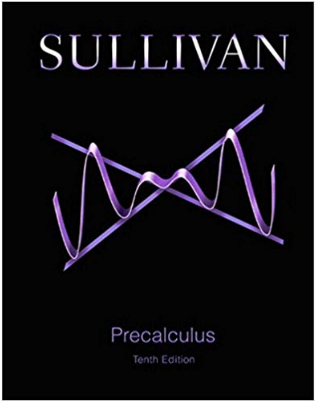
Precalculus 1 (Algebra)

Appendix A. Review

A.1. Algebra Essentials—Exercises, Examples, Proofs



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Page A11 Numbers 12, 14, 18, 22

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Solution. **(12)** By definition, the union of two sets is the set consisting of elements that belong to either set (or both sets). So $A \cup C = \{1, 3, 4, 5, 9\} \cup \{1, 3, 4, 6\} = \boxed{\{1, 3, 4, 5, 6, 9\}}$. □

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(14) By definition, the intersection of two sets is the set consisting of elements that belong to both sets. So
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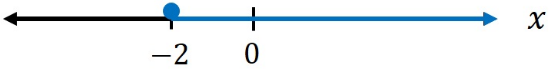

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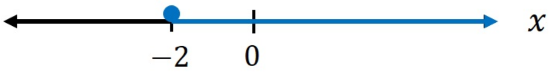

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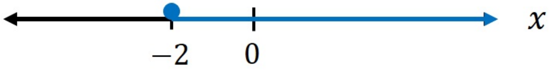

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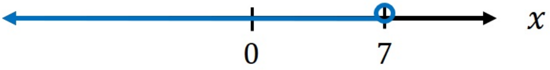

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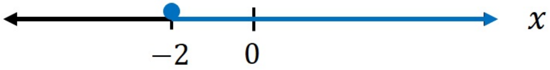

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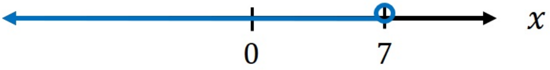

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Page A11 Numbers 46, 48, and 50

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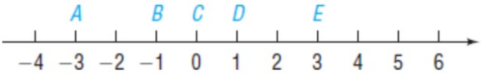

Solution. First, the coordinate of A is $x_A = -3$, the coordinate of B is $x_B = -1$, the coordinate of C is $x_C = 0$, the coordinate of D is $x_D = 1$, and the coordinate of E is $x_E = 3$. So by the definition of the distance between two points on the real number line in terms of coordinates, we have the following.

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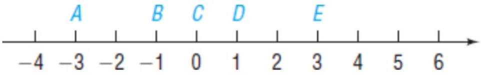

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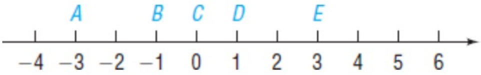

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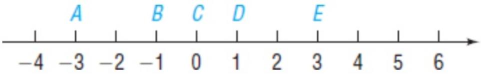

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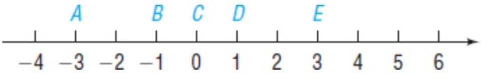

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Solution. Since the expression involves division, we must avoid division by 0 by Note A.1.D.

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Notice that for $x = 3$, $\dfrac{x^3}{x^2 - 1} = \dfrac{(3)^3}{(3)^2 - 1} = \dfrac{27}{8}$ and so $x = 3$ is in the domain of the expression.

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Notice that for $x = 1$, $x^2 - 1 = (1)^2 - 1 = 1 - 1 = 0$ and since the expression $x^2 - 1$ is in the denominator of the given expression, then $x = 1$ would produce division by 0 in the given expression so that $x = 1$ is not in the domain of the given expression.

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So of the given values, $\boxed{x = 3 \text{ and } x = 0 \text{ are in the domain}}$ of the expression. □

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Solution. As observed in Note A.1.D, at this stage the algebraic manipulations which must be avoided are division by 0 and square roots of negatives. In the given expression, there are no square roots but there is division.

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Solution. As observed in Note A.1.D, at this stage the algebraic manipulations which must be avoided are division by 0 and square roots of negatives. In the given expression, there are no square roots but there is division. So we find the "bad" value(s) of x by setting the denominator of the expression equal to 0: $x - 6 = 0$ or $x = 6$. So the domain of x is all real numbers except for 6. □

Page A12 Number 80

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Solution. As observed in Note A.1.D, at this stage the algebraic manipulations which must be avoided are division by 0 and square roots of negatives. In the given expression, there are no square roots but there is division. So we find the "bad" value(s) of x by setting the denominator of the expression equal to 0: $x - 6 = 0$ or $x = 6$. So the domain of x is all $\boxed{\text{real numbers except for 6}}$. □

Page A12 Numbers 90, 92, and 98

Page A12 Numbers 90, 92, and 98. Simplify each expression: **(90)** $4^{-2} \times 4^3$, **(92)** $(2^{-1})^{-3}$, and **(98)** $(-4x^2)^{-1}$.

Solution. By the the definition of a^{-n} where n is a positive integer and the Laws of Exponents, Theorem A.1.A, we have the following.

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