Test 1

1 Graphs

1.1 The Distance and Midpoint Formulas

In Problems 19-32, find the distance d between the points P_1 and P_2

25. $P_1 = (3, -4); P_2 = (4, 0)$

29. $P_1 = (-0.2, 0.3); P_2 = (2.3, 1.1)$ *In Problems 33-38, plot each point and form the triangle ABC. Show that the triangle is a right triangle. Find its area.*

33. (Same as 31 in 10th edition) $A = (-2, 5); B = (1, 3); C = (-1, 0)$

37. (Same as 35 in 10th edition) $A = (4, -3); B = (0, -3); C = (4, 2)$ *In Problems 39-46, find the midpoint of the line segment joining the points P_1 and P_2*

41. $P_1 = (3, -4); P_2 = (5, 4)$

45. (Same as 43 in 10th edition)$P_1 = (a, b); P_2 = (0, 0)$

49. Find all points having an x-coordinate of 3 whose distance from the point $(-2, -1)$ is 13.
    (a) By using the Pythagorean Theorem.
    (b) By using the distance formula.

53. Suppose that $A = (2, 5)$ are the coordinates of a point in the xy-plane.
    (a) Find the coordinates of the point if A is shifted 3 units to the left and 4 units down.
    (b) Find the coordinates of the point if A is shifted 2 units to the left and 8 units up.

57. **Geometry** The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side. Find the lengths of the medians of the triangle with vertices at $A = (0, 0), B = (6, 0)$ and $C = (4, 4)$.

73. ***Challenge Problem*** **Geometry** Gind the midpoint of each diagonal of a square with side of length s.Draw the conclusion that the diagonals of a square intersect at their midpoints. **Hint:** Use $(0, 0), (0, s), (s, 0)$, and (s, s) as the vertices of the square.

1.2 Graphs of Equations in Two-Variables; Intercepts; Symmetry

In Problems 57-72, list the intercepts and test for symmetry.

57. $y^2 = x + 16$

65. $x^3 - 64$

69. $\dfrac{4x}{x^2 + 16}$

85. ***Challenge Problem*** **Lemniscate** For a nonzero constant a, find the intercepts of the graph of $(x^2 + y^2)^2 = a^2(x^2 - y^2)$. Then test for symmetry with respect to the x-axis, the y-axis, and the origin.

1.3 Lines

In Problems 33-38, a point on a line and its slope are given. Find the point-slope form of the equation of the line.

37. $P = (-1, 3); m = 0$ *In Problems 39-44, the slope and a point on a line are given. Us this information to locate three additional points on the line. Answers may vary.* ***Hint:*** *It is not necessary to find the equation of the line. See Example 3.*

41. Slope $-\frac{3}{2}$; point $(2, -4)$

45. (See 39 in 10th edition)

49. (See 43 in 10th edition) *In Problems 53-78, find an equation for the line with the given properties. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever your prefer.*

53. Slope = 3; containing the point (-2,3)

57. Containing the points (1,3) and (-1,2)

61. x-intercept = -4; y-intercept = 4

65. Horizontal; containing the point (-3,2)

69. Parallel to the line $x - 2y = -5$; containing the point (0,0)

73. Perpendicular to the line $y = \frac{1}{2}x + 4$; containing the point (1,-2).

77. Perpendicular to the line $x = 8$; containing the point (3,4) *In Problems 79-98, find the slope and y-intercept of each line. Graph the line.*

81. $\frac{1}{2}y = x - 1$

85. $x + 2y = 4$

89. $x + y = 1$

93. $y = 5$

97. $2y - 3x = 0$ *In Problems 99-108, (a) find the intercepts of the graph of each equation and (b) graph the equation.*

101. $-4x + 5y = 40$

105. $\frac{1}{2}x + \frac{1}{3}y = 1$

109. Find an equation of the x-axis *In Problems 111-114, the equations of two lines are given. Determine whether the lines are parallel, perpendicular, or neither*

113. $y = 4x + 5$
     $y = -4x + 2$

117. (See 111 in 10th edition)

121. **Geometry** Use slopes and the distance formula to show that the quadrilateral who vertices are (0,0),(1,3),(4,2), and (3,-1) is a square.

125. **Cost of Driving a Car** The annual fixed costs of owning a small sedan are \$4252, assuming the car is completely paid for. The cost to drive the car is approximately 0.14 per mile. Write a linear equation that relates the cost C and the number x of miles driven annually.

129. **Measuring Temperature** The relationship between Celsius (°C) and Fahrenheit (°F) degrees of measuring temperature is linear. Find a linear equation relating °C and °F if 0°C corresponds to 32°F and 100°C corresponds to 212°F. Use the equation to find the Celsius measure of 70°F.

133. **Product Promotion** A cereal company finds that the number of people who will buy one of its products in the first month that the product is introduced is linearly related to the amount of money it spends on advertising. If it spends \$40,000 on advertising, then 100,000 boxes of cereal will be sold, and if it spends \$60,000, then 200,000 boxes will be sold.

    (a) Write a linear equation that relates the amount Aspent on advertising to the number x of boxes the company aims to sell.

    (b) How much expenditure on advertising is needed to sell 300,000 boxes of cereal?

    (c) Interpret the slope.

137. ***Challenge Problem*** Form a triangle using the points (0,0), (a,0), and (b,c), where $a > 0, b > 0$, and $c > 0$. Find the point of intersection of the three lines joining the midpoint of a side of the triangle to the opposite vertex.

1.4 Circles

In Problems 25-38, (a) find the center (h,k) and radius of each circle; (b) graph each circle; (c) find the intercepts, if any.

29. $x^2 + y^2 - 2x - 4y - 4 = 0$

33. $x^2 + y^2 - x + 2y + 1 = 0$

37. $2x^2 + 8x + 2y^2 = 0$

49. Find an equation of the line containing the centers of the two circles

$$(x - 4)^2 + (y + 2)^2 = 25$$

and

$$(x + 1)^2 + (y - 5)^2 = 16$$

53. (See 49 in 10th edition)

57. **Vertically Circular Building** Located in Al Raha, Abu Dhabi, the headquarters of property developing company Aldar is a vertically circular building with a diameter of 121 meters. The tip of the building is 110 meters above ground. Find an equation for the buildings outline if the center of the building is on the y-axis. *In Problems 59-62, find the standard form of the equation of each circle. (refer to the preceding discussion).*

61. Center (-1,3) and tangent to the line $y = 2$

2 Functions and Their Graphs

2.1 Functions

In Problems 17-18, a relation is expressed verbally is given.
(a) What is the domain and the range of the relation? (b) Express the relation using a mapping.
(c) Express the relation as a set of ordered pairs.

17. The density of a gas under constant pressure depends on temperature. Holding pressure constant at 14.5 pounds per square inch, a chemist measures the density of an oxygen sample at temperatures of 0,22, 40 , 70, and 100°C and obtains densities of 1.411, 1.305, 1.229, 1.121, and 1.031 kg/m^3, respectively.

    In Problems 19-30, find the domain and range of each relation. Then determine whether the relation represents a function

29. $\{(-1,8),(0,3),(2,-1),(4,3)\}$ *In Problems 31-42, determine whether the equation defines y as a function of x*

41. $|y| = 2x + 3$ *In Problems 51-70, find the domain of each function*

53. $f(x) = \dfrac{x+1}{2x^2+8}$

61. $p(x) = \dfrac{x}{|2x+3|-1}$

69. $M(t) = \sqrt[5]{\dfrac{t+1}{t^2-5t-14}}$

73. (See 69 in 10th edition)

77. (See 73 in 10th edition)

81. (See 77 in 10th edition) *In Problems 82-98, find the difference quotient of f; that is, find* $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$, *for each function. Be sure to simplify.*

85. $f(x) = x^2 - 4$

89. $f(x) = \dfrac{5}{4x-3}$

93. $f(x) = \sqrt{x-2}$

97. $Df(x) = \sqrt{4-x^2}$

101. If $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$, what is the value of A?

105. **Geometry** Express the area A of a rectangle as a function of the length x if the length of the rectangle is twice its width.

109. **Effect of Gravity on Earth** If a rock falls from a height of 20 meters on Earth, the height H (in meters) after x seconds is approximately

$$H(x) = 20 - 4.9x^2$$

   (a) What is the height of the rock when $x = 1$? When $x = 1.1$ seconds? When $x = 1.2$ seconds?

   (b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?

   (c) When does the rock strike the ground?

113. **Economics** The **participation rate** is the number of people in labor forced divided by the civilian population (excludes military). Let $L(x)$ represent the size of the labor force in year x and $P(x)$ arepresent the civilian population in year x. Determine a function that represents the participation rate R as a function of x.

117. **Profit Function** Suppose that the revenue R , in dollars, from selling x smartphones, in hundreds, is

$$R(x) = -1.2x^2 + 220x$$

   The cost C, in dollars, of selling x smartphones in hundreds is $C(x) = 0.005x^2 - 2x^2 + 65x + 500$

   (a) Find the profit function, $P(x) = R(x) - C(x)$

   (b) Find the profit if $x = 15$ hundred smartphones are sold.

   (c) Interpret $P(15)$.

121. ***Challenge Problem*** Find the difference quotient of the function $f(x) = \sqrt[3]{x}$.
   (Hint Factor using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ with $a = \sqrt[3]{x + h}$ and $b = \sqrt[3]{x}$.)

2.2 The Graph of a Function

In Problems 25-30, answer the questions about each function.

25. $f(x) = 3x^2 + x - 2$

   (a) Is the point (1,2) on the graph of f?

   (b) If $x = -2$, what is $f(x)$? What point is on the graph of f?

   (c) If $f(x) = -2$, what is x? What point(s) are on the graph of f?

   (d) What is the domain of f?

   (e) List the x-intercepts, if any, of the graph of f.

   (f) List the y-intercepts, if there is one, of the graph of f

29. $f(x) = \dfrac{12x^4}{x^2 + 1}$

   (a) Is the point (-1,6) on the graph of f?

   (b) If $x = 3$, what is $f(x)$? What point is on the graph of f?

   (c) If $f(x) = 1$, what is x? What point(s)are on the graph of f?

(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

(f) List the y-intercepts, if ther is one, of the graph of f.

41. ***Challenge Problem*** Suppose $f(x) = \sqrt{x} + 2$ and $g(x) = x^2 + n$. If $f(g(5)) = 4$, what is the value of $g(n)$?

Test 2

2.3 Properties of Functions

In Problems 37-48, determine algebraically whether each function is even, odd, or neither

41. $F(x) = \sqrt[3]{4x}$

45. $g(x) = \dfrac{1}{x^2 + 8}$

81. ***E. coli* Growth** A strain of *E. coli* Beu 397-recA441 is placed into a nutrient broth at 30° Celsius and allowd to grow. The data shown in the table are collected. The population is measured in grams and the time in hours. Since population P depends on time T, and each input corresponds to exactly on output, we can say that population is a function of time, so $P(t)$ represents the population at time t.

Time (Hours), t	Population (grams), P
0	0.09
2.5	0.18
3.5	0.26
4.5	0.35
6	0.5

(a) Find the average rate of change of the population from 0 to 2.5 hours.

(b) Find the average rate of change of the population from 4.5 to 6 hours.

(c) What is happening to the average rate of change as time passes?

Problems 85-92 require the following discussion of a secant line. The slope of the secant line containing the two points $(x, f(x))$ and $(x+h), f(x+h))$ on the graph of a function $y = f(x)$ may be given as

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{(x+h) - x)} = \frac{f(x+h) - f(x)}{h}, h \neq 0$$

(a) *Express the slope of the secant line of each function in terms of x and h. Be sure to simplify your answer.*

(b) *Find m_{sec} for $h = 0.5, 0.1$ and 0.01 at $x = 1$. What value does m_{sec} approach as h approaches 0?*

(c) *Find an equation for the secant line at $x = 1$ with $h = 0.01$.*

(d) *Use a graphing utility to graph f and the secant line found in part (c) in the same viewing window.*

85. $f(x) = 2x + 5$

89. $f(x) = 2x^2 - 3x + 1$

93. ***Challenge Problem*** **Mean Value Theorem** Suppose $f(x) = x^3 + 2x^2 - x + 6$. From Calculus, the Mean Value Theorem guarantees that there is at least one number in the open interval $(-1, 2)$ at which the value of the derivative of f, given by $f'(x) = 3x^2 + 4x - 1$ is equal to the average rate of change of f on the interval. Find all such number x in the interval.

2.4: Library of Function, Piecewise-Defined Functions

29. If $f(x) = \begin{cases} 2x+4 & \text{if } -3 \le x \le 1 \\ x^3 - 1 & \text{if } 1 < x \le 5 \end{cases}$ find:

    (a) $f(-2)$

    (b) $f(0)$

    (c) $f(1)$

    (d) $f(3)$

In Problems 31-42:

(a) *Find the domain of each function.*

(b) *Locate any intercepts.*

(c) *Graph each function.*

(d) *Based on the graph, find the range.*

41. $f(x) = \begin{cases} x^2 & \text{if } 0 < x \le 2 \\ x+2 & \text{if } 2 < x < 5 \\ 7 & \text{if } x \ge 5 \end{cases}$

49. (a) Graph $f(x) = \begin{cases} (x-1)^2 & \text{if } 0 \le x < 2 \\ -2x+10 & \text{if } 2 \le x \le 6 \end{cases}$

    (b) Find the domain of f.

    (c) Find the absolute maximum and the absolute minimum if they exist.

53. **Cost of Natural Gas** In March 2018, Spire, Inc. had the following rate schedule for natrual gas usage in single-family residences.

Monthly Service Charge	\$23.44
Delivery Charge	
First 20 therms	\$0.91686/therm
Over 30 therms	\$0
Natural Gas cost	
First 30 therms	\$0.26486/therm
Over 30 Therms	\$0.50897/therm

    (a) What is the charge for using 20 therms in a month?

(b) What is the charge for using 150 therms in a month?

(c) Develop a function that models the monthly charges C for x therms of gas.

(d) Graph the function found in part (c).

57. **Cost of Transporting Goods** A trucking company transports goods between Chicago and New York, a distance of 960 miles. The company's policy is to charge, for each pound, \$0.50 per mile for the first 100 miles, \$0.40 per mile for the next 300 miles, \$0.25 per mile for the next 400 miles, and no charge for the remaining 160 miles.

(a) Graph the relationship between the per-pound cost of transportation in dollars and mileage over the entire 960-mile route.

(b) Find the cost as a function of mileage for hauls between between 100 and 400 miles from Chicago.

(c) Find the cost as a function of mileage for hands for hauls between 400 and 800 miles from Chicago.

61. **Wind Chill** The wind chill factor represents the air temperature at a standard wind speed that would produce the same heat loss as the given temperature and wind speed. One formula for computing the equivalent temperature is

$$W = \begin{cases} t & 0 \le v < 1.79 \\ 33 - \frac{(10.45+10\sqrt{v}-v)(33-t)}{22.04} & 1.79 \le v \le 20 \\ 33 - 1.5958(33-t) & v > 20 \end{cases}$$

where v represents the wind speed (in meters per second) and t represents the air temperature ($^\circ C$). Compute the wind chill for the following:

(a) An air temperature of 10° C and a wind speed of 1 meter per second (m/sec)

(b) An air temperature of 10 ° C and a wind speed of 5 m/sec

(c) An air temperature of 10 ° C and a wind speed of 15 m/sec

(d) An air temperature of 10°C and a wind speed of 25 m/sec

(e) Explain the physical meaning of the equation corresponding to $0 \le v < 1.79$.

(f) Explain the physical meaning of the equation corresponding to $v > 20$.

2.5: Graphing Techniques: Transformations

In Problems 19-28, write the function whose graph is the graph of $y = x^3$, but is:

25. Vertically stretched by a factor of 5. *In Problems 29-32, find the function that is finally graphed after each of the following transformations is applied to the graph of $y = \sqrt{x}$*

29.
    1. Shift up 2 units
    2. Reflect about the x-axis
    3. Reflect about the y-axis

    In Problems 37-60, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function (for example, $y = x^2$) and show all the steps. Be sure to show at least three key points. Find the domain and the range.

37. $f(x) = x^2 - 1$

41. $h(x) = \sqrt{x+2}$

45. $g(x) = 4\sqrt{x}$

49. $f(x) = 2(x+1)^2 - 3$

53. $h(x) = \sqrt{-x} - 2$

57. $g(x) = 2|1-x|$

61. See 63 in 10th edition
    Mixed Practice *In Problems 65-72, complete the square of each quadratic expression. Then graph each function using graphing techniques. (If necessary, refer to Appendix A, Section A.3 to review completing the square)*

65. $f(x) = x^2 + 2x$

69. $f(x) = 2x^2 - 12x + 19$

73. Suppose that the x-intercepts of the graph of $y = f(x)$ are -5 and 3.
    (a) What are the x-intercepts of the graph of $y = f(x+2)$?
    (b) What are the x-intercepts of the graph of $y = f(x-2)$?
    (c) What are the x-intercepts of the graph of $y = 4f(x)$?
    (d) What are the x-intercepts of the graph $y = f(-x)$?

77. See 77 in 10th edition

81. Graph the following functions using transformations
    (a) $f(x) = \text{int}(-x)$
    (b) $g(x) = -\text{int}(x)$

85. See 83 in 10th edition

89. ***Challenge Problem*** If a function f is increasing on the intervals $[-3, 3]$ and $[11, 19]$ and decreasing on the interval $[3, 1]]$, determine the interval(s) on which $g(x) = -3f(2x-5)$ is increasing.

2.6 Mathematical Models: Building Functions

25. (e) What is the largest volume?

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3 Linear and Quadratic Function

3.1: Properties of Linear Functions and Linear Models

37. **Getting Towed** The cost C, in dollars, to tow a car is modeled by the function $C(x) = 2.5x + 85$, where x is the number of miles towed.

    (a) What is the cost of towing a car 40 miles?

    (b) If the cost of towing a car is $245, how many miles was it towed?

    (c) Suppose that you have only $150. What is the maximum number of miles that you can be towed?

    (d) What is the domain of C?

41. **Supply and Demand** Suppose that the quantity supplied S and the quantity demanded D of T-shirts at a concert are given by the following functions:

$$\begin{aligned} S(P) &= -600 + 50p \\ D(p) &= 1200 - 25p \end{aligned}$$

where p is the price of a T-shirt.

(a) Find the equilibrium price for T-shirts at this concert. What is the equilibrium quantity?

(b) Determine the prices for which quantity demanded is greater than quantity supplied.

(c) What do you think will eventually happen to the price of T-shirts if quantity demanded is greater than quantity supplied?

The point at which a company's profits equal zero is called the company's ***break-even point****. For Problems 45 and 46, let R represent a company's revenue, let C represent the company's costs, and let x represent the number of units produced and sold each day.*

(a) *Find the firm's break-even point; that is, find x so that $R = C$.*

(b) *Solve the inequality $R(x) > C(x)$ to find the units that represent a profit for the company.*

45.

$$\begin{aligned} R(x) &= 8x \\ C(x) &= 4.5x + 17,500 \end{aligned}$$

49. **Cost Function** The simplest function C is a linear function, $C(x) = mx + b$, where the y-intercept b represents the fixed cost of each item produced. Suppose that a small bicycle manufacturer has daily fixed costs of $1800, and each bicycle costs $90.

    (a) Write a linear model that expresses the costs C of manufacturing X bicycles in a day.

(b) Graph the model.
(c) What is the cost of manufacturing 14 bicycles in a day?
(d) How many bicycles could be manufactured for \$3780?

53. ***Challenge Problem*** **Temperature Conversion** The linear function $F(C) = \frac{9}{5}C + 32$ converts degrees Celsius to degrees Fahrenheit, and the linear function $R(f) = F + 495.67$ converts degrees Fahrenheit to degrees Rankine. Find a linear function that converts degrees Rankine to degrees Celsius.

3.3: Quadratic Functions and Their Properties Review

17. *See 15 in 10th edition*

21. *See 19 in 10th edition In Problems 23-30, (a) find the vertex and axis of symmetry of each quadratic function. (b) Determine whether the graph is concave up or concave down. (c) Graph the quadratic function.*

25. $f(x) = -2(x-3)^2 + 5$

29. $f(x) = -\frac{1}{3}\left(x - \frac{1}{2}\right)^2 - \frac{7}{6}$ *In Problems 31-42, graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting).* (**Hint:** If necessary, write f in the form $f(x) = a(x-h)^2 + k$.)

33. $f(x) = (x+2)^2 - 2$

37. $f(x) = 2x^2 - 4x + 1$

41. $f(x) = \frac{1}{2}x^2 + x - 1$ *In Problems 43-59, (a) find the vertex and the axis of symmetry of each quadratic function, and determine whether the graph is concave up or concave down. (b) Find the y-intercept and the x-intercepts, if any. (c) Use parts (a) and (b) to graph the function. (d) Find the domain and the range of the quadratic function. (e) Determine where the quadratic function is increasing and where it is decreasing. (f) Determine where $f(x) > 0$ and where $f(x) < 0$*

45. $f(x) = -x^2 - 6x$

49. $f(x) = x^2 + 2x + 1$

53. $f(x) = -2x^2 + 2x - 3$

57. $f(x) = -4x^2 - 6x + 2$

61. *See 51 in 10th Edition In Problems 65-72, determine, without graphing, whether the given quadratic function has a maximum value or a minimum value, and then find the value.*

65. $f(x) = 3x^2 + 24x$

69. $f(x) = -x^2 + 6x - 1$

73. The graph of the function $f(x) = ax^2 + bx + c$ has vertex at (0,2) and passes through the point (1,8). Find a, b, and c. *In Problems 76-80, for the given functions f and g.*

    (a) *Graph f and g on the same Cartesian Plan*

    (b) *Solve $f(x) = g(x)$*

    (c) *Use the result of part (b) to label the points of intersection of the graphs of f and g*

    (d) *Shade the region for which $f(x) > g(x)$; that is, the region below f and above g.*

77. $f(x) = -x^2 + 4$; $g(x) = -2x + 1$ *In Problems 81 and 82, use the fact that a quadratic function of the form $f(x) = ax^2 + bx + c$ with $b^2 - 4ac > 0$ may also be written in the form $f(x) = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the x-intercepts of the graph of the quadratic function.*

81. (a) Find quadratic functions whose x-intercepts are -3 and 1 with $a = 1; a = 2; a = -2; a = 5$.

    (b) How does the value of a affect the intercepts?

    (c) How does the value of a affect the axis of symmetry?

    (d) How does the value of a affect the vertex?

    (e) Compare the x-coordinate of the vertex with the midpoint of the x-intercepts. What might you conclude?

85. **Analyzing the Motion of a Projectile** A projectile is fired from a cliff 200 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 50 feet per second. The height h of the projectile above the water is modeled by

    $$h(x) = \frac{-32x^2}{50^2} + x + 200$$

    where x is the horizontal distance of the projectile from the face of the cliff.

    (a) At what horizontal distance from the face of the cliff is the height of the projectile a maximum?

    (b) Find the maximum height of the projectile.

    (c) At what horizontal distance from the face of the cliff will the projectile strike the water?

    (d) Graph the function$h$, $0 \leq x \leq 200$.

    (e) Use a graphing utility to verify the solutions found in parts (b) and (c).

    (f) When the height of the projectile is 100 feet above the water, how far is it from the cliff.

89. **Minimizing Marginal Cost** The **marginal cost** of a product can be thought of as the cost of producing one additional unit of output. For example, if the marginal cost of producing the 50th product is \$6.20, is costs \$6.20 to increase production from 49 to 50 units of output. Suppose the marginal cost C (in dollars) to produce x thousand digital music players is given by the function

    $$C(x) = x^2 0 - 140x + 7400$$

    (a) How many players should be produced to minimize the marginal cost?

    (b) What is the minimum marginal cost?

93. **Stopping Distance** An accepted relationship between stopping distance d (in feet), and the speed v of a car (in mph), is $d = 1.1v + 0.06v^2$ on dry, level concrete.

    (a) How many feet will it take a car traveling 45 mph to stop on dry level concrete.

    (b) If an accident occurs 200 feet ahead of you, what is the maximum speed you can be traveling to avoid being involved?

97. **Mixed Practice** Find the distance from the vertex of the parabola $g(x) = -3x^2 + 6x + 1$ to the center of the circle $x^2 + y^2 + 10x + 8y + 32 = 0$.

101. ***Challenge Problem*** **Increasing/Decreasing Function Test** Suppose $f(x) = x^3 - 7x^2 - 5x + 35$. From calculus, the derivative of f is given by $f'(x) = 3x^2 - 14x - 5$. The function f is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$. Determine where f is increasing and where f is decreasing.

Test 3

3.4: Build Quadratic Models from Verbal Descriptions and from Data

5. **Maximizing Revenue** The price p (in dollars) and the quantity x sold of a certain product satisfy the demand equation

$$x = -5p + 100$$

   (a) Find a model that expresses the revenue F as a function of p.
   (b) What is the domain of R? Assume R is nonnegative.
   (c) What price p maximizes revenue?
   (d) What is the maximum revenue?
   (e) How many units are sold at this price?
   (f) Graph R
   (g) What price should the company charge to earn at least \$480 in revenue?

13. **Constructing Rain Gutters** A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. *See the illustration in 15 in 10th edition.*

   (a) What depth will provide maximum cross-sectional area and allow the most water to flow?
   (b) What depths will allow at least 16 square inches of water to flow?

3.5: Inequalities Involving Quadratic Functions

Mixed Practice *In problems 23-30, use the given functions f and g*

(a) Solve $f(x) = 0$

(b) Solve $g(x) = 0$

(c) Solve $f(x) = g(x)$

(d) Solve $f(x) > 0$

(e) Solve $g(x) \leq 0$

(f) Solve $f(x) > g(x)$

(g) Solve $f(x) \geq 1$

25. $f(x) = -x^2 - x - 2$ $\qquad g(x) = 4x + 1$

29. $f(x) = x^2 - x - 2$ $\qquad g(x) = x^2 + x - 2$

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4 Polynomial and Rational Functions

4.1: Properties of Rational Function

In Problems 15-26, determine which functions are polynomial functions. For those that are, state the degree. For those that are not, state why not. Write each polynomial in standard form. Then identify the leading term and the constant.

17. $g(x) = \frac{2+3x^2}{5}$

21. $g(x) = x^{2/3} - x^{1/3} + 2$

25. $G(x) = 2(x-1)^2(x^2+1)$
    In Problems 27-40, use transformations of the graph of $y = x^4$ or $y = x^5$ to graph each function.

29. $f(x) = x^5 - 3$

33. $f(x) = -x^5$

37. $f(x) = 2(x+1)^4 + 1$
    In Problems 41-48, find a polynomial function whose real zeros and degree are given. Answers will vary depending on the choice of the leading coefficient.

41. Zeros: -1,1,3; degree 3

45. Zeros: -5,-2,3,5; degree 4
    In Problems 49-58, find a polynomial function with the given real zeros whose graph contains the given point.

49. Zeros: -5,-1,2,6
    Degree 3
    Point:(2,36)

53. Zeros: -3,1,4
    Degree 3
    y-intercept:36

57. Zeros: -5(multiplicity 2), 2(multiplicity 1), 4(multiplicity 1); degree 4; contains the point (3,128).

    In Problems 59-70, for each polynomial function.

    (a) *List each real zero and its multiplicity*
    (b) *Determine whether the graph crosses or touches the x-axis at each x-intercept*
    (c) *Determine the maximum number of turning points on the graph.*
    (d) *Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.*

61. $f(x) = 7(x^2+4)^2(x-5)^3$

65. $f(x) = (x-5)^3(x+4)^2$

69. $f(x) = -2x^2(x^2 - 2)$

73. *See 71 in 10th Edition*

77. *See 75 in 10th Edition*

81. *See 79 in 10th Edition*

85. ***Challenge Problem*** Find the real zeros of

$$f(x) = 3(x^2 - 1)(x^2 + 4x + 3)^2$$

and their multiplicity.

4.2: Graphing Polynomial Functions; Models

In Problems 5-22;31-42, graph each polynomial function by following Steps 1 through 5 on page (181 in 10th Edition)(191 in 11th Edition).

5. $f(x) = x^2(x - 3)$

9. $f(x) = -2(x + 2)(x - 2)^3$

13. $f(x) = x(1 - x)(2 - x)$

17. $f(x) = -2(x - 1)^2(x^2 - 16)$

21. $f(x) = x^2(x - 2)(x^2 + 3)$

33. $f(x) = x^3 + x^2 - 12x$

37. $f(x) = -x^5 - x^4 + x^3 + x^2$

41. $f(x) = \frac{1}{5}x^3 - \frac{4}{5}x^2 - 5x + 20$

4.3: Properties of Rational Functions

For 4.3 Homework, go to 4.2 Skill Building on Page 197 in 10th edition and follow same number unless otherwise stated.
In Problems 15-26, find the domain of each rational function.

21. $R(x) = \dfrac{x}{x^3 - 64}$

25. $R(x) = \dfrac{3(x^2 - x - 6)}{5(x^2 - 4)}$

53. $R(x) = \dfrac{6x^2 + 19x - 7}{3x - 1}$

4.4: The Graph of Rational Function

For Section 4.4 (11th Edition), see 4.3 (10th Edition) on page 211 in 10th edition and follow number to number.

4.5: Polynomial and Rational Inequalities

For Section 4.5 (11th Edition), see 4.4 (10th Edition) on page 211 in 10th edition and follow number to number unless otherwise stated. *In Problems 19-48, solve each inequality algebraically.*

25. $(x+2)(x-4)(x-6) \leq 0$

33. $3(x^2-2) < 2(x-1)^2 + x^2$

37. $\frac{(x-2)(x+2)}{x} \leq 0$

41. $\frac{x+4}{x-2} \leq 1$

45. $\frac{x+1}{x-3} \leq 2$

49. $\frac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0$

53. $6x - 5 < \frac{6}{x}$
    Mixed Practice *In Problems 55-58, (a) graph each function by hand, and (b) solve* $f(x) \geq 0$.

57. $f(x) = \frac{(x+4)(x^2-2x-3)}{x^2-x-6}$

61. What is the domain of the function $f(x) = \sqrt{x^4 - 16}$?
    In Problems 65-68, determine where the graph of f is below the graph of g by solving the inequality $f(x) \leq g(x)$. Graph f and g together.

65. $f(x) = x^4 - 1$
    $g(x) = -2x^2 + 2$

69. Where is the graph of $R(x) = \frac{x^4-16}{x^2-9}$ above the x-axis?

73. ***Challenge Problem*** **Bungee Jumping** Originating on Pentecost Island in the Pacific, the practive of a person jumping from a high place harnessed to a flexible attachment was introduced to Western culture in 1979 by the Oxford Univresity Dangerous Sport Club. One important parameter to know before attempting a bungee jump is the amount the cord will streth at the bottom of the fall. The stiffness of the cord is related to the amount of stretch by the equation

$$K = \frac{2W(S+L)}{S^2}$$

where	W	=	weight of the jumper (pounds)
	K	=	cord's stiffness (pound per foot)
	L	=	free length of the cord (feet)
	S	=	stretch (feet)

(a) A 150-pound person plans to jump off a ledge attached to a cord of length 42 feet. If the stiffness of the cord is no less than 16 pounds per foot, how much will the cord stretch?

(b) If safety requirements will not permit the jumper to get any closer than 3 feet to the ground, what is the minimum height required for the ledge in part (a)?

Test 4

4.6 The Real Zeros of a Polynomial Function

In Problems 11-20, use the Remainder Theorem to find the ramainder when $f(x)$ is divided by $x-c$. Then use the Factor Theorem to determine whether $x-c$ is a factor of $f(x)$.

13. $f(x) = 5x^4 - 20x^3 + x - 4; x - 2$

17. $f(x) = 4x^6 - 64x^4 + x^2 - 15; x + 4$
    In Problems 21-32, determine the maximum number of real zeros that each polynomial function may have. Then use Descartes' Rule of Signs to determine how many positive and how many negative real zeros each polynomial function may have. Do not attempt to find the zeros.

21. $f(x) = -4x^7 + x^3 - x^2 + 2$

25. $f(x) = -2x^3 + 5x^2 - x - 7$

29. $f(x) = x^5 + x^4 + x^2 + x + 1$
    In Problems 33-44, list the potential rational zeros of each polynomial function. Do not attempt to find the zeros.

33. $f(x) = 3x^4 - 3x^3 + x^2 - x + 1$

37. $f(x) = -9x^3 - x^2 + x + 3$

41. $f(x) = 2x^5 - x^3 + 2x^2 + 12$
    In Problems 45-56, use the Ration Zeros Theorem to find all the real zeros of each polynomial function. Use the zeros to factor f over the real numbers.

45. $f(x) = x^3 + 2x^2 - 5x - 6$

49. $f(x) = 2x^3 - 4x^2 - 10x + 20$

53. $f(x) = x^4 + x^3 - 3x^2 - x + 2$
    In Problems 57-68, solve each equation in the real number system.

57. $x^4 - x^3 + 2x^2 - 4x + 8 = 0$

61. $3x^3 - x^2 - 15x + 5 = 0$

65. $x^3 - \frac{2}{3}x^2 + \frac{8}{3}x + 1 = 0$
In Problems 69-78, find bounds on the real zeros of each polynomial function.

69. $f(x) = x^4 - 3x^2 - 4$

73. $f(x) = 3x^4 + 3x^3 - x^2 - 12x - 12$

77. $f(x) = -x^4 + 3x^3 - 4x^2 - 2x + 9$
In Problems 79-84, use the Intermediate Value Theorem to show that each polynomial function has a real zero in the given interval.

81. $f(x) = 2x^3 + 6x^2 - 8x + 2$; $[-5, -4]$
In Problems 85-88, each equation has a solution r in the interval indicated. Use the method of Example 10 to approximate this solution correct to two decimal places.

85. $8x^4 - 2x^2 + 5x - 1 = 0; 0 \leq r\ leq 1$
In Problems 89-92, each polynomial function has exactly one positive real zero. Use the method of Example 10 to approximate the zero correct to two decimal places.

89. $f(x) = x^3 + x^2 + x - 4$
Mixed Practice *In Problems 93-104, graph each polynomial function*

93. $f(x) = x^3 + 2x^2 - 5x - 6$

97. $f(x) = x^4 + x^2 - 2$

101. $f(x) = x^4 + x^3 - 3x^2 - x + 2$

105. Suppose that $f(x) = 3x^3 + 16x^2 + 3x - 10$. Find the zeros of $f(x + 3)$.

109. What is the
when $f(x) = 2x^{20} - 8x^{10} + x - 2$ is divided by $x - 1$?

113. One solution of the equation $x^3 - 8x^2 + 16x - 3 = 0$ is 3. Find the sum of the remaining solutions.

5 Exponential and Logarithmic Functions

5.1 Composite Functions

In Problems 13-22, for the given function f and g, find:
$(a)(f \circ g)(4)$ $(b)(g \circ f)(2)$ $(c)(f \circ f)(1)$ $(d)(g \circ g)(0)$

17. $f(x) = \sqrt{x}; g(x) = 5x$
    In Problems 23-38, for the given function f and g find:
    $(a)(f \circ g)(4)$ $(b)(g \circ f)(2)$ $(c)(f \circ f)(1)$ $(d)(g \circ g)(0)$
    State the domain of each composite function.

25. $f(x) = 3x - 1; g(x) = x^2$

33. $f(x) = \sqrt{x}; g(x) = 2x + 5$

73. Let $f(x) = ax + b$ and $g(x) = bx + a$, where a and b are integers. If $f(1) = 8$ and $f(g(20)) - g(f(20)) = -14$, find the product of a and b.

5.2 One-to-One functions: Inverse Function

29. *See number 47 in 10th edition.*
    In Problems 33-42, verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g.

33. $f(x) = 3x + 4; g(x) = \frac{1}{3}(x - 4)$

37. $f(x) = x^3 - 8; g(x) = \sqrt[3]{x + 8}$

41. $f(x) = \frac{1}{x}; g(x) = \frac{1}{x}$
    In Problems 43-54, the function f is one-to-one. (a) Find its inverse function f^{-1} and check your answer. (b) Find the domain and the range of f and f^{-1}. (c) Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

45. $f(x) = 4x + 2$

49. $f(x) = x^2 + 4, x \geq 0$

53. $f(x) = \frac{1}{x - 2}$
    In Problems 55-72, the function f is one-to-one. (a) Find its inverse function f^{-1} and check your answer. (b) Find the domain and the range of f and f^{-1}.

57. $f(x) = \frac{3x}{x + 2}$

61. $f(x) = \frac{3x + 4}{2x - 3}$

65. $f(x) = \frac{x^2 - 4}{2x^2}, x > 0$

69. $f(x) = \sqrt[3]{x^5 - 2}$

73. Use the graph of $y = f(x)$ given in problem 45 (in 10th edition) to evaluate the following:
    (a) $f(-1)$
    (b) $f(1)$
    (c) $f^{-1}(1)$
    (d) $f^{-1}(2)$

77. The domain of a one-to-one function f is $[5, \infty)$, and its range is $[-2, \infty)$. State the domain and the range of f^{-1}.

81. A function $y = f(x)$ is increasing on the interval $[0, 5]$. What conclusions can you draw about the graph of $y = f^{-1}(x)$?

85. A function f has an inverse function f^{-1}. If the graph of f lies in quadrant I, in which quadrant does the graph of f^{-1} lie?

89. *See note above 91 in 10th edition.*
    Vehicular Stopping Distance Taking into account reaction time, the distance d (in feet) that a car requires to come to a complete stop while traveling f miles per hour is given by the function

$$d(r) = 6.97r - 90.39$$

    (a) Express the speed r at which the car is traveling as a function of the distance d required to come to a complete stop.
    (b) Verify that $r = r(d)$ is the inverse of $d = d(r)$ by showing that $r(d(r)) = r$ and $d(r(d)) = d$.
    (c) Predict the speed that a car was traveling if the distance required to stop was 300 feet.

93. **Income Taxes** The function

$$T(g) = 1905 + 0.12(g - 19050)$$

    represents the 2018 federal income tax T (in dollars) due for a "married filing jointly" filer whose modified adjusted gross income is g dollars, where $19,050 < g \leq 77,400$.
    (a) What is the domain of the function T?
    (b) Given that the tax due T is an increasing linear function of modified adjusted gross income g, find the range of the function T.
    (c) Find adjusted gross income g as a function of federal income tax T. What are the domain and the range of this function?

97. ***Challenge Problem*** Given

$$f(x) = \frac{ax + b}{cx + d}$$

    find $f^{-1}(x)$. If $c \neq 0$, under what conditions on a, b, c, and d is $f = f^{-1}$?

5.3 Exponential Functions

Complete 35-129 EOO unless written below:

65. $6^x = 6^5$

81. $e^{2x} = e^{5x+12}$

133. ***Challenge Problem*** Solve: $3^{2x-1} - 4 \cdot 3^x + 9 = 0$

137. Do you think that there is a power function that increases more rapidly than an exponential function? Explain.

5.4 Logarithmic Function

In Problems 27-38, find the exact value of each logarithm without using a calculator.

29. $\log_7 49$
    In Problems 89-112, solve each equation.

93. $\log_x 16 = 2$

105. $\log_7(x^2 + 4) = 2$

137. ***Challenge Problem*** Solve: $\log_6(\log_2 x) = 1$

5.5 Properties of Logarithms

In Problems 13-28, use properties of logarithms to find the exact value of each expression. Do not use a calculator.

13. $\log_7 7^{29}$

17. $9^{\log_9 13}$

21. $\log_5 35 - \log_5 7$

25. $4^{\log_4 6 - \log_4 5}$
    In Problems 37-56, write each expression as a sum and/or difference of logarithms. Express powers as factors.

37. $\log_6 36x$

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5.6 Logarithmic and Exponential Equations

In Problems 5-44, solve each logarithmic equation. Express irrational solutions in exact forms.

9. $\log_4(x+4) = \log_4 15$

13. $\log_5|2x-1| = \log_5 13$

17. $3\log_2 x = -\log_9|5x-12|$

21. $\log x + \log(x+15) = 2$

25. $\log_2(x+7) + \log_2(x+8) = 1$

29. $\ln x + \ln(x+2) = 4$

33. $\log_{1/3}(x^2+x) - \log_{1/3}(x^2-x) = -1.$

37. $2\log_5(x-3) - \log_5 8 = \log_5 2$

41. $2\log_{13}(x+2) = \log_{13}(4x+7)$
    In Problems 45-72, solve each exponential equation. Express irrational solutions in exact form.

45. $2^{x-5} = 8$

49. $8^{-x} = 1.2$

53. $3^{1-2x} = 4^x$

57. $1.2^x = (0.5)^{-x}$

61. $2^{2x} + 2^x - 12 = 0$

65. $16^x + 4^{x+1} - 3 = 0$

69. $3 \cdot 4^x + 4 \cdot 2^x + 8 = 0.$

87. $f(x) = \log_2(x+3)$ and $g(x) = \log_2(3x+1)$.
    (a) Solve $f(x) = 3$. What point is on the graph of f?
    (b) Solve $g(x) = 4$. What point is on the graph of g?
    (c) Solve $f(x) = g(x)$. Do the graphs of f and g intersect? If so, where?
    (d) Solve $(f+g)(x) = 7$.
    (e) Solve $(f-g)(x) = 2$.

91. (a) Graph $f(x) = x^3$ and $g(x) = 10$ on the same Cartesian plane.
    (b) Shad the region bounded by the y-axis, $f(x) = 3^x$ and $g(x) = 10$ on the graph drawn in part (a).
    (c) Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

95. (a) Graph $f(x) = 2^x - 4$
    (b) Find the zero of f
    (c) Based on the graph, solve $f(x) < 0$.

99. **Depreciation** The value V of a Chevy Cruze LT that is t years old can be modeled by $V(t) = 19,200(0.82)^t$.
    (a) According to the model, whfen will the car be worth \$12,000?
    (b) According to the model, when will the car be worth \$9,000? According to the model, when will the car be worth \$3,000?

Challenge Problems *In Problems 101-105, solve each question. Express irrational solutions in exact form.*

103. $\log_2 x^{\log_2 x} = 4$

5.8: Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models

Same as 11th edition