2.1.77

Exercise 2.1.77 For $f(x) = 1 + \frac{1}{x}$ and $g(x) = \frac{1}{x}$, find (a) $(f+g)(x)$, (b) $(f-g)(x)$, (c) $(f \cdot g)(x)$, (d) $\left(\frac{f}{g}\right)(x)$, (e) $(f+g)(3)$, (f) $(f-g)(4)$, (g) $(f \cdot g)(2)$, and (h) $\left(\frac{f}{g}\right)(1)$.

Solution

We use the definitions of sum / difference / product / quotient of two functions.

(a) $(f+g)(x) = f(x) + g(x)$

$= \left(1 + \frac{1}{x}\right) + \left(\frac{1}{x}\right) = \boxed{1 + \frac{2}{x}}$.

(b) $(f-g)(x) = f(x) - g(x)$

$= \left(1 + \frac{1}{x}\right) - \left(\frac{1}{x}\right) = \boxed{1}$.

(c) $(f \cdot g)(x) = f(x) \cdot g(x)$

$= \left(1 + \frac{1}{x}\right)\left(\frac{1}{x}\right) = \boxed{\frac{1}{x} + \frac{1}{x^2}}$.

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(1 + 1/x)}{(1/x)}$

$= \left(1 + \frac{1}{x}\right) \cdot x = x + 1 \quad (\text{if } x \neq 0)$.

(e) $(f+g)(3) = 1 + \frac{2}{(3)}$ by part (a)

$= \boxed{\frac{5}{3}}$

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(f) $(f-g)(4) = \boxed{1}$ by part (b)

(g) $(f \cdot g)(2) = \frac{1}{(2)} + \frac{1}{(2)^2}$ by part (c)

$= \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{4}}$.

(h) $\left(\frac{f}{g}\right)(1) = (1) + 1$ by part (d)

$= \boxed{2}$. □