

Exercise 2.3.81 A strain of *E. coli* Ben 397-rec A411 is placed into a nutrient broth at 30° Celsius and allowed to grow. The data shown in the table are collected. The population is measured in grams and the time in hours. Since population P depends on time t , and each input corresponds to exactly one output, we can say that population is a function of time, so $P(t)$ represents the population at time t .

Time t (hours)	Population P (grams)
0	0.09
2.5	0.18
3.5	0.26
4.5	0.35
6	0.50

- (a) Find the average rate of change of the population from 0 to 2.5 hours.
 (b) Find the average rate of change of the population from 4.5 to 6 hours.
 (c) What is happening to the average rate of change as time passes.

Solution

(a) The average rate of change of function P from $t=a$ to $t=b$ is $\frac{P(b) - P(a)}{b - a}$, so

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continued

with $a=0$ and $b=2.5$ we have the rate of change

$$\frac{P(2.5) - P(0)}{(2.5) - (0)} = \frac{(0.18) - (0.09)}{(2.5) - (0)}$$
$$= \frac{0.09}{2.5} = \boxed{0.036 \text{ grams/hour}}$$

(b) With $a=4.5$ and $b=6$ we have the rate of change

$$\frac{P(6) - P(4.5)}{(6) - (4.5)} = \frac{(0.50) - (0.35)}{(6) - (4.5)}$$
$$= \frac{0.15}{1.5} = \boxed{0.1 \text{ grams/hour}}$$

(c) Based on the average rates of change computed in (a) and (b), we see that the average rate of change is increasing as time increases. (This means that the population is "accelerating."). \square