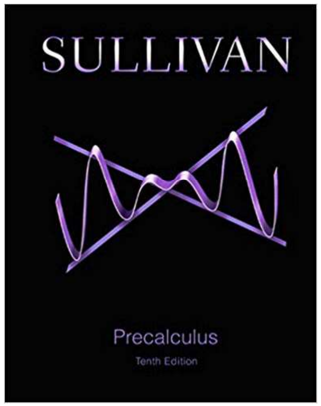
Precalculus 1 (Algebra)

Appendix A. Review

A.3. Polynomials—Exercises, Examples, Proofs



Page A30 Number 24

Page A30 Number 24. Tell whether the expression $3x^2 + 4$ is a monomial. If it is, name the variable and the coefficient and give the degree of the monomial. If it is not a monomial, state why not.

Solution. The expression $3x^2 + 4$ is $\boxed{\text{not a monomial}}$, but is a binomial which is the sum of the two monomials $3x^2$ and 4. Notice that monomial $3x^2$ is degree 2 with coefficient 3 and variable x. □

Page A30 Numbers 48 and 50

Page A30 numbers 48 and 50. Multiply, as indicated and express your answer as a single polynomial in standard form: **(48)** $(3x + 1)(2x + 1)$, and **(50)** $(x - 1)(x + 1)$.

Solution. (48) Applying FOIL we have $(3x + 1)(2x + 1) = (3x)(2x) + (3x)(1) + (1)(2x) + (1)(1) = 6x^2 + 3x + 2x + 1 = \boxed{6x^2 + 5x + 1}$. □

(50) Applying FOIL we have $(x - 1)(x + 1) = (x)(x) + (x)(1) + (-1)(x) + (-1)(1) = x^2 + x - x - 1 = \boxed{x^2 - 1}$. □

Page A30 Numbers 66 and 74

Page A30 Numbers 66 and 74. Find the quotient and the remainder. Check your work by verifying that (Quotient)(Divisor) + remainder = Dividend. **(66)** Divide $5x^4 - x^2 + x - 2$ by $x^2 + 2$, and **(74)** divide $1 - x^2 + x^4$ by $x^2 - x + 1$.

Solution. (66) We perform long division:

$$\begin{array}{r|l} & 5x^2 - 11 \\ \hline x^2 + 2 & 5x^4 - x^2 + x - 2 \\ & 5x^4 + 10x^2 \\ \hline & -11x^2 + x - 2 \\ & -11x^2 - 22 \\ \hline & x + 20 \end{array}$$

So $\boxed{\text{the quotient is } 5x^2 - 11 \text{ and the remainder is } x + 20}$. To check: $(5x^2 - 11)(x^2 + 2) + (x + 20) = (5x^2)(x^2) + (5x^2)(2) + (-11)(x^2) + (-11)(2) + x + 20 = 5x^4 + 10x^2 - 11x^2 - 22 + x + 20 = 5x^4 - x^2 + x - 2$. □

Page A30 Numbers 66 and 74 (continued)

Solution (continued). (74) We perform long division:

$$
\begin{array}{r|rrrrrrrr}
 & & & & x^2 & + & x & - & 1 \\ \hline
x^2-x+1 & x^4 & & - & x^2 & & & + & 1 \\
 & x^4 & - x^3 & + & x^2 & & & & \\ \hline
 & & x^3 & - & 2x^2 & & & & \\
 & & x^3 & - & x^2 & + & x & & \\ \hline
 & & & - & x^2 & - & x & + & 1 \\
 & & & - & x^2 & + & x & - & 1 \\ \hline
 & & & & & - & 2x & + & 2
\end{array}
$$

So $\boxed{\text{the quotient is } x^2+x-1 \text{ and the remainder is } -2x+2}$. To check: $(x^2+x-1)(x^2-x+1)+(-2x+2) = x^2(x^2-x+1)+x(x^2-x+1)-1(x^2-x+1)-2x+2 = x^4-x^3+x^2+x^3-x^2+x-x^2+x-1-2x+2 = x^4-x^2+1$. □

Page A30 Number 78

Page A30 Number 78. Factor completely x^2-9. If it cannot be factored, say it is prime.

Solution. The special product "Difference of Two Square" states $(x-a)(x+a) = x^2-a^2$. The right hand side of this equation with $a=3$ is x^2-9, so we can factor x^2-9 as $\boxed{(x-3)(x+3)}$. □

Page A30 Number 83

Page A30 Number 83. Factor completely $x^2-10x+21$. If it cannot be factored, say it is prime.

Solution. We look for integers a and b such that $a+b=B=-10$ and $ab=C=21$. From the condition $ab=21$ we see that we must use ± 1 and ± 21, or ± 3 and ± 7. Since $(-3)+(-7)=-10$, we take $a=-3$ and $b=-7$ to get $x+a=x-3$ and $x+b=x-7$. Then $\boxed{x^2-10x+21=(x-3)(x-7)}$. □

Page A30 Number 124

Page A30 Number 124. Factor completely x^4+x^3+x+1. If it cannot be factored, say it is prime.

Solution. We first factor by grouping: $x^4+x^3+x+1 = (x^4+x^3)+(x+1) = x^3(x+1)+1(x+1) = (x^3+1)(x+1)$. Since $x^3+1 = x^3+1^3$ is a Sum of Two Cubes, we have by one of the "special products" that $x^3+a^3 = (x+a)(x^2-ax+a^2)$ and so (with $a=1$) we know $x^3+1=(x+1)(x^2-x+1)$. Hence we can factor as $x^4+x^3+x+1 = (x^3+1)(x+1) = (x+1)(x^2-x+1)(x+1) = \boxed{(x^2-x+1)(x+1)^2}$.

Notice that x^2-x+1 cannot be factored over the integers since this would require integers a and b with $ab=1$ and $a+b=-1$, which cannot happen. □

Page A30 Number 98

Page A30 Number 98. Factor completely $8x^2 + 6x - 2$. If it cannot be factored, say it is prime.

Solution. First, we factor out the common factor 2 to get $2(4x^2 + 3x - 1)$. Now in $Ax^2 + Bx + C = 4x^2 + 3x - 1$, we have $AC = (4)(-1) = -4$. Now we need integers a and b such that $ab = AC = -4$ and $a + b = B = 3$. So we can take $a = 4$ and $b = -1$. This gives $Ax^2 + Bx + C = Ax^2 + ax + bx + C = 4x^2 + 4x - x - 1 = 4x(x+1) - (x+1) = (4x-1)(x+1)$. Hence $\boxed{8x^2 + 6x - 2x = 2(4x-1)(x+1)}$. □

Page A30 Number 100

Page A30 Number 100. Factor completely $x^4 - 1$. If it cannot be factored, say it is prime.

Solution. The special product "Difference of Two Square" states $(x-a)(x+a) = x^2 - a^2$. We have $x^4 - 1 = (x^2)^2 - 1^2$ so we factor to get $(x^2-1)(x^2+1)$. But $x^2 - 1$ is also a difference of two squares and $x^2 - 1 = (x-1)(x+1)$. Hence $x^4 - 1 = (x^2-1)(x^2+1) = \boxed{(x^2+1)(x-1)(x+1)}$. Notice that $x^2 + 1$ is prime by Theorem A.3.B, so it does not factor. □

Page A31 Number 126

Page A31 Number 126. Determine the number that should be added to complete the square of the expression $p^2 + 14p$. Then factor the expression.

Solution. We know $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$. So with $b = 14$ we add $(b/2)^2 = (14/2)^2 = 7^2 = 49$ to the given expression to get $\boxed{p^2 + 14p + 49 = (p + (14/2))^2 = (p+7)^2}$. □