2.4.61

Exercise 2.4.61 The wind chill factor represents the air temperature at a standard wind speed that would produce the same heat loss as the given temperature and wind speed. One formula for computing the equivalent temperature is

$$W = \begin{cases} t & 0 \le v < 1.79 \\ 33 - \dfrac{(10.45 + 10\sqrt{v} - v)(33 - t)}{22.04} & 1.79 \le v \le 20 \\ 33 - 1.5958\,(33 - t) & v > 20 \end{cases}$$

where v represents the wind speed (in meters per second) and t represents the air temperature (°C). Compute the wind chill for the following:

(a) An air temperature of 10°C and a wind speed of 1 m/sec.

(b) An air temperature of 10°C and a wind speed of 5 m/sec.

(c) An air temperature of 10°C and a wind speed of 15 m/sec.

(d) An air temperature of 10°C and a wind speed of 25 m/sec.

(e) Explain the physical meaning of the equation corresponding to $0 \le v < 1.79$.

(f) Explain the physical meaning of the equation corresponding to $v > 20$.

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(a) With $t = 10$ and $v = 1$, we use the first "piece" of W to get $\boxed{W = t = 10^\circ C}$.

(b) With $t = 10$ and $v = 5$, we use the second "piece" of W to get

$$W = 33 - \frac{(10.45 + 10\sqrt{(5)} - (5))\,(33 - (10))}{22.04}$$

$$= 33 - \frac{(5.45 + 10\sqrt{5})\,(23)}{22.04} \approx \boxed{4^\circ C}.$$

(c) With $t = 10$ and $v = 15$, we use the second "piece" of W to get

$$W = 33 - \frac{(10.45 + 10\sqrt{(15)} - (15))\,(33 - (10))}{22.04}$$

$$= 33 - \frac{(10\sqrt{15} - 4.55)\,(23)}{22.04} \approx \boxed{-2.7^\circ C}.$$

(d) With $t = 10$ and $v = 25$, we use the third "piece" of W to get

$$W = 33 - 1.5958\,(33 - (10)) \approx \boxed{-3.7^\circ C}.$$

(e) Since $W = t$ for $0 \leq v < 1.79$, this means that wind speed of less than or equal to 1.79 m/sec yields a wind chill equal to the temperature.

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(f) Since $W = 33 - 1.5958(33 - t)$ for $v > 20$, this means that the impact of the wind on wind chill reaches a maximum coefficient of 1.5958 times the amount that the temperature is above 33°C.

In fact, with $v = 20$ m/sec,

$$\frac{10.45 + 10\sqrt{v} - v}{22.04} \approx 1.5958.$$

In fact, when $v = 1.79$ m/sec,

$$\frac{10.45 + 10\sqrt{v} - v}{22.04} \approx 1.00,$$ so that wind chill is a continuous function of wind speed v. □