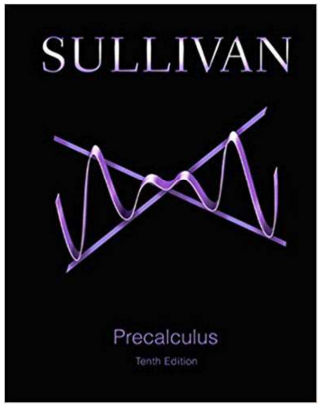
Precalculus 1 (Algebra)

Appendix A. Review

A.1. Algebra Essentials—Exercises, Examples, Proofs



Page A11 Numbers 12, 14, 18, 22

Page A11 Numbers 12, 14, 18, 22. Let the universal set be $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let $A = \{1, 3, 4, 5, 9\}$, $B = \{2, 4, 6, 7, 8\}$, and $C = \{1, 3, 4, 6\}$. Find **(12)** $A \cup C$, **(14)** $A \cap C$, **(18)** $\overline{C} = C^c$, and **(22)** $\overline{B} \cap \overline{C} = B^c \cap C^c$.

Solution. (12) By definition, the union of two sets is the set consisting of elements that belong to either set (or both sets). So $A \cup C = \{1, 3, 4, 5, 9\} \cup \{1, 3, 4, 6\} = \boxed{\{1, 3, 4, 5, 6, 9\}}$. □

(14) By definition, the intersection of two sets is the set consisting of elements that belong to both sets. So $A \cap C = \{1, 3, 4, 5, 9\} \cap \{1, 3, 4, 6\} = \boxed{\{1, 3, 4\}}$. □

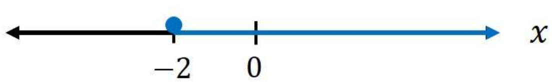
(18) By definition, the complement of a set is the set consisting of all elements in the universal set that are not in the given set. So $\overline{C} = C^c = \{0, 2, 5, 7, 8, 9\}$. □

(22) Similar to (18), we have $\overline{B} = B^c = \{0, 1, 3, 5, 9\}$. So $\overline{B} \cap \overline{C} = B^c \cap C^c = \{0, 1, 3, 5, 9\} \cap \{0, 2, 5, 7, 8, 9\} = \boxed{\{0, 5, 9\}}$. □

Page A11 Numbers 42 and 44

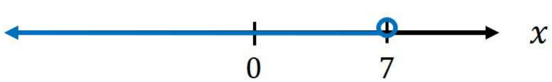
Page A11 Numbers 42 and 44. Graph the numbers x on the real number line which satisfy **(42)** $x \geq -2$, and **(44)** (modified) $x < 7$.

Solution. (42) To graph all x where $x \geq -2$, we graph $x = -2$ with a solid disk and shade the real number line to the right of -2:



□

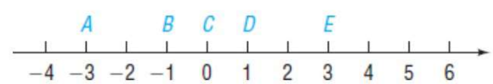
(44) To graph all x where $x < 7$, we graph $x = 7$ with a circle (since the inequality is strict) and shade the real number line to the left of 7:



□

Page A11 Numbers 46, 48, and 50

Page A11 Numbers 46, 48, and 50. Use the given real number line to compute each distance: **(46)** $d(C, A)$, **(48)** $d(C, E)$, and **(50)** $d(D, B)$.



Solution. First, the coordinate of A is $x_A = -3$, the coordinate of B is $x_B = -1$, the coordinate of C is $x_C = 0$, the coordinate of D is $x_D = 1$, and the coordinate of E is $x_E = 3$. So by the definition of the distance between two points on the real number line in terms of coordinates, we have the following.

(46) $d(C, A) = |x_A - x_C| = |(-3) - (0)| = |-3| = \boxed{3}$. □

(48) $d(C, E) = |x_E - x_C| = |(3) - (0)| = |3| = \boxed{3}$. □

(50) $d(D, B) = |x_B - x_D| = |(-1) - (1)| = |-2| = \boxed{2}$. □

Page A11 Numbers 52 and 54, Page A12 numbers 64 and 68

Page A11 Numbers 52 and 54, Page A12 numbers 64 and 68. Evaluate each expression if $x = -2$ and $y = 3$: **(52)** $3x + y$, **(54)** $-2x + xy$, **(64)** $|y|/y$, and **(68)** $3|x| + 2|y|$.

Solution. We substitute the given values for x and y.

(52) $3x + y = 3(-2) + (3) = -6 + 3 = \boxed{-3}$. □

(54) $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = \boxed{-2}$. □

(64) $|y|/y = |(3)|/(3) = 3/3 = \boxed{1}$. Notice that $|x|/x = |(-2)|/(-2) = 2/(-2) = \boxed{-1}$. □

(68) $3|x| + 2|y| = 3|(-2)| + 2|(3)| = 3(2) + 2(3) = 6 + 6 = \boxed{12}$. □

Page A12 Number 74

Page A12 Number 74. Determine which of the values (a) $x = 3$, (b) $x = 1$, (c) $x = 0$, (d) $x = -1$, is in the domain of variable x in the expression $\dfrac{x^3}{x^2 - 1}$.

Solution. Since the expression involves division, we must avoid division by 0 by Note A.1.D.

Notice that for $x = 3$, $\dfrac{x^3}{x^2 - 1} = \dfrac{(3)^3}{(3)^2 - 1} = \dfrac{27}{8}$ and so $x = 3$ is in the domain of the expression.

Notice that for $x = 1$, $x^2 - 1 = (1)^2 - 1 = 1 - 1 = 0$ and since the expression $x^2 - 1$ is in the denominator of the given expression, then $x = 1$ would produce division by 0 in the given expression so that $x = 1$ is not in the domain of the given expression.

Page A12 Number 74 (continued)

Page A12 Number 74. Determine which of the values (a) $x = 3$, (b) $x = 1$, (c) $x = 0$, (d) $x = -1$, is in the domain of variable x in the expression $\dfrac{x^3}{x^2 - 1}$.

Solution (continued). Notice that for $x = 0$, $\dfrac{x^3}{x^2 - 1} = \dfrac{(0)^3}{(0)^2 - 1} = \dfrac{0}{-1} = 0$ and so $x = 0$ is in the domain of the expression.

Notice that for $x = -1$, $x^2 - 1 = (-1)^2 - 1 = 1 - 1 = 0$ and since the expression $x^2 - 1$ is in the denominator of the given expression, then $x = -1$ would produce division by 0 in the given expression so that $x = -1$ is not in the domain of the given expression.

So of the given values, $\boxed{x = 3 \text{ and } x = 0 \text{ are in the domain}}$ of the expression. □

Page A12 Number 80

Page A12 Number 80. Determine the domain of the variable x in the expression $\dfrac{x - 2}{x - 6}$.

Solution. As observed in Note A.1.D, at this stage the algebraic manipulations which must be avoided are division by 0 and square roots of negatives. In the given expression, there are no square roots but there is division. So we find the "bad" value(s) of x by setting the denominator of the expression equal to 0: $x - 6 = 0$ or $x = 6$. So the domain of x is all $\boxed{\text{real numbers except for 6}}$. □

Page A12 Numbers 90, 92, and 98

Page A12 Numbers 90, 92, and 98. Simplify each expression:
(90) $4^{-2} \times 4^3$, **(92)** $(2^{-1})^{-3}$, and **(98)** $(-4x^2)^{-1}$.

Solution. By the the definition of a^{-n} where n is a positive integer and the Laws of Exponents, Theorem A.1.A, we have the following.

(90) $4^{-2} \times 4^3 = \frac{1}{4^2} \times 4^3 = \frac{4^3}{4^2} = 4^{3-2} = 4^1 = \boxed{4}$. □

(92) $(2^{-1})^{-3} = 2^{(-1)(-3)} = 2^3 = 2 \times 2 \times 2 = \boxed{8}$. □

(98)
$(-4x^2)^{-1} = (-4)^{-1}(x^2)^{-1} = \frac{1}{-4}x^{(2)(-1)} = \frac{1}{-4}x^{-2} = \frac{1}{-4}\frac{1}{x^2} = \boxed{\frac{1}{-4x^2}}$. □