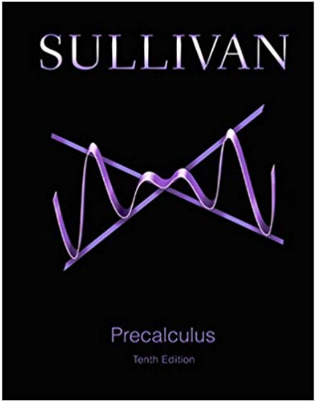
Precalculus 1 (Algebra)

Appendix A. Review

A.6. Solving Equations—Exercises, Examples, Proofs



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Page A51 Number 16

Page A51 Number 16. Solve $3x = -24$.

Solution. Dividing both sides by 3 gives the equivalent equation $(3x)/3 = (-24)/3$ or $\boxed{x = -8}$. The solution set is $\boxed{\{-8\}}$. □

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Page A51 Number 24

Page A51 Number 24. Solve $3 - 2x = 2 - x$.

Solution. Adding $2x$ to both sides (to get the x's on one side) gives the equivalent equation $(3 - 2x) + 2x = (2 - x) + 2x$ or $3 = 2 + x$. Subtracting 2 from both sides (to isolate x) of this gives the equivalent equation $(3) - 2 = (2 + x) - 2$ or $1 = x$. So $\boxed{x = 1}$ and the solution set is $\boxed{\{1\}}$. □

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Page A51 Number 28

Page A51 Number 28. Solve $5 - (2x - 1) = 10$.

Solution. First, we simplify the left hand side to get $5 - (2x - 1) = 5 - 2x + 1 = 6 - 2x$. So an equivalent equation is $6 - 2x = 10$. Adding $2x$ to both sides of this equation (to get the x's on one side) gives the equivalent equation $(6 - 2x) + 2x = 10 + 2x$ or $6 = 10 + 2x$.

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Page A51 Number 36

Page A51 Number 36. Solve $(x+2)(x-3) = (x-3)^2$.

Solution. We try to use the Zero-Product Property, so we want 0 on one side. We subtract $(x+2)(x-3)$ from both sides to get the equivalent equation $(x+2)(x-3) - (x+2)(x-3) = (x-3)^2 - (x+2)(x-3)$ or $0 = (x-3)^2 - (x+2)(x-3)$.

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Solution. We try to use the Zero-Product Property, so we want 0 on one side. We subtract $(x+2)(x-3)$ from both sides to get the equivalent equation $(x+2)(x-3) - (x+2)(x-3) = (x-3)^2 - (x+2)(x-3)$ or $0 = (x-3)^2 - (x+2)(x-3)$. Factoring out the common factor $x-3$ we get the equivalent equation $0 = (x-3)\left((x-3) - (x+2)\right)$ or $(x-3)(-5) = 0$. Dividing both sides by -5 gives $(x-3)(-5)/(-5) = 0/(-5)$ or $x - 3 = 0$. Adding 3 to both sides we have $x = 3$. So $\boxed{x = 3}$ and the solution set is $\boxed{\{3\}}$. □

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Page A51 Number 48

Page A51 Number 48. Solve $\dfrac{1}{2x+3}+\dfrac{1}{x-1}=\dfrac{1}{(2x+3)(x-1)}$.

Solution. First, we get a common denominator on the left hand side:
$\dfrac{1}{2x+3}+\dfrac{1}{x-1}=\dfrac{1}{2x+3}\left(\dfrac{x-1}{x-1}\right)+\dfrac{1}{x-1}\left(\dfrac{2x+3}{2x+3}\right)=$
$\dfrac{x-1}{(2x+3)(x-1)}+\dfrac{2x+3}{(x-1)(2x+3)}=\dfrac{(x-1)+(2x+3)}{(x-1)(2x+3)}=$
$\dfrac{3x+2}{(x-1)(2x+3)}$. So the original equation is equivalent to
$\dfrac{3x+2}{(x-1)(2x+3)}=\dfrac{1}{(2x+3)(x-1)}$.

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$\dfrac{1}{2x+3}+\dfrac{1}{x-1}=\dfrac{1}{2x+3}\left(\dfrac{x-1}{x-1}\right)+\dfrac{1}{x-1}\left(\dfrac{2x+3}{2x+3}\right)=$
$\dfrac{x-1}{(2x+3)(x-1)}+\dfrac{2x+3}{(x-1)(2x+3)}=\dfrac{(x-1)+(2x+3)}{(x-1)(2x+3)}=$
$\dfrac{3x+2}{(x-1)(2x+3)}$. So the original equation is equivalent to
$\dfrac{3x+2}{(x-1)(2x+3)}=\dfrac{1}{(2x+3)(x-1)}$. Since the denominators are the same, then the quotients can only be equal if the numerators are equal. So we need $3x+2=1$ (speeding up the process a little) or $3x=-1$ or $x=-1/3$. So $\boxed{x=-1/3}$ and the solution set is $\boxed{\{-1/3\}}$. □

Page A51 Number 48

Page A51 Number 48. Solve $\frac{1}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)}$.

Solution. First, we get a common denominator on the left hand side:
$\frac{1}{2x+3} + \frac{1}{x-1} = \frac{1}{2x+3}\left(\frac{x-1}{x-1}\right) + \frac{1}{x-1}\left(\frac{2x+3}{2x+3}\right) =$
$\frac{x-1}{(2x+3)(x-1)} + \frac{2x+3}{(x-1)(2x+3)} = \frac{(x-1)+(2x+3)}{(x-1)(2x+3)} =$
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Page A51 Number 54

Page A51 Number 54. Solve $|1 - 2z| = 3$.

Solution. The equation $|1 - 2z| = 3$ is satisfied if either $1 - 2z = 3$ or $1 - 2z = -3$, so we find the solution set by solving these equations separately. For $1 - 2z = 3$, we have $-2 = 2z$ or $z = -1$. For $1 - 2z = -3$, we have $4 = 2z$ or $z = 2$. So we have $\boxed{\text{either } z = -1 \text{ or } z = 2}$ and the solution set is $\boxed{\{-1, 2\}}$. □

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Page A51 Number 74

Page A51 Number 74. Solve $x(x+1) = 12$.

Solution. First we multiply the left side out to get $x(x+1) = x^2 + x$, and then the given equation is equivalent to $x^2 + x = 12$ or $x^2 + x - 12 = 0$. We can factor to get the equivalent equation
$x^2 + x - 12 = (x-3)(x+4) = 0$.

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Solution. First we multiply the left side out to get $x(x+1) = x^2 + x$, and then the given equation is equivalent to $x^2 + x = 12$ or $x^2 + x - 12 = 0$. We can factor to get the equivalent equation $x^2 + x - 12 = (x-3)(x+4) = 0$. So by the Zero-Product Property, either $x - 3 = 0$ or $x + 4 = 0$; that is, $\boxed{\text{either } x = 3 \text{ or } x = -4}$. So the solution set is $\boxed{\{-4, 3\}}$. □

Page A51 Number 86

Page A51 Number 86. Solve $(3x-2)^2=4$ by taking square roots.

Solution. Taking square roots of both sides of the given equation gives $\sqrt{(3x-2)^2}=\sqrt{4}$ or $|3x-2|=2$. (BEWARE that $\sqrt{x^2}=|x|$ by Note A.1.E., it does not simply equal x.)

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Page A52 Number 92

Page A52 Number 92. Solve $2x^2 - 3x - 1 = 0$ by completing the square.

Solution. First, we need to consider
$2x^2 - 3x - 1 = 2\left(x^2 - \frac{3}{2}x - \frac{1}{2}\right) = 0$. To complete the square on the expression in the parentheses, we recall that
$x^2 - bx + (b/2)^2 = (x - b/2)^2$ and see that we have $b = 3/2$ or $b/2 = 3/4$ here, and so $(b/2)^2 = (3/4)^2 = 9/16$. So we have
$x^2 - \frac{3}{2}x + \frac{9}{16} = \left(x - \frac{3}{4}\right)^2$.

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Page A52 Number 92 (continued 1)

Solution (continued). Now we take square roots of both sides of $\left(x-\frac{3}{4}\right)^2=\frac{17}{16}$ to get $\sqrt{\left(x-\frac{3}{4}\right)^2}=\sqrt{\frac{17}{16}}$ or $\left|x-\frac{3}{4}\right|=\sqrt{\frac{17}{16}}=\frac{\sqrt{17}}{\sqrt{16}}=\frac{\sqrt{17}}{4}$. That is, either $x-\frac{3}{4}=\frac{\sqrt{17}}{4}$ or $x-\frac{3}{4}=-\frac{\sqrt{17}}{4}$. Solving $x-\frac{3}{4}=\frac{\sqrt{17}}{4}$ gives $x=\frac{3}{4}+\frac{\sqrt{17}}{4}=\frac{3+\sqrt{17}}{4}$. Solving $x-\frac{3}{4}=-\frac{\sqrt{17}}{4}$ gives $x=\frac{3}{4}-\frac{\sqrt{17}}{4}=\frac{3-\sqrt{17}}{4}$. So either $x=\frac{3+\sqrt{17}}{4}$ or $x=\frac{3-\sqrt{17}}{4}$.

Page A52 Number 92 (continued 2)

Page A52 Number 92. Solve $2x^2 - 3x - 1 = 0$ by completing the square.

Solution (continued). That is, $\boxed{x = \frac{3 \pm \sqrt{17}}{4}}$ and the solution set is

$$\boxed{\left\{\frac{3-\sqrt{17}}{4}, \frac{3+\sqrt{17}}{4}\right\}}.$$

□

Note. We could have used the quadratic formula (with $a = 2$, $b = -3$, and $c = -1$) and got the same answer:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$$

Page A52 Number 92 (continued 2)

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Solution (continued). That is, $\boxed{x = \dfrac{3 \pm \sqrt{17}}{4}}$ and the solution set is

$$\boxed{\left\{ \frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4} \right\}}.$$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$$

Theorem A.6.A

Theorem A.6.A. The Quadratic Formula. Consider the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$. If $b^2 - 4ac < 0$ then this equation has no real solution. If $b^2 - 4ac \geq 0$ then the real solution(s) of this equation is (are) given by the *quadratic formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity $b^2 - 4ac$ is called the *discriminant*.

Proof. We derive the quadratic formula by completing the square:

$$\begin{aligned} ax^2 + bx + c &= 0,\ a \neq 0 \\ \frac{ax^2 + bx + c}{a} &= \frac{0}{a} \text{ (divide both sides by } a\text{)} \\ x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \text{ (simplify)} \end{aligned}$$

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Theorem A.6.A (continued 1)

Proof (continued).

$$\begin{aligned}
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\
x^2 + \frac{b}{a}x &= -\frac{c}{a} \text{ (subtract } c/a \text{ from both sides)} \\
x^2 + \frac{b}{a}x + \left(\frac{b/a}{2}\right)^2 &= -\frac{c}{a} + \left(\frac{b/a}{2}\right)^2 \text{ (complete the square on the left} \\
&\quad \text{and keep equation balanced)} \\
x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= -\frac{c}{a} + \frac{b^2}{4a^2} \text{ (simplify)} \\
\left(x + \frac{b}{2a}\right)^2 &= -\frac{c}{a} + \frac{b^2}{4a^2} \text{ (factor the perfect square)} \\
\left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \text{ (get common denominator)}
\end{aligned}$$

Theorem A.6.A (continued 2)

Proof (continued).

$$\begin{aligned}
\left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \text{ (simplify)} \\
x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \text{ (Square Root Method)} \\
x &= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \text{ (subtract } b/2a \text{ from both sides)} \\
x &= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2|a|} \text{ (simplify)} \\
x &= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \text{ (the } \pm \text{ takes care of any negative}
\end{aligned}$$

signs absorbed by the absolute value)

Theorem A.6.A (continued 3)

Theorem A.6.A. The Quadratic Formula. Consider the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$. If $b^2 - 4ac < 0$ then this equation has no real solution. If $b^2 - 4ac \geq 0$ then the real solution(s) of this equation is (are) given by the *quadratic formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity $b^2 - 4ac$ is called the *discriminant*.

Proof (continued).

$$\begin{aligned} x &= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (simplify)} \end{aligned}$$

□

Page A52 Number 98

Page A52 Number 98. Use the quadratic formula to find the real solutions, if any, for $2x^2 + 5x + 3 = 0$.

Solution. With $a = 2$, $b = 5$, and $c = 3$ we have from the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm 1}{4}.$$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm 1}{4}.$$

So we have that
$\boxed{\text{either } x = (-5+1)/4 = -1 \text{ or } x = (-5-1)/4 = -6/4 = -3/2}$ and the solution set is $\boxed{\{-1, -3/2\}}$. □

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So we have that $\boxed{\text{either } x = (-5+1)/4 = -1 \text{ or } x = (-5-1)/4 = -6/4 = -3/2}$ and the solution set is $\boxed{\{-1, -3/2\}}$. □

Page A52 Number 104

Page A52 Number 104. Use the quadratic formula to find the real solutions, if any, for $x^2 + \sqrt{2}x - 2 = 0$.

Solution. With $a = 1$, $b = \sqrt{2}$, and $c = -2$ we have from the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(\sqrt{2}) \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}.$$

So we have that $\boxed{\text{either } x = \frac{-\sqrt{2} - \sqrt{10}}{2} \text{ or } \frac{-\sqrt{2} + \sqrt{10}}{2}}$ and the solution set is $\boxed{\left\{ \frac{-\sqrt{2} - \sqrt{10}}{2}, \frac{-\sqrt{2} + \sqrt{10}}{2} \right\}}$. □

Page A52 Number 104

Page A52 Number 104. Use the quadratic formula to find the real solutions, if any, for $x^2 + \sqrt{2}x - 2 = 0$.

Solution. With $a = 1$, $b = \sqrt{2}$, and $c = -2$ we have from the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(\sqrt{2}) \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}.$$

So we have that $\boxed{\text{either } x = \frac{-\sqrt{2} - \sqrt{10}}{2} \text{ or } \frac{-\sqrt{2} + \sqrt{10}}{2}}$ and the solution set is $\boxed{\left\{ \frac{-\sqrt{2} - \sqrt{10}}{2}, \frac{-\sqrt{2} + \sqrt{10}}{2} \right\}}$. □

Page A52 Number 110

Page A52 Number 110. Use the discriminant to determine whether $2x^2 - 3x - 4 = 0$ has two unequal real solutions, a repeated real solution, or no real solution without solving the equation.

Solution. With $a = 2$, $b = -3$, and $c = -4$, the discriminant is $b^2 - 4ac = (-3)^2 - 4(2)(-4) = 9 + 32 = 41$. Since $b^2 - 4ac > 0$ then by Note A.6.C, $\boxed{\text{there are two unequal real solutions}}$. □

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