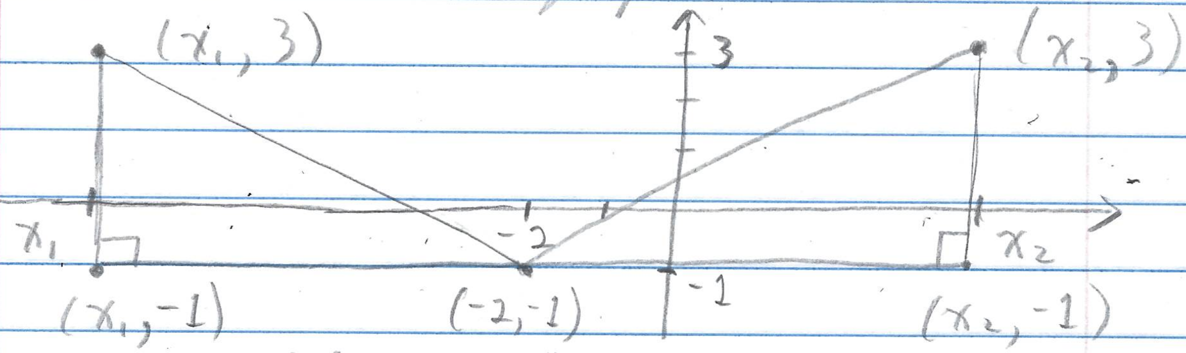
1.1.49 Modified

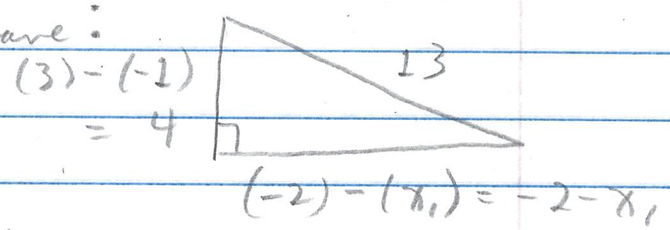
<u>Exercise 1.1.49</u> (Modified) Find all points having a y-coordinate of 3 whose distance from the point $(-2,-1)$ is 13:
(a) by using the Pythagorean Theorem, and
(b) by using the distance formula.

<u>Solution</u>

(a) We have the graph:



On the left we have:



and so by the Pythagorean Theorem,

$$(13)^2 = (4)^2 + (-2-x_1)^2$$ or

$169 = 16 + (-2-x_1)^2$ or $153 = (-2-x_1)^2$

or $\sqrt{153} = \sqrt{(-2-x_1)^2} = |-2-x_1| = -2-x_1$

since $x_1 < -2$ and so $0 < -2-x_1$.

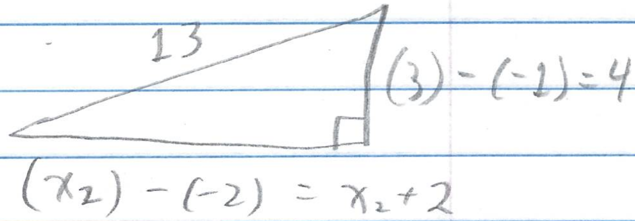
So $-2-x_1 = \sqrt{153}$ or $x_1 = -2-\sqrt{153}$.

Hence, the point on the left

$\boxed{(x_1, 3) = (-2-\sqrt{153}, 3)}$ is 13 units from point $(-2,-1)$.

(1.1.49 modified continued)

The right triangle is



and so by the Pythagorean Theorem,

$$(13)^2 = (4)^2 + (x_2+2)^2 \text{ or } 169 = 16 + (x_2+2)^2$$

or $\sqrt{153} = \sqrt{(x_2+2)^2} = |x_2+2| = x_2+2$ since $x_2 > -2$ and so $x_2+2 > 0$.

So $x_2 + 2 = \sqrt{153}$ or $x_2 = -2 + \sqrt{153}$.

Hence the point on the right $\boxed{(x_2, 3) = (-2+\sqrt{153}, 3)}$ is 13 units from point $(-2,-1)$. $\square$

(b) With $(x, y) = (x, 3)$ a point distance 13 from point $(-2,-1)$, we have by the distance formula:

$$13 = \sqrt{(x-(-2))^2 + ((3)-(-1))^2}$$

$$= \sqrt{(x+2)^2 + (4)^2}.$$

Then $(13)^2 = \left(\sqrt{(x+2)^2+16}\right)^2$

or $169 = (x+2)^2 + 16$ or $153 = (x+2)^2$ or $\sqrt{153} = \sqrt{(x+2)^2} = |x+2|$. Therefore $x+2 = \pm\sqrt{153}$ or $x = -2 \pm \sqrt{153}$.

So the two desired points are

$$\boxed{(-2-\sqrt{153}, 3) \text{ and } (-2+\sqrt{153}, 3)}$$

(as also shown in part (a)). $\square$