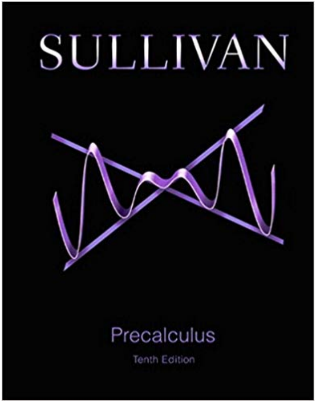
Precalculus 1 (Algebra)

Appendix A. Review

A.5. Rational Expressions—Exercises, Examples, Proofs



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Page A41 Number 8

Page A41 Number 8. Reduce to lowest terms: $\dfrac{4x^2 + 8x}{12x + 24}$.

Solution. We factor the numerator and denominator and cancel common expressions (with the added condition that we have not canceled expressions which are 0) to get:

$$\begin{aligned}\frac{4x^2 + 8x}{12x + 24} &= \frac{4x(x+2)}{12(x+2)} \\ &= \frac{4x}{12} \text{ if } x \neq -2 \\ &= \frac{x}{3} \text{ if } x \neq -2.\end{aligned}$$

So in lowest terms we have $\boxed{x/3 \text{ where } x \neq -2}$. □

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Page A41 Number 16

Page A41 Number 16. Perform the indicated operation and simplify the result. Leave your answer in factored form: $\dfrac{3}{2x} \times \dfrac{x^2}{6x+10}$.

Solution. To multiply quotients, we multiply the numerators together and multiply the denominators together (by Note A.5.A(1)) to get:

$$\frac{3}{2x} \times \frac{x^2}{6x+10} = \frac{3x^2}{2x(6x+10)}.$$

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Cancelling the common x term we get

$$\frac{3}{2x} \times \frac{x^2}{6x+10} = \frac{3x^2}{2x(6x+10)} = \frac{3x}{2(6x+10)} \text{ if } x \neq 0.$$

We can also factor out a 2 from $6x+10$ to get

$$\boxed{\frac{3}{2x} \times \frac{x^2}{6x+10} = \frac{3x}{4(3x+5)} \text{ if } x \neq 0}.$$

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Page A41 Number 19. Perform the indicated operation and simplify the result. Leave your answer in factored form: $\dfrac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}}$.

Solution. Division by a quotient is equivalent to multiplication by its reciprocal (by Note A.5.A(2)), so we have

$$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} = \frac{8x}{x^2-1} \times \frac{x+1}{10x} = \frac{8x(x+1)}{(x^2-1)10x}.$$

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Since $x^2 - 1$ is a difference of two squares (namely, x and 1), then $x^2 - 1 = (x-1)(x+1)$ and

$$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} = \frac{8x(x+1)}{(x^2-1)10x} = \frac{8x(x+1)}{(x-1)(x+1)10x}.$$

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Solution (continued). ...

$$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} = \frac{8x(x+1)}{(x^2-1)10x} = \frac{8x(x+1)}{(x-1)(x+1)10x}.$$

Cancelling the x and $x+1$ expressions and noting that these cannot be 0 (that is, $x \neq 0$ and $x \neq -1$) and canceling the common multiple of 2 gives

$$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} = \boxed{\frac{4}{5(x-1)} \text{ if } x \neq -1 \text{ and } x \neq 0}.$$

Page A42 Number 28

Page A42 Number 28. Perform the indicated operation and simplify the result. Leave your answer in factored form: $\dfrac{x}{x-3} - \dfrac{x+1}{x^2+5x-24}$.

Solution. Notice that $x^2+5x-24 = (x-3)(x+8)$ so we can factor the denominator in the second expression and find a common denominator to do the subtraction:

$$\frac{x}{x-3} - \frac{x+1}{x^2+5x-24} = \frac{x}{x-3} - \frac{x+1}{(x-3)(x+8)}$$

$$= \frac{x}{x-3}\left(\frac{x+8}{x+8}\right) - \frac{x+1}{(x-3)(x+8)} = \frac{x(x+8)}{(x-3)(x+8)} - \frac{x+1}{(x-3)(x+8)}$$

$$= \frac{x(x+8)-(x+1)}{(x-3)(x+8)} = \frac{x^2+8x-x-1}{(x-3)(x+8)} = \boxed{\frac{x^2+7x-1}{(x-3)(x+8)}}.$$

(Notice that x^2+7x-1 does not factor over the integers.) □

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Solution. Notice that $x^2 + 5x - 24 = (x-3)(x+8)$ so we can factor the denominator in the second expression and find a common denominator to do the subtraction:

$$\frac{x}{x-3} - \frac{x+1}{x^2+5x-24} = \frac{x}{x-3} - \frac{x+1}{(x-3)(x+8)}$$

$$= \frac{x}{x-3}\left(\frac{x+8}{x+8}\right) - \frac{x+1}{(x-3)(x+8)} = \frac{x(x+8)}{(x-3)(x+8)} - \frac{x+1}{(x-3)(x+8)}$$

$$= \frac{x(x+8)-(x+1)}{(x-3)(x+8)} = \frac{x^2+8x-x-1}{(x-3)(x+8)} = \boxed{\frac{x^2+7x-1}{(x-3)(x+8)}}.$$

(Notice that $x^2 + 7x - 1$ does not factor over the integers.) $\square$

Page A42 Number 32

Page A42 Number 32. Perform the indicated operation and simplify the result. Leave your answer in factored form:
$\dfrac{2}{(x+2)^2(x-1)} - \dfrac{6}{(x+2)(x-1)^2}$.

Solution. We get a common denominator to do the subtraction:

$$\begin{aligned}
&\frac{2}{(x+2)^2(x-1)} - \frac{6}{(x+2)(x-1)^2} \\
&= \frac{2}{(x+2)^2(x-1)}\left(\frac{x-1}{x-1}\right) - \frac{6}{(x+2)(x-1)^2}\left(\frac{x+2}{x+2}\right) \\
&= \frac{2(x-1)}{(x+2)^2(x-1)^2} - \frac{6(x+2)}{(x+2)^2(x-1)^2} = \frac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2} \\
&= \frac{2x-2-6x-12}{(x+2)^2(x-1)^2} = \frac{-4x-14}{(x+2)^2(x-1)^2} = \boxed{\frac{-2(x+7)}{(x+2)^2(x-1)^2}}.
\end{aligned}$$

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$$\frac{2}{(x+2)^2(x-1)} - \frac{6}{(x+2)(x-1)^2}$$
$$= \frac{2}{(x+2)^2(x-1)}\left(\frac{x-1}{x-1}\right) - \frac{6}{(x+2)(x-1)^2}\left(\frac{x+2}{x+2}\right)$$
$$= \frac{2(x-1)}{(x+2)^2(x-1)^2} - \frac{6(x+2)}{(x+2)^2(x-1)^2} = \frac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2}$$
$$= \frac{2x-2-6x-12}{(x+2)^2(x-1)^2} = \frac{-4x-14}{(x+2)^2(x-1)^2} = \boxed{\frac{-2(x+7)}{(x+2)^2(x-1)^2}}.$$

□

Page A42 Number 34

Page A42 Number 34. Perform the indicated operation and simplify the result. Leave your answer in factored form: $\dfrac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}}$.

Solution. The LCM in both the numerator and denominator is x^2 so we multiply by x^2/x^2 to get

$$\frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}} = \frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}}\left(\frac{x^2}{x^2}\right) = \frac{4x^2+\frac{x^2}{x^2}}{3x^2-\frac{x^2}{x^2}} = \frac{4x^2+1}{3x^2-1}, \text{ if } x \neq 0.$$

So

$$\frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}} = \boxed{\frac{4x^2+1}{3x^2-1} \text{ if } x \neq 0}.$$

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Page A42 Number 36

Page A42 Number 36. Perform the indicated operation and simplify the result. Leave your answer in factored form: $\dfrac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}}$.

Solution. We first get common denominators in order to do the subtraction:

$$\frac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}} = \frac{\frac{2x+5}{x}\left(\frac{x-3}{x-3}\right) - \frac{x}{x-3}\left(\frac{x}{x}\right)}{\frac{x^2}{x-3}\left(\frac{x+3}{x+3}\right) - \frac{(x+1)^2}{x+3}\left(\frac{x-3}{x-3}\right)}$$

$$= \frac{\frac{(2x+5)(x-3)}{x(x-3)} - \frac{(x)(x)}{x(x-3)}}{\frac{x^2(x+3)}{(x-3)(x+3)} - \frac{(x+1)^2(x-3)}{(x-3)(x+3)}} = \frac{\frac{(2x+5)(x-3)-x^2}{x(x-3)}}{\frac{x^2(x+3)-(x+1)^2(x-3)}{(x-3)(x+3)}}$$

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Solution. We first get common denominators in order to do the subtraction:

$$\frac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}} = \frac{\frac{2x+5}{x}\left(\frac{x-3}{x-3}\right) - \frac{x}{x-3}\left(\frac{x}{x}\right)}{\frac{x^2}{x-3}\left(\frac{x+3}{x+3}\right) - \frac{(x+1)^2}{x+3}\left(\frac{x-3}{x-3}\right)}$$

$$= \frac{\frac{(2x+5)(x-3)}{x(x-3)} - \frac{(x)(x)}{x(x-3)}}{\frac{x^2(x+3)}{(x-3)(x+3)} - \frac{(x+1)^2(x-3)}{(x-3)(x+3)}} = \frac{\frac{(2x+5)(x-3)-x^2}{x(x-3)}}{\frac{x^2(x+3)-(x+1)^2(x-3)}{(x-3)(x+3)}}$$

Page A42 Number 36 (continued)

Solution (continued). Division by a quotient is the same as multiplication by the reciprocal (by Note A.5.A(2)), so we next have

$$\frac{\frac{2x+5}{x}-\frac{x}{x-3}}{\frac{x^2}{x-3}-\frac{(x+1)^2}{x+3}} = \frac{(2x+5)(x-3)-x^2}{x(x-3)} \times \frac{(x-3)(x+3)}{x^2(x+3)-(x+1)^2(x-3)}$$

$$= \frac{(2x^2-6x+5x-15)-x^2}{x(x-3)} \times \frac{(x-3)(x+3)}{(x^3+3x)-(x^2+2x+1)(x-3)}$$

$$= \frac{x^2-x-15}{x(x-3)} \times \frac{(x-3)(x+3)}{x^3+3x-(x^3+2x^2+x-3x^2-6x-3)}$$

$$= \frac{x^2-x-15}{x(x-3)} \times \frac{(x-3)(x+3)}{x^2+8x+3} = \boxed{\frac{(x^2-x-15)(x+3)}{x(x^2+8x+3)} \text{ if } x \neq 3}.$$

Notice that x^2-x-15 and x^2+8x+3 are irreducible over the integers. □