(2.3.41)

Exercise 2.3.41 Determine algebraically whether $F(x) = \sqrt[3]{4x}$ is even, odd, or neither.

Solution

To test if F is even, we check if F is symmetric with respect to the y-axis. So we replace x with $-x$ and get
$F(-x) = \sqrt[3]{4(-x)} = \sqrt[3]{4(-1)(x)} = -\sqrt[3]{4x}$.
Now $-\sqrt[3]{4x}$ and $\sqrt[3]{4x}$ are not equivalent functions since with $x = 2$ we have
$-\sqrt[3]{4(2)} = -\sqrt[3]{8} = -2 \neq 2 = \sqrt[3]{8} = \sqrt[3]{4(2)}$.
Hence F is not symmetric with respect to the y-axis and $\boxed{F \text{ is not even}}$.

To test if F is odd, we check if F is symmetric with respect to the origin. So we replace x with $-x$ and get
$F(-x) = \sqrt[3]{4(-x)} = \sqrt[3]{4(-1)(x)} = -\sqrt[3]{4x} = -F(x)$,
so F is symmetric with respect to the origin and is $\boxed{\text{an odd function.}}$ □