2.4.41

Exercise 2.4.41 Consider

$$f(x) = \begin{cases} x^2 & \text{if } 0 < x \leq 2 \\ x+2 & \text{if } 2 < x < 5 \\ 7 & \text{if } x \geq 5. \end{cases}$$

(a) Find the domain. (b) Locate any intercepts. (c) Graph. (d) Find the range.

Solution

(a) The domain of f is the set of x values where f is defined, so (in interval notation) the domain is $(0,2] \cup (2,5) \cup [5,\infty) = (0,\infty)$.

(b) For the y-intercept, we set $x = 0$. But f is not defined at $x = 0$, so there is no y-intercept.

For x-intercepts, we set each piece of f equal to 0 and see if the resulting x-value is in the appropriate interval for that piece. Setting $x^2 = 0$ implies $x = 0$, but f is undefined at $x = 0$. Setting $x + 2 = 0$ implies $x = -2$. But f is undefined at $x = -2$. Setting $7 = 0$, of course, yields no such x. So there is no x-intercept.

(c) We know the graph of $y = x^2$ from the Library of functions. We know the graph of $y = x + 2$ since it is a line with

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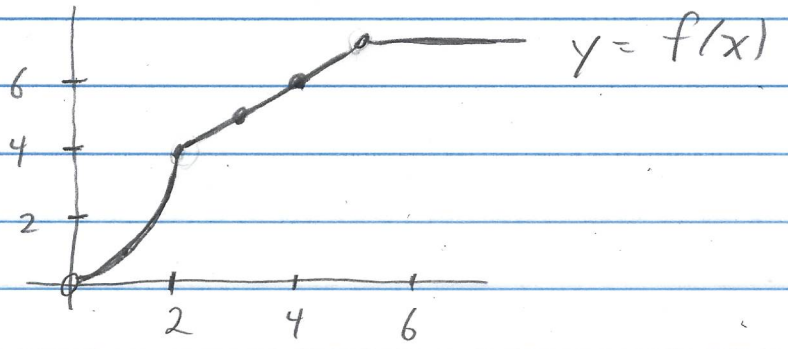
slope 1 containing the points (3, 5) and (4, 6) (say). The graph of $y = 7$ is a constant function. Graphing these functions over the appropriate intervals yields the graph of f:



(d) The range of f is the set of y-values for points on the graph. Notice that there are no points (x, y) on the graph for $y \leq 0$ nor for $y > 7$. **The range is $(0, 7]$.**