

Exercise 2.1.37 Does the equation $x = y^2$ determine y as a function of x ?

Solution

If y is a function of x then for each x in the domain of the relation, there corresponds EXACTLY ONE y value.

Notice that if $x = 9$ then we need $9 = y^2$. So:

$$\textcircled{1} \quad 9 = y^2 \text{ or}$$

$$y^2 - 9 = 0 \text{ or}$$

$$(y+3)(y-3) = 0 \text{ or}$$

$$y = -3 \text{ or } y = +3$$

$$\textcircled{2} \quad 9 = y^2 \text{ or}$$

$$\sqrt{9} = \sqrt{y^2} \text{ or}$$

$$3 = |y| \text{ or}$$

$$y = \pm 3.$$

$$\textcircled{3} \quad 9 = y^2 \text{ or}$$

$$y^2 = 9 \text{ or}$$

$$y = \pm \sqrt{9} \text{ or}$$

$$y = \pm 3.$$

Therefore, since for $x = 9$ we have TWO corresponding y -values (namely, -3 and $+3$) then NO, y is NOT a function of x . \square

NOTE The equation $x = y^2$ is equivalent to the equation $y = \pm \sqrt{x}$.