Exercise 2.4.40 Consider $f(x) = \begin{cases} 2-x & \text{if } -3 \le x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

(a) Find the domain.

Solution

Well, f is defined for $-3 \le x < 1$ and $x > 1$. So the domain is $[-3,1) \cup (1,\infty)$.

(b) Locate intercepts.

Solution

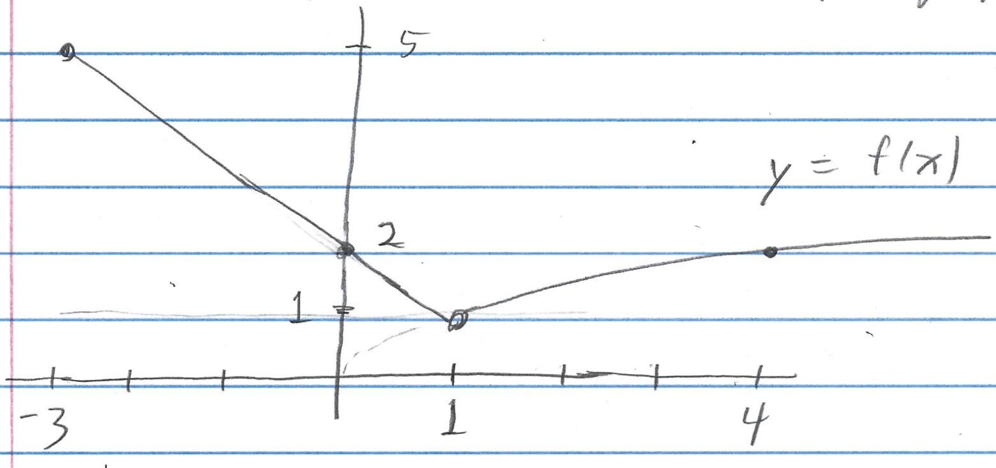
For the y-intercept, set $x=0$ and we get $f(0) = 2-(0) = 2$ and the $\boxed{y\text{-intercept is } 2.}$ For the x-intercept, set $f(x)=0$. BUT f has 2 pieces, so: $2-x=0$ gives $x=2$; but when $x=2$, we don't use this piece. So this piece does not produce an x-intercept. Next, consider $\sqrt{x}=0$ or $x=0$. But when $x=0$ we don't use this piece. So this piece does not produce an x-intercept and $\boxed{f \text{ has no } x\text{-intercepts.}}$

(c) Graph $y=f(x)$.

Solution

We graph $y = 2-x$ for $x \in [-3,1)$ and $y=\sqrt{x}$ for $x \in (1,\infty)$. To graph $y=2-x$, we notice that $x=-3$ implies $y=2-(3)=5$.

When $x = 0$, $y = 2-(0) = 2$ (and this is the y-intercept of f). We know the graph of $y = \sqrt{x}$ from the Library of Functions. Also, when $x = 4$, $y = \sqrt{4} = 2$. The graph of $y = f(x)$ is



(d) Based on the graph, find the range.

Solution

Notice the 1 is not in the domain of f, but every value greater than 1 is in the range (since $\sqrt{x}$ is unbounded above). So $\boxed{\text{the range is } (1, \infty).}$

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