1.4.41

Exercise 1.4.41 Find the standard form of the circle with endpoints of the diameter at $(1, 4)$ and $(-3, 2)$.

Solution

With $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (-3, 2)$ then the diameter d is (by the distance formula):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-3) - (1))^2 + ((2) - (4))^2}$$

$$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}.$$

So the radius is $r = d/2 = \sqrt{5}$.

The center of the circle is (by the midpoint formula) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{(1) + (-3)}{2}, \frac{(4) + (2)}{2} \right)$

$$= (-1, 3) = (h, k).$$

So the standard form of the circle is

$$(x - h)^2 + (y - k)^2 = r^2$$ or

$$(x - (-1))^2 + (y - (3))^2 = (\sqrt{5})^2$$

or $\boxed{(x + 1)^2 + (y - 3)^2 = 5}$. $\square$