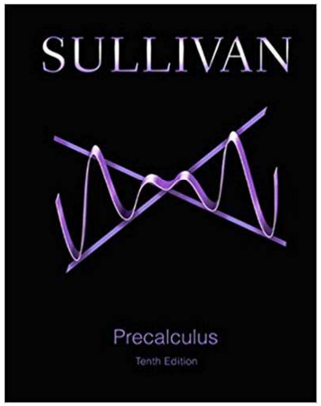
Precalculus 1 (Algebra)

Appendix A. Review

A.10. *n*th Roots; Rational Exponents—Exercises, Examples, Proofs



Page A88 Numbers 12, 14, 22, 30, 36, and 38

Page A88 Numbers 12, 14, 22, 30, 36, and 38. Simplify each expression: **(12)** $\sqrt[4]{16}$, **(14)** $\sqrt[3]{-1}$, **(22)** $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}$, **(30)** $\left(\sqrt[3]{3}\sqrt{10}\right)^4$, **(36)** $2\sqrt{12} - 3\sqrt{27}$, and **(38)** $\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right)$.

Solution. (12) For $\sqrt[4]{16}$, we seek a nonnegative real number a that when raised to the 4th power gives 16, $a^4 = 16$. We have $a = 2$ since $2^4 = 2 \times 2 \times 2 \times 2 = 16$, and so $\boxed{\sqrt[4]{16} = 2}$.

(12) For $\sqrt[3]{-1}$, we seek a real number a that when raised to the 3rd power gives -1, $a^3 = -1$. We have $a = -1$ since $(-1)^3 = (-1) \times (-1) \times (-1) = -1$, and so $\boxed{\sqrt[3]{-1} = -1}$.

Page A88 Numbers 12, 14, 22, 30, 36, and 38 (continued 1)

Solution. (22) We first simplify under the radical and have, by properties of exponents, $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \sqrt[3]{\dfrac{3}{81}\dfrac{x}{x^4}\dfrac{y^2}{y^2}} = \sqrt[3]{\dfrac{1}{27}\dfrac{1}{x^3}(1)}$ if $y \neq 0$. Now by properties of roots (Notes A.10.A), $\sqrt[3]{\dfrac{1}{27}\dfrac{1}{x^3}(1)} = \sqrt[3]{\dfrac{1}{27}}\sqrt[3]{\dfrac{1}{x^3}}$ and since $(1/3)^3 = 1/27$ and $(1/x)^3 = 1/x^3$ then $\sqrt[3]{\dfrac{1}{27}}\sqrt[3]{\dfrac{1}{x^3}} = \dfrac{1}{3}\dfrac{1}{x} = \dfrac{1}{3x}$. So $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \boxed{\dfrac{1}{3x} \text{ if } y \neq 0}$. □

(30) By properties of exponents, $\left(\sqrt[3]{3}\sqrt{10}\right)^4 = (\sqrt[3]{3})^4(\sqrt{10})^4 = (\sqrt[3]{3})^3\sqrt[3]{3}(\sqrt{10})^2(\sqrt{10})^2$. Now $(\sqrt[3]{3})^3 = 3$ and $(\sqrt{10})^2 = 10$, so
$\left(\sqrt[3]{3}\sqrt{10}\right)^4 = (\sqrt[3]{3})^3\sqrt[3]{3}(\sqrt{10})^2(\sqrt{10})^2 = (3)\sqrt[3]{3}(10)(10) = \boxed{300\sqrt[3]{3}}$. □

Page A88 Numbers 12, 14, 22, 30, 36, and 38 (continued 2)

Solution. (36) We express the numbers under the radicals as products to get $2\sqrt{12} - 3\sqrt{27} = 2\sqrt{3 \times 2^2} - 3\sqrt{3^3}$ so that we can apply properties of roots (Notes A.10.A) to get
$2\sqrt{12} - 3\sqrt{27} = 2\sqrt{2^2 \times 3} - 3\sqrt{3^3} = 2\sqrt{2^2}\sqrt{3} - 3\sqrt{3^2}\sqrt{3}$. Since $\sqrt{2^2} = 2$ and $\sqrt{3^2} = 3$, then
$2\sqrt{12} - 3\sqrt{27} = 2(2\sqrt{3}) - 3(3\sqrt{3}) = (4-9)\sqrt{3} = \boxed{-5\sqrt{3}}$. □

(38) By FOIL we have
$\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right) = (\sqrt{5})(\sqrt{5}) + (\sqrt{5})(3) + (-2)(\sqrt{5}) + (-2)(3) =$
$\sqrt{5}^2 + (3-2)\sqrt{5} - 6 = 5 + \sqrt{5} - 6 = \boxed{-1+\sqrt{5}}$. □

Page A88 numbers 52 and 58

Page A88 numbers 52 and 58. Rationalize the denominator of each expression. (Assume that all variables are positive when they appear.) **(52)** $\dfrac{-\sqrt{3}}{\sqrt{8}}$ and **(58)** $\dfrac{-3}{\sqrt{5}+4}$.

Solution. (52) To get a rational denominator, we multiply by a version of 1 that eliminates the radical term in the denominator:

$$\frac{-\sqrt{3}}{\sqrt{8}} = \frac{-\sqrt{3}}{\sqrt{8}}(1) = \frac{-\sqrt{3}}{\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right) = \frac{-\sqrt{3}\sqrt{8}}{\sqrt{8}\sqrt{8}} = \frac{-\sqrt{3}\sqrt{2\times 2^2}}{(\sqrt{8})^2}$$

$$= \frac{-\sqrt{3}\sqrt{2}\sqrt{2^2}}{8} = \frac{-2\sqrt{2}\sqrt{3}}{8} = \frac{-\sqrt{2}\sqrt{3}}{4} = \boxed{\frac{-\sqrt{6}}{4}}.$$

□

Page A88 numbers 52 and 58 (continued)

(58) $\dfrac{-3}{\sqrt{5}+4}$.

Solution. (58) To get a rational denominator, we multiply by a version of 1 that eliminates the radical term in the denominator. We do so by taking advantage of the identity $a^2 - b^2 = (a-b)(a+b)$:

$$\frac{-3}{\sqrt{5}+4} = \frac{-3}{\sqrt{5}+4}(1) = \frac{-3}{\sqrt{5}+4}\left(\frac{\sqrt{5}-4}{\sqrt{5}-4}\right)$$

$$= \frac{-3(\sqrt{5}-4)}{(\sqrt{5}+4)(\sqrt{5}-4)} = \frac{-3(\sqrt{5}-4)}{(\sqrt{5})^2-(4)^2} = \frac{-3\sqrt{5}+12}{(5)-16}$$

$$= \frac{-3\sqrt{5}+12}{-11} = \frac{-(3\sqrt{5}-12)}{-(11)} = \boxed{\frac{3\sqrt{5}-12}{11}}.$$

□

Page A88 Number 66

Page A88 Number 66. Solve the equation $\sqrt{12-x} = x$.

Solution. First, notice that x is a square root and so $x \geq 0$. We square both sides to eliminate the square root, so $(\sqrt{12-x})^2 = x^2$ or $12 - x = x^2$ or $x^2 + x - 12 = 0$ or (factoring) $(x-3)(x+4) = 0$. So we initially have the solutions of $x = 3$ and $x = -4$. But we first observed that $x \geq 0$, so the only solution is $\boxed{x = 3}$. □

Note. When squaring both sides of an equation, we loose negative signs and run the risk of introducing *extraneous roots* (see Note A.6.B). Here, the value $x = -4$ is an extraneous root and, in fact, not a solution to the original equation.

Page A88 Numbers 80 and 88

Page A88 Numbers 80 and 88. Simplify each expression. Express your answer so that only positive exponents occur. Assume that the variables are positive. **(80)** $-25^{-1/2}$, and **(88)** $\dfrac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}}$.

Solution. (80) Since a negative exponent indicates a reciprocal, we have $-25^{-1/2} = \dfrac{-1}{25^{1/2}}$. Since an exponent of 1/2 indicates a square root, then we have $-25^{-1/2} = \dfrac{-1}{\sqrt{25}} = \dfrac{-1}{\sqrt{5^2}} = \boxed{\dfrac{-1}{5}}$. □

(88) By the properties of roots, we have $\dfrac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}} = \dfrac{x^{1/4}y^{1/4}(x^2)^{1/2}(y^2)^{1/2}}{(x^2)^{3/4}y^{3/4}} = \dfrac{x^{1/4}y^{1/4}x^1y^1}{x^{3/2}y^{3/4}} = x^{1/4}x^1x^{-3/2}y^{1/4}y^1y^{-3/4} = x^{1/4+1-3/2}y^{1/4+1-3/4} = x^{-1/4}y^{1/2} = \boxed{\dfrac{y^{1/2}}{x^{1/4}}}$. □

Page A89 Number 112

Page A89 Number 112. Consider $6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2}$ where $x \geq 3/4$. Factor and express your answer so that only positive exponents occur.

Solution. We can factor out the lowest powers of $(6x+1)$ and $(4x-3)$, which are $(6x+1)^{1/3}$ and $(4x-3)^{1/2}$:

$$\begin{aligned} & 6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2} \\ &= 6(6x+1)^{1/3}(4x-3)^{1/2}\left((4x-3)^{3/2-1/2} + (6x+1)^{4/3-1/3}\right) \\ &= 6(6x+1)^{1/3}(4x-3)^{1/2}\left((4x-3) + (6x+1)\right) \\ &= \boxed{6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2)}. \end{aligned}$$

□