(2.1.57)

Exercise 2.1.57 Find the domain of

$$F(x) = \frac{x-2}{x^3+x}.$$

Solution

We must avoid (1) division by 0, and (2) square of negative. There are no square roots, BUT there is division. We cannot have $x^3 + x = 0$. So consider $x^3 + x = 0$ or $x(x^2+1) = 0$. So $x \neq 0$ and $x^2 + 1 \neq 0$; but $x^2 + 1 = 0$ would imply $x^2 = -1$ and this can't happen (with real numbers).

Therefore the only thing that "goes wrong" is division by 0 when $x = 0$. So

the domain is all real numbers except $x = 0$:

$$(-\infty, 0) \cup (0, \infty)$$

□