(1.4.29)

Exercise 1.4.29 Consider the circle given by the equation $x^2+y^2-2x-4y-4=0$. (a) Find the center (h,k) and radius r; (b) graph the circle; and (c) find the intercepts, if any.

Solution

Recall that to complete the square (see Appendix A3) we have

$$(x^2+bx)+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2.$$

(a) So to complete the square for x^2-2x we need to add $((-2)/2)^2=1$, and to complete the square for y^2-4y we need to add $((-4)/2)^2=4$. So completing the square (in order to put the formula in the standard form of a circle) we have:

$$x^2-2x+(1)+y^2-4y+(4)-4=(1)+(4) \text{ or}$$

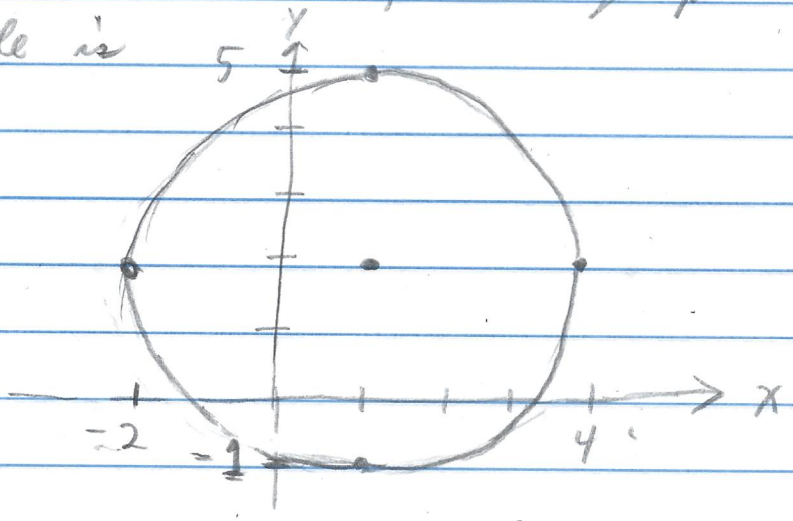
$$(x-1)^2+(y-2)^2-4=5 \text{ or}$$

$$(x-1)^2+(y-2)^2=9. \text{ This is in}$$

the form $(x-h)^2+(y-k)^2=r^2$ where the $\boxed{\text{center is } (h,k)=(1,2)}$ and the radius $\boxed{r=\sqrt{9}=3}$.

1.4.29 continued

(b) The center is $(h, k) = (1, 2)$ and the radius is $r = 3$, the graph of the circle is



(c) For the x-intercepts we set $y = 0$ and consider $(x-1)^2 + ((0)-2)^2 = 9$ or $(x-1)^2 + 4 = 9$ or $(x-1)^2 = 5$ or $(x-1) = \pm\sqrt{5}$ or $x = 1 \pm \sqrt{5}$. So the x-intercepts are $(1-\sqrt{5}, 0)$ and $(1+\sqrt{5}, 0)$.

For the y-intercepts we set $x = 0$ and consider $((0)-1)^2 + (y-2)^2 = 9$ or $1 + (y-2)^2 = 9$ or $(y-2)^2 = 8$ or $y - 2 = \pm\sqrt{8} = \pm 2\sqrt{2}$ or $y = 2 \pm 2\sqrt{2}$. So the y-intercepts are $(0, 2-2\sqrt{2})$ and $(0, 2+2\sqrt{2})$. □