Exercise 1.4.65 The Greek method for finding the equation of the tangent line to a circle uses the fact that at any point on a circle, the line containing the center and the tangent line are perpendicular. Use this method to find an equation of the tangent line to the circle $x^2+y^2=9$ at the point $(1, 2\sqrt{2})$.

Solution

The center of the circle $x^2+y^2=9$ is $(0,0)$. With line L_1 as the line through the center $(0,0)$ and the point $(1, 2\sqrt{2})$, the slope of L_1 is

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{(2\sqrt{2})-(0)}{(1)-(0)} = 2\sqrt{2}.$$

With L_2 as the line tangent to the circle at point $(1, 2\sqrt{2})$ and with m_2 as the slope of L_2, we have by the Greek method that L_1 and L_2 are perpendicular. So by the Criterion for Perpendicular Lines (Theorem 1.3.F), $m_2 = -1/m_1 = -1/(2\sqrt{2})$, or $m_2 = \frac{-1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-\sqrt{2}}{4}$. By the point slope form, the equation for line L_2 is $y-y_1 = m_2(x-x_1)$ or $y-(2\sqrt{2}) = \frac{-\sqrt{2}}{4}(x-(1))$ or $y = \frac{-\sqrt{2}}{4}x + \frac{\sqrt{2}}{4} + 2\sqrt{2}$ or $\boxed{y = \frac{-\sqrt{2}}{4}x + \frac{9\sqrt{2}}{4}}$. □