Precalculus 1 (Algebra), Pandemic Inspired Problems

Note. This document involves use data from the ETSU COVID-19 Dashboard. This website provides the number of "Active Positive Cases" among ETSU students and employees. It is updated each weekday. An "active positive case" is defined as: "Individuals who have received and reported positive COVID-19 test results and are currently in isolation." Details on the collection of the data are given on the Dashboard website. Throughout the semester, we will apply the material of our Precalculus 1 (Algebra) class to this data. This will illustrate the strengths (and weaknesses) of the material as applied to contemporary real-world data that affects us all.

Section 1.3. Lines

Note. The example presented next is similar to Exercise 1.3.126 in the class notes.

Example A. We will number the weeks of the semester, starting with week #1 (which began August 23, 2021) and ending with week #16 (which begins December 6, 2021; this is finals week). According to the ETSU COVID-19 Dashboard, there were 15 active positive cases on August 23 (at the beginning of Week 1). One week later on August 30 there were 61 active positive cases (at the beginning of Week 2).

(a) Write a linear equation that relates the number of active positive cases y to the number of the week x.

(b) Find the intercepts of the graph of your equation.

(c) Do these intercepts have a meaningful interpretation?

(d) Use your equation to estimate the number of active positive cases at the end of Wednesday in the first week. (The actual number of cases on that day was 38.)

(e) Use your equation to estimate the number of active cases at the beginning of finals week (this would be Week 16 when $x = 16$).

Solution. **(a)** We use the given information to determine two points. In Week 1 there are 15 active cases, so we let the first point be $(x_1, y_1) = (1, 15)$. In Week 2 there are 61 active cases, so we let the second point be $(x_2, y_2) = (2, 61)$. This allows to use the formula for slope to compute:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(61) - (15)}{(2) - (1)} = \frac{46}{1} = 46 \text{ cases/week.}$$

Based on the point-slope formula of a line, $y - y_1 = m(x - x_1)$, we have

$$y - (15) = (46)(x - (1)) \text{ or } y - 15 = 46x - 46 \text{ or } \boxed{y = 46x - 31}.$$

(b) To find the x-intercept, we set $y = 0$. This gives $y = 46x - 31 = 0$ and so the $\boxed{x\text{-intercept is } (31/46, 0) \approx (0.67, 0)}$. To find the y intercept, we set $x = 0$. This gives $y = 46x - 31 = 46(0) - 31 = -31$ and so the $\boxed{y\text{-intercept is } (0, -31)}$.

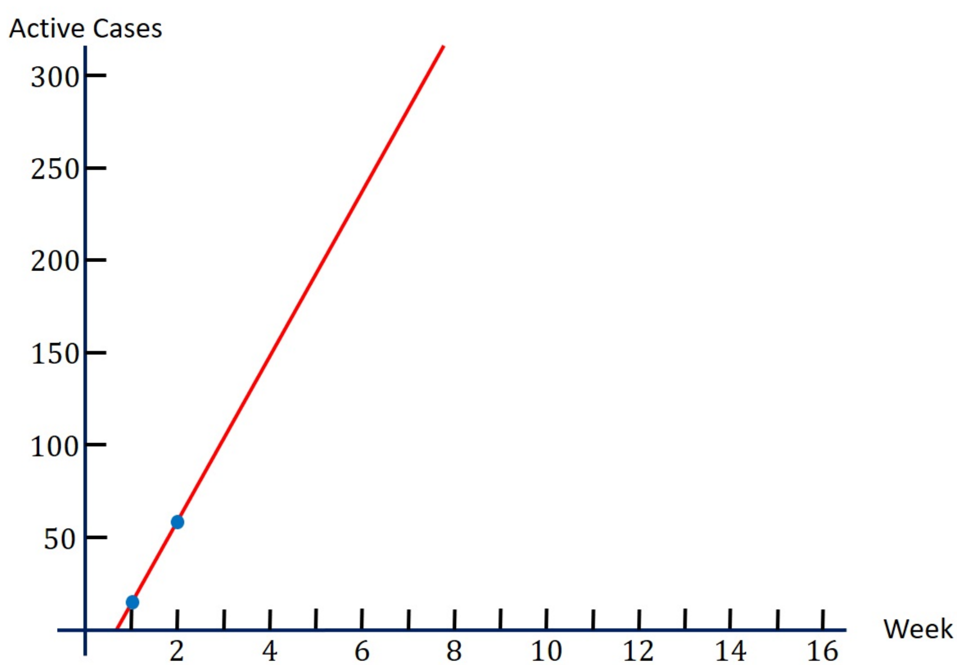
(c) The x-intercept $(0.67, 0)$ implies that there were 0 cases at week 0.67 (which would translate into $(0.67)(7) \approx 4.7$ days into Week 0. This means that our linear equation predicts 0 cases on Friday, August 20 (that is, about 2.3 days before

August 23). This is numerically meaningful (though with no cases present, the virus cannot spread so there are some biological concerns about the meaningfulness of this intercept). The y-intercept $(0, -31)$ implies that at week 0 (which would correspond to August 16, 2021) there are -31 active positive cases. Since a quantity cannot be negative, then this is not meaningful.

(d) We consider Week 1 as beginning at the start of Monday, August 23 so at the end of Wednesday in the first week, we have the corresponding x value of $x = 1 + 3/7 = 10/7$ (measured in weeks). So at this point in time, we have from our equation that the number of positive active cases is $\boxed{y = 46(10/7) - 31 \approx 34.7}$. Since the actual number of cases was 38, our model is fairly close!

(e) At Week 1 when $x = 16$, our equation predicts $y = 46(16) - 31 = \boxed{705}$ active positive cases.

The graph of our equation is:



In part (d) we have *interpolated* between two known data points, and the result was fairly accurate. In part (e) here we have *extrapolated* from the first two weeks what the behavior will be several weeks into the future. This is likely not a reliable prediction since our linear model is very elementary and it is unlikely that the number of active cases will respond in in such a way.

Section 2.3. Properties of Functions

Note. The example presented next is similar to Exercise 2.3.82 in the class notes.

Example B. Continuing Example A, we have for the first five weeks of classes the following active positive case numbers:

Week	**Cases**
1	15
2	61
3	74
4	35
5	27

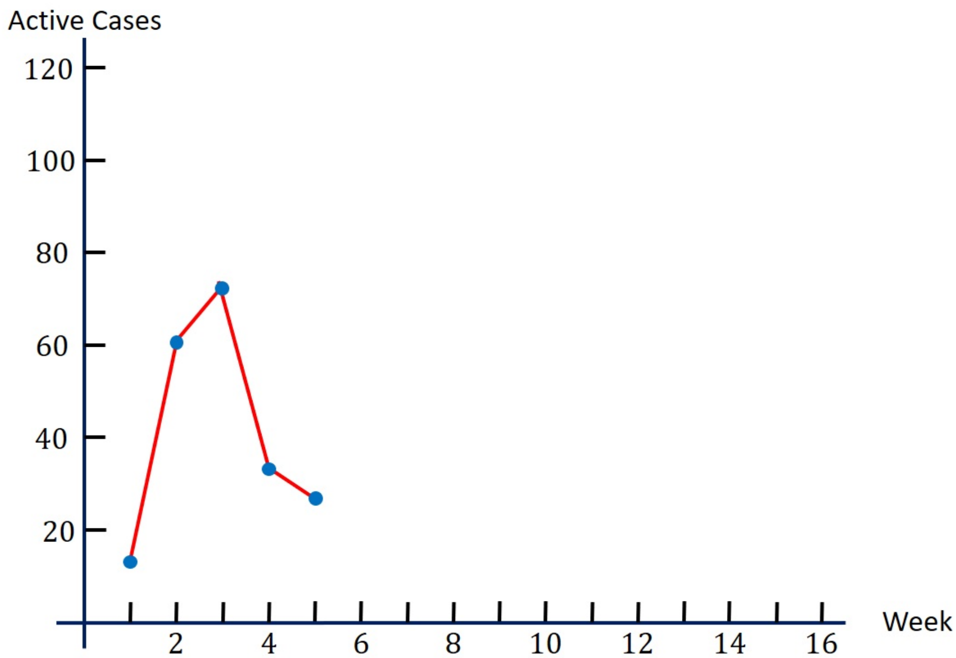
Let C be the number of cases as a function of x, the number of the week.

(a) Plot the points as in Example A. Draw line segments joining the the points corresponding to consecutive weeks.

(b) Find the average rate of change for for each week.

(c) What appears to be happening to the average rate of change as time passes?

Solution. **(a)** The points and corresponding line segments are as given below. Notice that the scale of the y axis has changed from Example A.



(b) The average rate of change of function C from a to b is $\dfrac{C(b)-C(a)}{b-a}$. So we have:

From Week #1 to Week #2:

$$\text{Rate of Change} = \frac{C(2)-C(1)}{(2)-(1)} = \frac{(61)-(15)}{(2)-(1)} = 46 \text{ cases/week.}$$

From Week #2 to Week #3:

$$\text{Rate of Change} = \frac{C(3)-C(2)}{(3)-(2)} = \frac{(74)-(61)}{(3)-(2)} = 13 \text{ cases/week.}$$

From Week #3 to Week #4:

$$\text{Rate of Change} = \frac{C(4)-C(3)}{(4)-(3)} = \frac{(35)-(74)}{(4)-(3)} = -39 \text{ cases/week.}$$

From Week #4 to Week #5:

$$\text{Rate of Change} = \frac{C(5)-C(4)}{(5)-(4)} = \frac{(27)-(35)}{(5)-(4)} = -8 \text{ cases/week.}$$

(c) Though the number of cases went up from Week #1 to Week #3, the rate at which they went up decreased. from the Week #3 to Week #4 the rate went negative, and remained negative (but decreased in magnitude) from Week #4 to Week #5. It appears that the number of positive active cases is remaining small (which is evidence for the success of the mask mandate).

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