. Write each expression as a single logarithm: **(62)** $\log(x^2 + 3x + 2) - 2\log(x + 1)$ and **(70)** $3\log_5(3x + 1) - 2\log_5(2x - 1) - \log_5 x$.

Solution. (62) Applying the parts of Theorem 5.5.A, we have

$$\begin{aligned}
\log(x^2 + 3x + 2) - 2\log(x + 1) &= \log(x^2 + 3x + 2) - \log(x + 1)^2 \\
&\quad \text{by Theorem 5.5.A(5)} \\
&= \log \frac{x^2 + 3x + 2}{(x + 1)^2} \text{ by Theorem 5.5.A(4)} \\
&= \log \frac{(x + 2)(x + 1)}{(x + 1)^2} = \boxed{\log \frac{x + 2}{x + 1}}.
\end{aligned}$$

□

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Solution. (62) Applying the parts of Theorem 5.5.A, we have

$$\begin{aligned} \log(x^2+3x+2)-2\log(x+1) &= \log(x^2+3x+2)-\log(x+1)^2 \\ &\quad \text{by Theorem 5.5.A(5)} \\ &= \log\frac{x^2+3x+2}{(x+1)^2} \text{ by Theorem 5.5.A(4)} \\ &= \log\frac{(x+2)(x+1)}{(x+1)^2} = \boxed{\log\frac{x+2}{x+1}}. \end{aligned}$$

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Page 305 Numbers 62 and 70 (continued)

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Solution (continued). (70) Applying the parts of Theorem 5.5.A, we have

$$
\begin{aligned}
& 3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x \\
= \;& \log_5(3x+1)^3 - \log_5(2x-1)^2 - \log_5 x \\
& \text{ by Theorem 5.5.A(5)} \\
= \;& \log_5 \frac{(3x+1)^3}{(2x-1)^2} - \log_5 x \text{ by Theorem 5.5.A(4)} \\
= \;& \boxed{\log_5 \frac{(3x+1)^3}{x(2x-1)^2}} \text{ by Theorem 5.5.A(4).}
\end{aligned}
$$

Page 305 Numbers 62 and 70 (continued)

Page 305 Numbers 62 and 70. Write each expression as a single logarithm: **(62)** $\log(x^2 + 3x + 2) - 2\log(x + 1)$ and **(70)** $3\log_5(3x + 1) - 2\log_5(2x - 1) - \log_5 x$.

Solution (continued). (70) Applying the parts of Theorem 5.5.A, we have

$$\begin{aligned} & 3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x \\ = \ & \log_5(3x+1)^3 - \log_5(2x-1)^2 - \log_5 x \\ & \text{ by Theorem 5.5.A(5)} \\ = \ & \log_5 \frac{(3x+1)^3}{(2x-1)^2} - \log_5 x \text{ by Theorem 5.5.A(4)} \\ = \ & \boxed{\log_5 \frac{(3x+1)^3}{x(2x-1)^2}} \text{ by Theorem 5.5.A(4).} \end{aligned}$$

□

Theorem 5.5.C. Change-of-Base Formula

Theorem 5.5.C. Change-of-Base Formula. If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then $\log_a M = \dfrac{\log_b M}{\log_b a}$.

Proof. Let $y = \log_a M$. Translating this logarithmic expression into an exponential expression gives $a^y = M$. Then

$$\begin{aligned} \log_b a^y &= \log_b M, \text{ and} \\ y \log_b a &= \log_b M \text{ by Theorem 5.5.A(5), and so} \\ y &= \frac{\log_b M}{\log_b a}, \text{ or} \\ \log_a M &= \frac{\log_b M}{\log_b a}, \end{aligned}$$

as claimed. □

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as claimed. □

Page 306 Number 72

Page 306 Number 72. Use the Change-of-Base Formula and a calculator to evaluate $\log_5 18$. Round your answer to three decimal places.

Solution. The Change-of-Base Formula (Theorem 5.5.C) states $\log_a M = \dfrac{\log_b M}{\log_b a}$. We choose $b = e$ and have $a = 5$ and $M = 18$ to get

$$\log_5 18 = \frac{\log_e 18}{\log_e 5} = \frac{\ln 18}{\ln 5} \approx \frac{2.890371758}{1.609437912} \approx \boxed{1.796}.$$

□

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$$\log_5 18 = \frac{\log_e 18}{\log_e 5} = \frac{\ln 18}{\ln 5} \approx \frac{2.890371758}{1.609437912} \approx \boxed{1.796}.$$

□

Page 306 Number 98

Page 306 Number 98. Find the value of $\log_2 4 \log_4 6 \log_6 8$.

Solution. The Change-of-Base Formula (Theorem 5.5.C) states $\log_a M = \dfrac{\log_b M}{\log_b a}$. We change the base of each logarithm to e to get

$$\log_2 4 \log_4 6 \log_6 8 = \frac{\ln 4}{\ln 2}\frac{\ln 6}{\ln 4}\frac{\ln 8}{\ln 6} = \frac{\ln 8}{\ln 2} = \frac{\ln 2^3}{\ln 2} = \frac{3 \ln 2}{\ln 2} = \boxed{3},$$

where the second to last equality is justified by Theorem 5.5.A(5). □

Page 306 Number 98

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where the second to last equality is justified by Theorem 5.5.A(5). □

Page 306 Number 102

Page 306 Number 102. Show that
$\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)=0$.

Solution. We have

$$\begin{aligned}
&\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)\\
&= \log_a\left(\left(\sqrt{x}+\sqrt{x-1}\right)\left(\frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}}\right)\right)\\
&\quad +\log_a\left(\left(\sqrt{x}-\sqrt{x-1}\right)\left(\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}\right)\right)\\
&= \log_a\left(\frac{x-(x-1)}{\sqrt{x}-\sqrt{x-1}}\right)+\log_a\left(\frac{x-(x-1)}{\sqrt{x}+\sqrt{x-1}}\right)\\
&= \log_a\left(\frac{1}{\sqrt{x}-\sqrt{x-1}}\right)+\log_a\left(\frac{1}{\sqrt{x}+\sqrt{x-1}}\right)
\end{aligned}$$

Page 306 Number 102

Page 306 Number 102. Show that $\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)=0$.

Solution. We have

$$\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)$$

$$\begin{aligned}
&= \log_a\left(\left(\sqrt{x}+\sqrt{x-1}\right)\left(\frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}}\right)\right) \\
&\quad +\log_a\left(\left(\sqrt{x}-\sqrt{x-1}\right)\left(\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}\right)\right) \\
&= \log_a\left(\frac{x-(x-1)}{\sqrt{x}-\sqrt{x-1}}\right)+\log_a\left(\frac{x-(x-1)}{\sqrt{x}+\sqrt{x-1}}\right) \\
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\end{aligned}$$

Page 306 Number 102 (continued)

Page 306 Number 102. Show that
$\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)=0.$

Solution (continued).

$$\begin{aligned} &= \log_a\left(\frac{1}{\sqrt{x}-\sqrt{x-1}}\frac{1}{\sqrt{x}+\sqrt{x-1}}\right) \text{ by Theorem 5.5.A(3)} \\ &= \log_a\left(\frac{1}{x-(x-1)}\right) \\ &= \log_a\frac{1}{1} = \log_a 1 = 0 \text{ since } a^0 = 1, \end{aligned}$$

as claimed. □