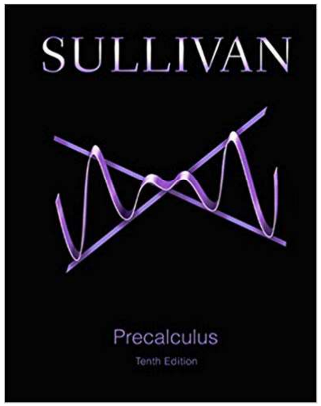
Precalculus 1 (Algebra)

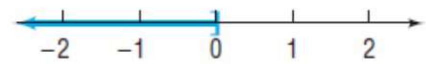
Appendix A. Review

A.9. Interval Notation; Solving Inequalities—Exercises, Examples, Proofs



Page A79 Number 18

Page A79 Number 18. Express the graph shown in blue using interval notation and as an inequality involving x.



Solution. Here, a square bracket is used to indicate inclusion of an endpoint. So in interval notation, the blue points are in the interval $\boxed{(-\infty, 0]}$. As an inequality, this is $\boxed{x \le 0}$. □

Page A79 Number 30

Page A79 Number 30. Write the inequality $-2 < x < 0$ using interval notation, and illustrate it using the real number line.

Solution. As an interval the inequality corresponds to $\boxed{(-2, 0)}$. On the real number line the inequality gives the points in blue:

□

Page A79 Number 32

Page A79 Number 32. Write the inequality $x \le 5$ using interval notation, and illustrate it using the real number line.

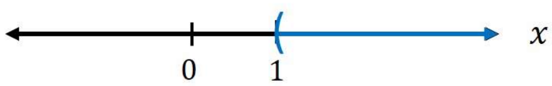
Solution. As an interval the inequality corresponds to $\boxed{(-\infty, 5]}$. On the real number line the inequality gives the points in blue:

□

Page A79 Number 34

Page A79 Number 34. Write the inequality $x > 1$ using interval notation, and illustrate it using the real number line.

Solution. As an interval the inequality corresponds to $\boxed{(1, \infty)}$. On the real number line the inequality gives the points in blue:



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Page A79 Number 38

Page A79 Number 38. Write the interval $[0, 1)$ as an inequality involving x, and illustrate it using the real number line.

Solution. As an inequality the interval corresponds to $\boxed{0 \leq x < 1}$. On the real number line the inequality gives the points in blue:

□

Page A79 Number 42

Page A79 Number 42. Write the interval $(-8, \infty)$ as an inequality involving x, and illustrate it using the real number line.

Solution. As an inequality the interval corresponds to $\boxed{x > -8}$. On the real number line the inequality gives the points in blue:

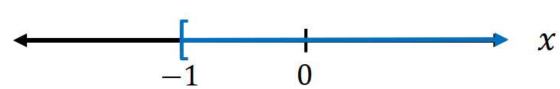
□

Page A79 Number 60

Page A79 Number 60. Solve the inequality $2 - 3x \leq 5$. Express your answer using set notation and interval notation. Graph the solution set.

Solution. Adding $3x$ to both sides of the inequality gives $(2 - 3x) + 3x \leq (5) + 3x$ or $2 \leq 5 + 3x$. Subtracting 5 from both sides of this new inequality gives $(2) - 5 \leq (5 + 3x) - 5$ or $-3 \leq 3x$, from which we have (dividing both sides by 3) $-1 \leq x$ or $\boxed{x \geq -1}$. In set notation, this is $\boxed{\{x \in \mathbb{R} \mid x \geq -1\}}$.

On the real number line the inequality gives the points in blue:



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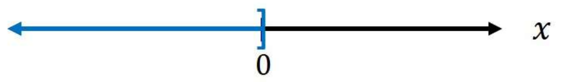
Page A79 Number 68

Page A79 Number 68. Solve the inequality $8 - 4(2 - x) \le -2x$. Express your answer using set notation and interval notation. Graph the solution set.

Solution. We multiply out the left side of the inequality to get $8 - 8 + 4x \le -2x$ or $4x \le -2x$. Adding $2x$ to both sides gives $(4x) + 2x \le (-2x) + 2x$ or $6x \le 0$ or (dividing both sides by 6) $x \le 0$. In set notation, this is $\boxed{\{x \in \mathbb{R} \mid x \le 0\}}$ and in interval notation $\boxed{(-\infty, 0]}$.

On the real number line the inequality gives the points in blue:



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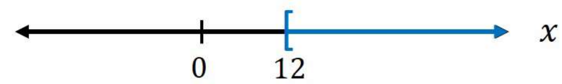
Page A79 Number 72

Page A79 Number 72. Solve the inequality $\frac{x}{3} \ge 2 + \frac{x}{6}$. Express your answer using set notation and interval notation. Graph the solution set.

Solution. We can multiply both sides of the inequality by 6 to get $6\left(\frac{x}{3}\right) \ge 6\left(2 + \frac{x}{6}\right)$ or $\frac{6x}{3} \ge 12 + \frac{6x}{6}$ or $2x \ge 12 + x$. Subtracting x from both sides gives $(2x) - x \ge (12 + x) - x$ or $x \ge 12$. In set notation, this is $\boxed{\{x \in \mathbb{R} \mid x \ge 12\}}$ and in interval notation $\boxed{[12, \infty)}$.

On the real number line the inequality gives the points in blue:



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Page A80 Number 86

Page A80 Number 86. Solve the inequality $\frac{1}{3} < \frac{x+1}{2} \le \frac{2}{3}$. Express your answer using set notation and interval notation. Graph the solution set.

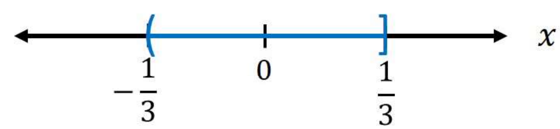
Solution. We multiply through by 6 to get $6\left(\frac{1}{3}\right) < 6\left(\frac{x+1}{2}\right) \le 6\left(\frac{2}{3}\right)$ or $\frac{6}{3} < \frac{6(x+1)}{2} \le \frac{(6)(2)}{3}$ or $2 < 3(x+1) \le 4$. Dividing by 3 now gives $\frac{2}{3} < x + 1 \le \frac{4}{3}$. Subtracting 1 throughout gives $\frac{2}{3} - 1 < (x+1) - 1 \le \frac{4}{3} - 1$ or $-\frac{1}{3} < x \le \frac{1}{3}$. In interval notation this is $\boxed{(-1/3, 1/3]}$ and in set notation $\boxed{\{x \in \mathbb{R} \mid -1/3 < x \le 1/3\}}$.

Page A80 Number 86 (continued)

Page A80 Number 86. Solve the inequality $\frac{1}{3} < \frac{x+1}{2} \le \frac{2}{3}$. Express your answer using set notation and interval notation. Graph the solution set.

Solution (continued). ...In interval notation this is $\boxed{(-1/3, 1/3]}$ and in set notation $\boxed{\{x \in \mathbb{R} \mid -1/3 < x \le 1/3\}}$.

On the real number line the inequality gives the points in blue:



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Page A80 Number 92

Page A80 Number 92. Solve the inequality $0 < (3x+6)^{-1} < \frac{1}{3}$. Express your answer using set notation and interval notation. Graph the solution set.

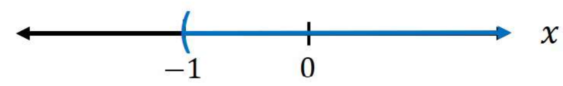
Solution. We need both $0 < (3x+6)^{-1}$ and $(3x+6)^{-1} < \frac{1}{3}$. For $0 < (3x+6)^{-1}$, we have by the Reciprocal Property of Inequalities that $(3x+6) > 0$ or $(3x+6) - 6 > 0 - 6$ or $3x > -6$ or $3x/3 > -6/3$ or $x > -2$ or $-2 < x$. For $(3x+6)^{-1} < \frac{1}{3}$ we have $\frac{1}{3x+6} < \frac{1}{3}$. We already know that $x > -2$ so that $3x + 6 > 3(-2) + 6 = 0$ or $3x + 6$ is positive. So we multiply both sides of $\frac{1}{3x+6} < \frac{1}{3}$ by the positive quantity $3x + 6$ to get $\frac{1}{3x+6}(3x+6) < \frac{1}{3}(3x+6)$ or $1 < x + 2$ or $1 - 2 < x$ or $-1 < x$.

Page A80 Number 92 (continued)

Page A80 Number 92. Solve the inequality $0 < (3x+6)^{-1} < \frac{1}{3}$. Express your answer using set notation and interval notation. Graph the solution set.

Solution (continued). Combining both $-2 < x$ and $-1 < x$ we see that we must have $-1 < x$ (or $x > -1$). In set notation we have $\boxed{\{x \in \mathbb{R} \mid x > -1\}}$ or in interval notation $\boxed{(-1, \infty)}$.

On the real number line the inequality gives the points in blue:



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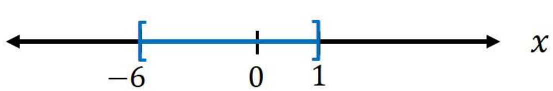
Page A80 Number 98

Page A80 Number 98. Solve the inequality $|2x+5| \leq 7$. Express your answer using set notation and interval notation. Graph the solution set.

Solution. The inequality $|2x+5| \leq 7$ is equivalent to $-7 \leq 2x+5 \leq 7$. Subtracting 5 throughout gives $(-7) - 5 \leq (2x+5) - 5 \leq (7) - 5$ or $-12 \leq 2x \leq 2$. Dividing by 2 throughout gives $-12/2 \leq 2x/2 \leq 2/2$ or $-6 \leq x \leq 1$. In interval notation this is $\boxed{[-6, 1]}$ and in set notation it is $\boxed{\{x \in \mathbb{R} \mid -6 \leq x \leq 1\}}$.

On the real number line the inequality gives the points in blue:



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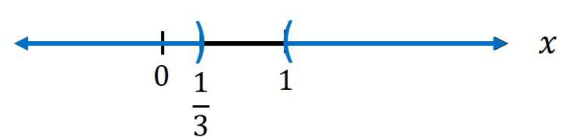
Page A80 Number 100

Page A80 Number 100. Solve the inequality $|2-3x| > 1$. Express your answer using set notation and interval notation. Graph the solution set.

Solution. The inequality $|2-3x| > 1$ is equivalent to $2 - 3x < -1$ or $2 - 3x > 1$. Solving $2 - 3x < -1$ gives $2 - (-1) < 3x$ or $3 < 3x$ or $1 < x$ or $x > 1$. Solving $2 - 3x > 1$ gives $2 - (1) > 3x$ or $1 > 3x$ or $1/3 > x$ or $x < 1/3$. In set notation we have $\boxed{\{x \in \mathbb{R} \mid \text{ either } x < 1/3 \text{ or } x > 1\}}$. In interval notation we have $\boxed{(-\infty, 1/3) \cup (1, \infty)}$.

On the real number line the inequality gives the points in blue:



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Page A81 Number 123

Page A81 Number 123. In your Economics 101 class, you have scores of 68, 82, 87, and 89 on the first four of five tests. To get a grade of B, the average of the first five test scores must be greater than or equal to 80 and less than 90.

(a) Solve an inequality to find the range of the score that you need on the last test to get a B.

(b) What score do you need if the fifth test counts double?

Solution. We let T_5 represent your grade on the fifth test.
(a) Your average on the five tests is $(68 + 82 + 87 + 89 + T_5)/5 = (326 + T_5)/5$. To get an average greater than or equal to 80 and less than 90 you need $80 \leq (326 + T_5)/5 < 90$ or (multiplying through by 5) $5(80) \leq 5(326 + T_5)/5 < 5(90)$ or $400 \leq 326 + T_5 < 450$. Subtracting 326 throughout gives $400 - 326 \leq (326 + T_5) - 326 < 450 - 326$ or $74 \leq T_5 < 124$. So you need to make a grade of $\boxed{\text{at least 74 (and at most, presumably, 100)}}$.

Page A81 Number 123 (continued)

Page A81 Number 123. In your Economics 101 class, you have scores of 68, 82, 87, and 89 on the first four of five tests. To get a grade of B, the average of the first five test scores must be greater than or equal to 80 and less than 90.

(a) Solve an inequality to find the range of the score that you need on the last test to get a B.

(b) What score do you need if the fifth test counts double?

Solution (continued). (b) If the fifth test counts double (now giving effectively 6 tests) then your average is $(68 + 82 + 87 + 89 + 2T_5)/6 = (326 + 2T_5)/6$. To get an average greater than or equal to 80 and less than 90 you need $80 \leq (326 + 2T_5)/6 < 90$ or, similar to part (a), $6(80) \leq 6(326 + 2T_5)/6 < 6(90)$ or $480 \leq 326 + 2T_5 < 540$ or $480 - 326 \leq 326 + 2T_5 - 326 < 540 - 326$ or $154 \leq 2T_5 < 214$ or $154/2 \leq 2T_5/2 < 214/2$ or $77 \leq T_5 < 107$. So you need to make a grade of $\boxed{\text{at least 77 (and at most, presumably, 100)}}$. □