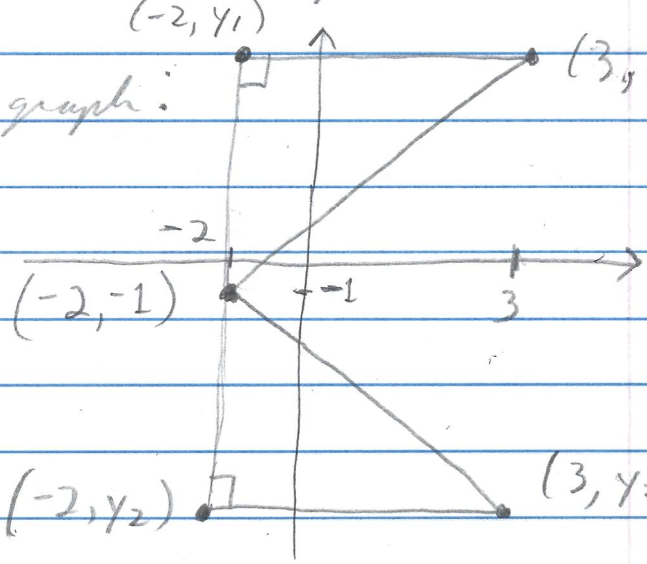
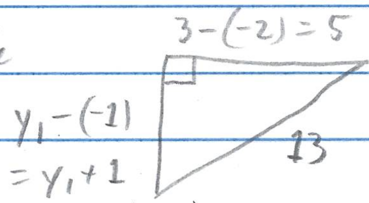
<u>Exercise 1.1.49</u> Find all points having an x-coordinate of 3 whose distance from the point $(-2,-1)$ is 13:
(a) by using the Pythagorean Theorem, and
(b) by using the distance formula.

<u>Solution</u>

We have the graph:



In the upper right triangle we have



and so by the Pythagorean Theorem,

$(13)^2 = (5)^2 + (y_1+1)^2$ or $169 = 25 + (y_1+1)^2$

or $144 = (y_1+1)^2$ or $\sqrt{144} = \sqrt{(y_1+1)^2}$ or

$12 = |y_1+1| = y_1+1$ since $y_1 > -1$ and $y_1+1>0$.

So $y_1 = 12-1 = 11$. Hence, the point at the top $\boxed{(3,y_1) = (3,11)}$ is 13 units from point $(-2,-1)$.

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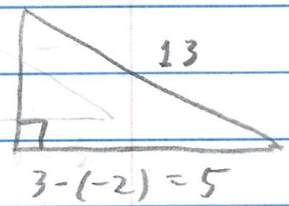
cont.

In the lower right triangle we have $(-1)-y_2 = -1-y_2$



and so by the Pythagorean Theorem,
$(13)^2 = (5)^2 + (-1-y_2)^2$ or $144 = (-1-y_2)^2$
or $\sqrt{144} = \sqrt{(-1-y_2)^2}$ or $12 = |-1-y_2| = -1-y_2$
since $y_2 < -1$ and $0 < -1-y_2$. So
$y_2 = (-1)-(12) = -13$. Hence, the point
at the bottom $\boxed{(3, y_2) = (3,-13)}$ is 13 units
from point $(-2,-1)$. $\square$

(b) With $(x,y) = (3,y)$ a point distance 13 from
point $(-2,-1)$, we have by the distance
formula: $(13) = \sqrt{((3)-(-2))^2 + ((y)-(-1))^2}$
$= \sqrt{(5)^2 + (y+1)^2}$.
Then $(13)^2 = \left(\sqrt{25+(y+1)^2}\right)^2$ or $169 = 25 + (y+1)^2$
or $144 = (y+1)^2$ or $\sqrt{144} = \sqrt{(y+1)^2} = |y+1|$.
So $|y+1| = 12$ and $y+1 = \pm 12$ or $y = -1 \pm 12$,
and either $y = -13$ or $y = 11$. So the two
desired points are $\boxed{(3,-13) \text{ and } (3,11)}$

(as shown in part (a)). $\square$