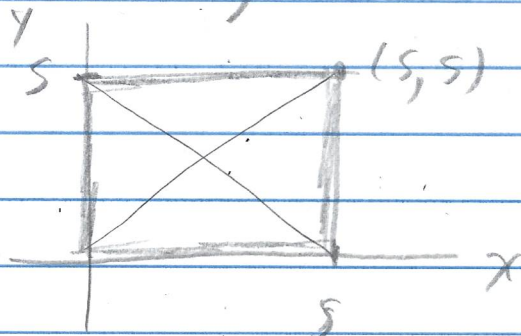


Exercise 1.1, 73 Find the midpoint of each diagonal of a square with side of length s . Draw the conclusion that the diagonals of a square intersect at their midpoints.
 HINT: Use $(0,0)$, $(0,s)$, $(s,0)$, and (s,s) as the vertices of the square.

Solution

Following the hint, we have



The midpoint of the diagonal from $(x_1, y_1) = (0,0)$ to $(x_2, y_2) = (s,s)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0+s}{2}, \frac{0+s}{2} \right) = \left(\frac{s}{2}, \frac{s}{2} \right).$$

The midpoint of the diagonal from $(x_1, y_1) = (s,0)$ to $(x_2, y_2) = (0,s)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{s+0}{2}, \frac{0+s}{2} \right) = \left(\frac{s}{2}, \frac{s}{2} \right).$$

Therefore the diagonals of this square (and, since $s > 0$ is arbitrary, any square) intersect at their midpoints, since $\left(\frac{s}{2}, \frac{s}{2} \right)$ is the midpoint of each diagonal. \square