Precalculus 1 (Algebra)

Chapter 5. Exponential and Logarithmic Functions

5.8. Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models—Exercises, Examples, Proofs

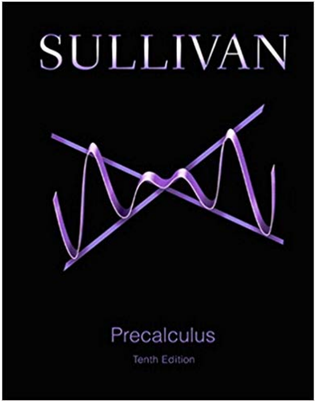

Table of contents

1. Page 332 Number 2. Growth of Bacteria
2. Page 332 Number 4. Radioactive Decay
3. Page 332 Number 14. Newton's Law of Cooling
4. Page 333 Number 26. Population of an Endangered Species

Page 332 Number 2

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution. **(a)** The number of bacteria at $t = 0$ is $N(0) = 1000e^{0.01(0)} = 1000e^{0} = 1000(1) = \boxed{1000}$. Notice that this is N_0 if we write $N(t) = N_0 e^{kt}$. □

Page 332 Number 2

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution. **(a)** The number of bacteria at $t = 0$ is $N(0) = 1000e^{0.01(0)} = 1000e^0 = 1000(1) = \boxed{1000}$. Notice that this is N_0 if we write $N(t) = N_0e^{kt}$. □

(b) The growth rate is parameter k where $N(t) = N_0e^{kt}$, so the growth rate is $\boxed{0.01}$. □

Page 332 Number 2

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution. **(a)** The number of bacteria at $t = 0$ is $N(0) = 1000e^{0.01(0)} = 1000e^0 = 1000(1) = \boxed{1000}$. Notice that this is N_0 if we write $N(t) = N_0 e^{kt}$. □

(b) The growth rate is parameter k where $N(t) = N_0 e^{kt}$, so the growth rate is $\boxed{0.01}$. □

(c) When $t = 4$ hours, the number of bacteria is $N(4) = 1000e^{0.01(4)} = \boxed{1000e^{0.04} \approx 1041}$. □

Page 332 Number 2

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution. (a) The number of bacteria at $t = 0$ is $N(0) = 1000e^{0.01(0)} = 1000e^0 = 1000(1) = \boxed{1000}$. Notice that this is N_0 if we write $N(t) = N_0e^{kt}$. □

(b) The growth rate is parameter k where $N(t) = N_0e^{kt}$, so the growth rate is $\boxed{0.01}$. □

(c) When $t = 4$ hours, the number of bacteria is $N(4) = 1000e^{0.01(4)} = \boxed{1000e^{0.04} \approx 1041}$. □

Page 332 Number 2 (continued)

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution (continued). (d) The number of bacteria will be 1700 when t satisfies $N(t) = 1000e^{0.01t} = 1700$, or $e^{0.01t} = 1700/1000 = 1.7$.

Page 332 Number 2 (continued)

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution (continued). (d) The number of bacteria will be 1700 when t satisfies $N(t) = 1000e^{0.01t} = 1700$, or $e^{0.01t} = 1700/1000 = 1.7$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 1.7$, or $0.01t = \ln 1.7$, or $\boxed{t = 100 \ln 1.7 \approx 53 \text{ hours}}$. □

Page 332 Number 2 (continued)

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution (continued). (d) The number of bacteria will be 1700 when t satisfies $N(t) = 1000e^{0.01t} = 1700$, or $e^{0.01t} = 1700/1000 = 1.7$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 1.7$, or $0.01t = \ln 1.7$, or $\boxed{t = 100 \ln 1.7 \approx 53 \text{ hours}}$. □

(e) Since the initial number of bacteria is 1000, then the number has doubled when the number of bacteria is 2000. So we solve for t in the equation $N(t) = 1000e^{0.01t} = 2000$, or $e^{0.01t} = 2000/1000 = 2$.

Page 332 Number 2 (continued)

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution (continued). **(d)** The number of bacteria will be 1700 when t satisfies $N(t) = 1000e^{0.01t} = 1700$, or $e^{0.01t} = 1700/1000 = 1.7$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 1.7$, or $0.01t = \ln 1.7$, or $\boxed{t = 100 \ln 1.7 \approx 53 \text{ hours}}$. □

(e) Since the initial number of bacteria is 1000, then the number has doubled when the number of bacteria is 2000. So we solve for t in the equation $N(t) = 1000e^{0.01t} = 2000$, or $e^{0.01t} = 2000/1000 = 2$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 2$, or $0.01t = \ln 2$, or $\boxed{t = 100 \ln 2 \approx 69 \text{ hours}}$. □

Page 332 Number 2 (continued)

Page 332 Number 2. The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$. **(a)** Determine the number of bacteria at $t = 0$ hours. **(b)** What is the growth rate of the bacteria? **(c)** What is the population after 4 hours? **(d)** When will the number of bacteria reach 1700? **(e)** When will the number of bacteria double?

Solution (continued). **(d)** The number of bacteria will be 1700 when t satisfies $N(t) = 1000e^{0.01t} = 1700$, or $e^{0.01t} = 1700/1000 = 1.7$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 1.7$, or $0.01t = \ln 1.7$, or $\boxed{t = 100 \ln 1.7 \approx 53 \text{ hours}}$. □

(e) Since the initial number of bacteria is 1000, then the number has doubled when the number of bacteria is 2000. So we solve for t in the equation $N(t) = 1000e^{0.01t} = 2000$, or $e^{0.01t} = 2000/1000 = 2$. So we take a natural logarithm of both sides to get $\ln e^{0.01t} = \ln 2$, or $0.01t = \ln 2$, or $\boxed{t = 100 \ln 2 \approx 69 \text{ hours}}$. □

Page 332 Number 4

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(a)** What is the decay rate of iodine-131? **(b)** How much iodine-131 is left after 9 days? **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution. **(a)** The decay rate is parameter k where $A(t) = A_0 e^{kt}$, so the decay rate is $\boxed{-0.087}$. □

Page 332 Number 4

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(a)** What is the decay rate of iodine-131? **(b)** How much iodine-131 is left after 9 days? **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution. **(a)** The decay rate is parameter k where $A(t) = A_0 e^{kt}$, so the decay rate is $\boxed{-0.087}$. □

(b) When $t = 9$ days, the amount of iodine-131 is $A(9) = 100e^{-0.087(9)} = \boxed{100e^{-0.783} \approx 45.70 \text{ grams}}$. □

Page 332 Number 4

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(a)** What is the decay rate of iodine-131? **(b)** How much iodine-131 is left after 9 days? **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution. **(a)** The decay rate is parameter k where $A(t) = A_0 e^{kt}$, so the decay rate is $\boxed{-0.087}$. □

(b) When $t = 9$ days, the amount of iodine-131 is
$A(9) = 100e^{-0.087(9)} = \boxed{100e^{-0.783} \approx 45.70 \text{ grams}}$. □

Page 332 Number 4 (continued)

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution (continued). (c) The amount of iodine-131 will be 70 grams when t satisfies $A(t) = 100e^{-0.087t} = 70$, or $e^{-0.087t} = 70/100 = 0.7$. So we take a natural logarithm of both sides to get $\ln e^{-0.087t} = \ln 0.7$, or $-0.087t = \ln 0.7$, or $\boxed{t = -(\ln 0.7)/0.087 \approx 4.10 \text{ days}}$. □

Page 332 Number 4 (continued)

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution (continued). (c) The amount of iodine-131 will be 70 grams when t satisfies $A(t) = 100e^{-0.087t} = 70$, or $e^{-0.087t} = 70/100 = 0.7$. So we take a natural logarithm of both sides to get $\ln e^{-0.087t} = \ln 0.7$, or $-0.087t = \ln 0.7$, or $\boxed{t = -(\ln 0.7)/0.087 \approx 4.10 \text{ days}}$. □

(d) The half-life is the time it takes the original amount, $A_0 = 100$ grams, to decay down to $A = 50$ grams. So we set $A(t) = 100e^{-0.087t} = 50$ and solve for t.

Page 332 Number 4 (continued)

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution (continued). (c) The amount of iodine-131 will be 70 grams when t satisfies $A(t) = 100e^{-0.087t} = 70$, or $e^{-0.087t} = 70/100 = 0.7$. So we take a natural logarithm of both sides to get $\ln e^{-0.087t} = \ln 0.7$, or $-0.087t = \ln 0.7$, or $\boxed{t = -(\ln 0.7)/0.087 \approx 4.10 \text{ days}}$. □

(d) The half-life is the time it takes the original amount, $A_0 = 100$ grams, to decay down to $A = 50$ grams. So we set $A(t) = 100e^{-0.087t} = 50$ and solve for t. We have $e^{-0.087t} = 50/100 = 1/2$ and by taking a natural logarithm of both sides we get $\ln e^{-0.087t} = \ln 1/2$ or $-0.087t = \ln 1/2$, or $\boxed{t = -(\ln 1/2)/0.087 \approx 7.97 \text{ days}}$. □

Page 332 Number 4 (continued)

Page 332 Number 4. Iodine-131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine-131. **(c)** When will 70 grams of iodine-131 be left? **(d)** What is the half-life of iodine-131?

Solution (continued). (c) The amount of iodine-131 will be 70 grams when t satisfies $A(t) = 100e^{-0.087t} = 70$, or $e^{-0.087t} = 70/100 = 0.7$. So we take a natural logarithm of both sides to get $\ln e^{-0.087t} = \ln 0.7$, or $-0.087t = \ln 0.7$, or $\boxed{t = -(\ln 0.7)/0.087 \approx 4.10 \text{ days}}$. □

(d) The half-life is the time it takes the original amount, $A_0 = 100$ grams, to decay down to $A = 50$ grams. So we set $A(t) = 100e^{-0.087t} = 50$ and solve for t. We have $e^{-0.087t} = 50/100 = 1/2$ and by taking a natural logarithm of both sides we get $\ln e^{-0.087t} = \ln 1/2$ or $-0.087t = \ln 1/2$, or $\boxed{t = -(\ln 1/2)/0.087 \approx 7.97 \text{ days}}$. □

Page 332 Number 14

Page 332 Number 14. A thermometer reading 72°F is placed in a refrigerator where the temperature is a constant 38°F. **(a)** If the thermometer reads 60°F after 2 minutes, what will it read after 7 minutes? **(b)** How long will it take before the thermometer reads 39°F? **(c)** Determine the time that must elapse before the thermometer reads 45°F. **(d)** What do you notice about the temperature as time passes?

Solution. Recall that Newton's Law of Cooling states $u(t) = T + (u_0 - T)e^{kt}$ where $k < 0$. Here we measure temperature in degrees Fahrenheit and time in minutes. We are given an initial temperature of $u_0 = 72°\text{F}$ and a temperature of the medium of $T = 38°\text{F}$, so that for this problem $u(t) = 38 + (72 - 38)e^{kt} = 38 + 34e^{kt}$

(a) We can find k based on the given fact that $u(2) = 38 + 34e^{k(2)} = 60$, or $34e^{2k} = 22$, or $e^{2k} = 22/34$, or $\ln e^{2k} = \ln 22/34$, or $2k = \ln 22/34$, or $k = \frac{1}{2}\ln 22/34 \approx -0.218$. Hence, we have $u(t) = 38 + 34e^{-0.218t}$. So when $t = 7$ minutes, the thermometer will read $\boxed{u(7) = 38 + 34e^{-0.218(7)} \approx 45.4°\text{F}}$. □

Page 332 Number 14

Page 332 Number 14. A thermometer reading 72°F is placed in a refrigerator where the temperature is a constant 38°F. **(a)** If the thermometer reads 60°F after 2 minutes, what will it read after 7 minutes? **(b)** How long will it take before the thermometer reads 39°F? **(c)** Determine the time that must elapse before the thermometer reads 45°F. **(d)** What do you notice about the temperature as time passes?

Solution. Recall that Newton's Law of Cooling states $u(t) = T + (u_0 - T)e^{kt}$ where $k < 0$. Here we measure temperature in degrees Fahrenheit and time in minutes. We are given an initial temperature of $u_0 = 72°\text{F}$ and a temperature of the medium of $T = 38°\text{F}$, so that for this problem $u(t) = 38 + (72 - 38)e^{kt} = 38 + 34e^{kt}$

(a) We can find k based on the given fact that $u(2) = 38 + 34e^{k(2)} = 60$, or $34e^{2k} = 22$, or $e^{2k} = 22/34$, or $\ln e^{2k} = \ln 22/34$, or $2k = \ln 22/34$, or $k = \frac{1}{2}\ln 22/34 \approx -0.218$. Hence, we have $u(t) = 38 + 34e^{-0.218t}$. So when $t = 7$ minutes, the thermometer will read

$\boxed{u(7) = 38 + 34e^{-0.218(7)} \approx 45.4°\text{F}}$. □

Page 332 Number 14 (continued)

Page 332 Number 14. A thermometer reading 72°F is placed in a refrigerator where the temperature is a constant 38°F. **(b)** How long will it take before the thermometer reads 39°F? **(c)** Determine the time that must elapse before the thermometer reads 45°F. **(d)** What do you notice about the temperature as time passes?

Solution (continued). (b) The temperature will be 39°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 39$, or $34e^{-0.218t} = 1$ or $e^{-0.218t} = 1/34$ or $\ln e^{-0.218t} = \ln 1/34$, or $-0.218t = \ln 1/34$, or

$\boxed{t = -(\ln 1/34)/0.218 \approx 16.18 \text{ minutes}}$. □

(c) The temperature will be 45°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 45$, or $34e^{-0.218t} = 7$ or $e^{-0.218t} = 7/34$ or $\ln e^{-0.218t} = \ln 7/34$, or $-0.218t = \ln 7/34$, or

$\boxed{t = -(\ln 7/34)/0.218 \approx 7.25 \text{ minutes}}$. □

Page 332 Number 14 (continued)

Page 332 Number 14. A thermometer reading 72°F is placed in a refrigerator where the temperature is a constant 38°F. **(b)** How long will it take before the thermometer reads 39°F? **(c)** Determine the time that must elapse before the thermometer reads 45°F. **(d)** What do you notice about the temperature as time passes?

Solution (continued). (b) The temperature will be 39°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 39$, or $34e^{-0.218t} = 1$ or $e^{-0.218t} = 1/34$ or $\ln e^{-0.218t} = \ln 1/34$, or $-0.218t = \ln 1/34$, or

$\boxed{t = -(\ln 1/34)/0.218 \approx 16.18 \text{ minutes}}$. □

(c) The temperature will be 45°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 45$, or $34e^{-0.218t} = 7$ or $e^{-0.218t} = 7/34$ or $\ln e^{-0.218t} = \ln 7/34$, or $-0.218t = \ln 7/34$, or

$\boxed{t = -(\ln 7/34)/0.218 \approx 7.25 \text{ minutes}}$. □

(d) As time passes, the thermometer reading gets closer to the temperature of the refrigerator. In fact, for $t \to \infty$ we have $u(t) \to 38$. □

Page 332 Number 14 (continued)

Page 332 Number 14. A thermometer reading 72°F is placed in a refrigerator where the temperature is a constant 38°F. **(b)** How long will it take before the thermometer reads 39°F? **(c)** Determine the time that must elapse before the thermometer reads 45°F. **(d)** What do you notice about the temperature as time passes?

Solution (continued). (b) The temperature will be 39°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 39$, or $34e^{-0.218t} = 1$ or $e^{-0.218t} = 1/34$ or $\ln e^{-0.218t} = \ln 1/34$, or $-0.218t = \ln 1/34$, or

$\boxed{t = -(\ln 1/34)/0.218 \approx 16.18 \text{ minutes}}$. □

(c) The temperature will be 45°F when t satisfies $u(t) = 38 + 34e^{-0.218t} = 45$, or $34e^{-0.218t} = 7$ or $e^{-0.218t} = 7/34$ or $\ln e^{-0.218t} = \ln 7/34$, or $-0.218t = \ln 7/34$, or

$\boxed{t = -(\ln 7/34)/0.218 \approx 7.25 \text{ minutes}}$. □

(d) As time passes, the thermometer reading gets closer to the temperature of the refrigerator. In fact, for $t \to \infty$ we have $u(t) \to 38$. □

Page 333 Number 26

Page 333 Number 26. Environmentalists often capture an endangered species and transport the species to a controlled environment where the species can produce offspring and regenerate its population. Suppose that six American bald eagles are captured, transported to Montana, and set free. Based on experience, the environmentalists expect the population to grow according to the model $P(t) = \dfrac{500}{1 + 83.33e^{-0.162t}}$ where t is measured in years. **(a)** Determine the carrying capacity of the environment. **(b)** What is the growth rate of the bald eagle? **(c)** What is the population after 3 years? **(d)** When will the population be 300 eagles? **(e)** How long does it take for the population to reach one-half of the carrying capacity?

Solution. Recall that the logistic model gives population size as $P(t) = \dfrac{c}{1 + ae^{-bt}}$. So here, we have $c = 500$, $a = 83.33$, and $b = 0.162$.

(a) The carrying capacity is parameter c, so the $\boxed{\text{carrying capacity is } c = 500}$. □

Page 333 Number 26

Page 333 Number 26. Environmentalists often capture an endangered species and transport the species to a controlled environment where the species can produce offspring and regenerate its population. Suppose that six American bald eagles are captured, transported to Montana, and set free. Based on experience, the environmentalists expect the population to grow according to the model $P(t) = \dfrac{500}{1 + 83.33e^{-0.162t}}$ where t is measured in years. **(a)** Determine the carrying capacity of the environment. **(b)** What is the growth rate of the bald eagle? **(c)** What is the population after 3 years? **(d)** When will the population be 300 eagles? **(e)** How long does it take for the population to reach one-half of the carrying capacity?

Solution. Recall that the logistic model gives population size as $P(t) = \dfrac{c}{1 + ae^{-bt}}$. So here, we have $c = 500$, $a = 83.33$, and $b = 0.162$.

(a) The carrying capacity is parameter c, so the $\boxed{\text{carrying capacity is } c = 500}$. □

Page 333 Number 26 (continued 1)

Solution (continued). (b) The growth rate is b (if $b > 0$), so the $\boxed{\text{growth rate is } b = 0.162}$.

(c) When $t = 3$ years, the population will be $P(3) = \dfrac{500}{1 + 83.33e^{-0.162(3)}} = \dfrac{500}{1 + 83.33e^{-0.486}} \approx 9.57$. Rounding up, when $t = 3$ years, there will be $\boxed{\text{10 eagles in the population}}$.

Page 333 Number 26 (continued 1)

Solution (continued). (b) The growth rate is b (if $b > 0$), so the $\boxed{\text{growth rate is } b = 0.162}$.

(c) When $t = 3$ years, the population will be $P(3) = \dfrac{500}{1 + 83.33e^{-0.162(3)}} = \dfrac{500}{1 + 83.33e^{-0.486}} \approx 9.57$. Rounding up, when $t = 3$ years, there will be $\boxed{\text{10 eagles in the population}}$.

(d) The population will be 300 when $P(t) = \dfrac{500}{1 + 83.33e^{-0.162t}} = 300$, or when $\dfrac{1}{1 + 83.33e^{-0.162t}} = 300/500 = 3/5$, or $1 + 83.33e^{-0.162t} = 5/3$, or $83.33e^{-0.162t} = 2/3$, or $e^{-0.162t} = (2/3)/83.33$, or $\ln e^{-0.162t} = \ln 2/((3)(83.33))$, or $-0.162t = \ln 2/249.99$, or $\boxed{t = -(\ln 2/249.99)/0.162 \approx 29.80 \text{ years}}$.

Page 333 Number 26 (continued 1)

Solution (continued). (b) The growth rate is b (if $b > 0$), so the $\boxed{\text{growth rate is } b = 0.162}$.

(c) When $t = 3$ years, the population will be
$P(3) = \dfrac{500}{1 + 83.33e^{-0.162(3)}} = \dfrac{500}{1 + 83.33e^{-0.486}} \approx 9.57$. Rounding up, when $t = 3$ years, there will be $\boxed{\text{10 eagles in the population}}$.

(d) The population will be 300 when $P(t) = \dfrac{500}{1 + 83.33e^{-0.162t}} = 300$, or when $\dfrac{1}{1 + 83.33e^{-0.162t}} = 300/500 = 3/5$, or $1 + 83.33e^{-0.162t} = 5/3$, or $83.33e^{-0.162t} = 2/3$, or $e^{-0.162t} = (2/3)/83.33$, or $\ln e^{-0.162t} = \ln 2/((3)(83.33))$, or $-0.162t = \ln 2/249.99$, or $\boxed{t = -(\ln 2/249.99)/0.162 \approx 29.80 \text{ years}}$.

Page 333 Number 26 (continued 2)

Solution (continued). (e) The population reaches one-half of the carrying capacity when the population is size $500/2 = 250$, so we consider $P(t) = \dfrac{500}{1 + 83.33e^{-0.162t}} = 250$. This gives $\dfrac{1}{1 + 83.33e^{-0.162t}} = 250/500 = 1/2$, or $1 + 83.33e^{-0.162t} = 2$, or $83.33e^{-0.162t} = 1$, or $e^{-0.162t} = 1/83.33$, or $\ln e^{-0.162t} = \ln 1/83.33$, or $-0.162t = \ln 1/83.33$, or $\boxed{t = -(\ln 1/83.33)/0.162 \approx 27.30 \text{ years}}$. □