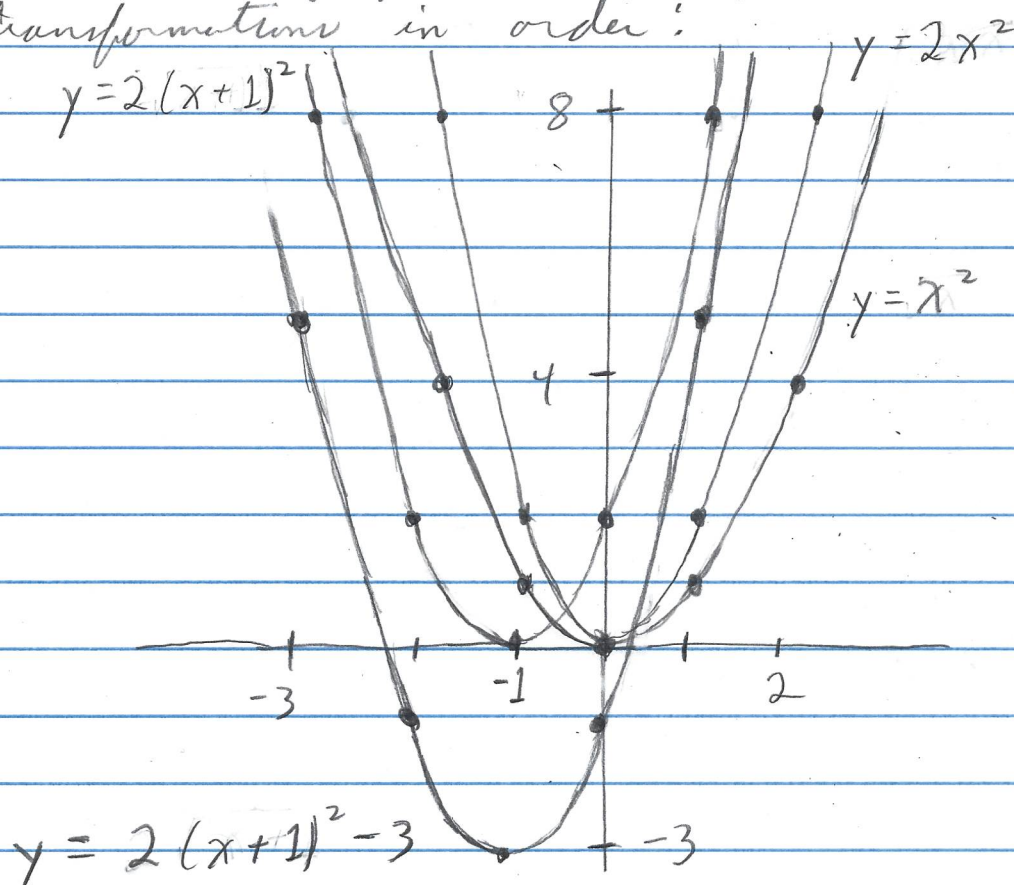


Exercise 2.5.49 Graph $f(x) = 2(x+1)^2 - 3$.

Start with the basic function $y = x^2$ and show all steps. Find the domain and range.

Solution

Starting with x^2 , we introduce a vertical stretch by a factor of $a = 2$ to get $2x^2$. Next we shift left by 1-unit by replacing x with $x-h$ where $h = -1$; this gives $2(x-(-1))^2 = 2(x+1)^2$. Finally we shift vertically down 3 units by adding $k = -3$ to the previous function to get $f(x) = 2(x+1)^2 - 3$. So we start with the graph of $y = x^2$ and apply these transformations in order:



2.5.49
continued

The domain of $f(x) = 2(x+1)^2 - 3$ is all real numbers, as we see from the function (there is no division by 0 and no square roots of negatives). We also see from the graph. So the domain is $\mathbb{R} = (-\infty, \infty)$

We see from the graph that the absolute minimum is -3 and that the graph includes a point of the form (x, y) for all $y \geq -3$ (and some corresponding x). So the range of f is $[-3, \infty)$. \square