

Exercise 2.5.79 Suppose  $(1, 3)$  is a point on the graph of  $y = f(x)$ .

- (a) What point is on the graph of  $y = f(x+3) - 5$ ?  
 (b) What point is on the graph of  $y = -2f(x-2) + 1$ ?  
 (c) What point is on the graph of  $y = f(2x+3)$ ?

Solution

Since  $(1, 3)$  is a point on the graph of  $y = f(x)$ , then  $f(1) = 3$ .

(a) Since we know  $f(1)$ , then we can find  $f(x+3) - 5$  when  $x+3=1$  or when  $x = -2$ .

This gives  $y = f((-2)+3) - 5 = f(1) - 5 = 3 - 5 = -2$ .

So the point  $(x, y) = (-2, -2)$  is on the graph of  $y = f(x+3) - 5$ .

(b) We have  $x-2=1$  when  $x=3$ . So with  $x=3$  we have  $y = -2f(3-2) + 1 = -2f(1) + 1 = -2(3) + 1 = -5$ .

So the point  $(x, y) = (3, -5)$  is on the graph of  $y = -2f(x-2) + 1$ .

(c) We have  $2x+3=1$  when  $x = -1$ .

This gives  $y = f(2(-1)+3) = f(1) = 3$ .

So the point  $(x, y) = (-1, 3)$  is on the graph of  $y = f(2x+3)$ .  $\square$

Note We can also consider the given functions of transformations of  $y = f(x)$  and use these transformations to find a point on the graph corresponding to point  $(1, 3)$ .