Exercise 2.5.31 Find the function that is finally graphed after each of the following transformations is applied to the graph of $y = \sqrt{x}$ in the order stated:
(1) Vertical stretch by a factor of 3,
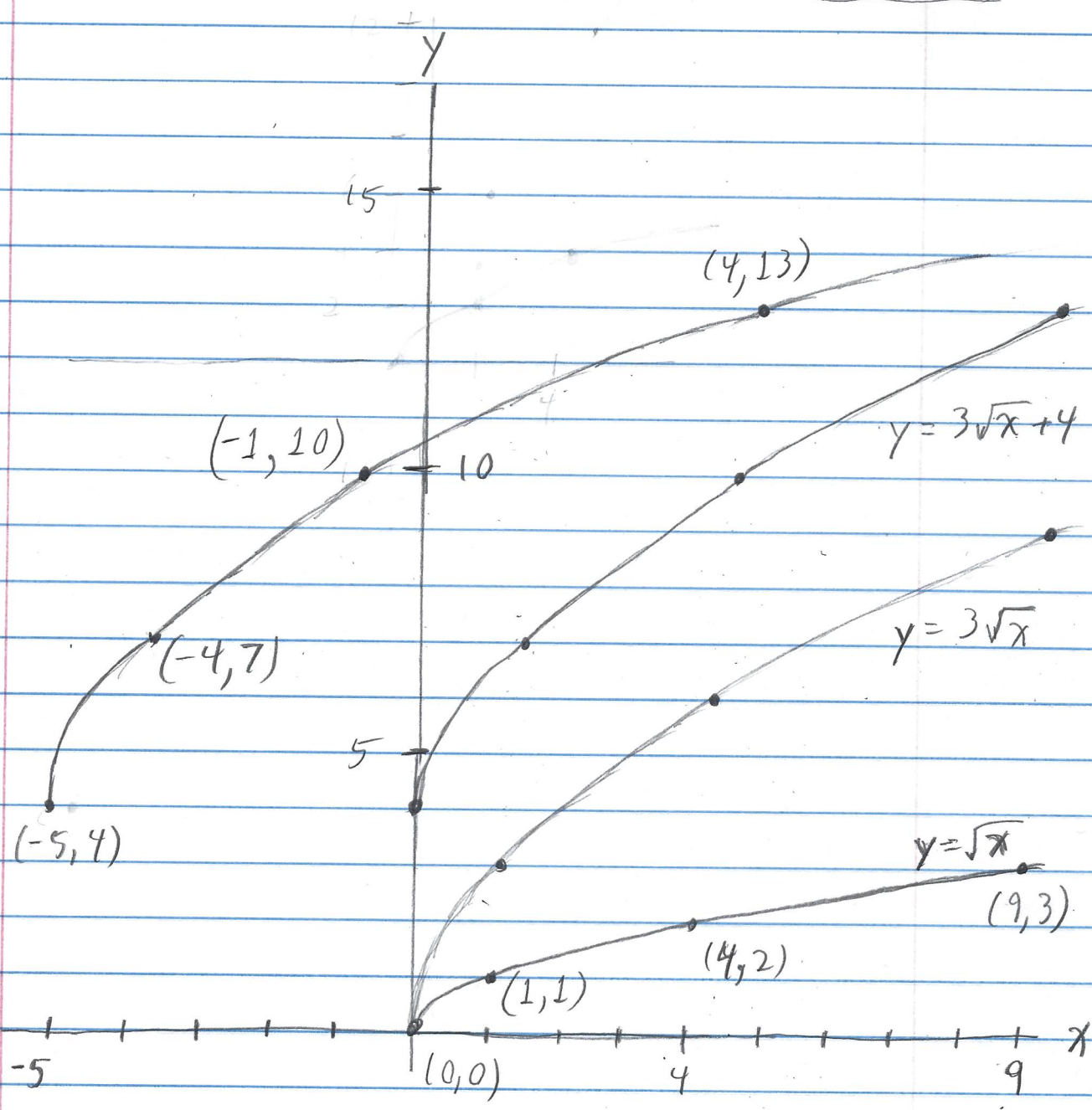(2) Shift up 4 units, (3) Shift left 5 units.

Solution

To vertically stretch the graph of $y = f(x)$, we consider the new function $y = a f(x)$ where $a > 1$. So we first consider $3\sqrt{x}$.

To shift the graph of $y = f(x)$ up k units, we consider the new function $y = f(x) + k$ where $k > 0$. So we let $k = 4$ and second consider $3\sqrt{x} + 4$.

To shift the graph of $y = f(x)$ horizontally left h units we consider the new function $y = f(x - h)$ where $h < 0$. So we let $h = -5$ and finally consider $f(x) = 3\sqrt{x - (-5)} + 4$, or $\boxed{f(x) = 3\sqrt{x + 5} + 4}$.

We start with the graph of $y = \sqrt{x}$ and the "key points" $(0, 0)$, $(1, 1)$, $(4, 2)$, and $(9, 2)$. Applying the three transformations gives the following:

y

15

$(4,13)$

$(-1,10)$

10

$y = 3\sqrt{x} + 4$

$(-4,7)$

$y = 3\sqrt{x}$

$(-5,4)$

5

$y = \sqrt{x}$

$(9,3)$

$(4,2)$

$(1,1)$

$(0,0)$

-5

4

9

x