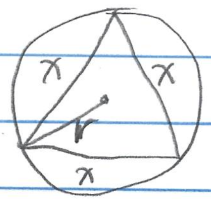
2.6.17

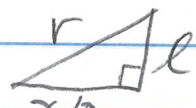
Exercise 2.6.17 An equilateral triangle is inscribed in a circle of radius r.



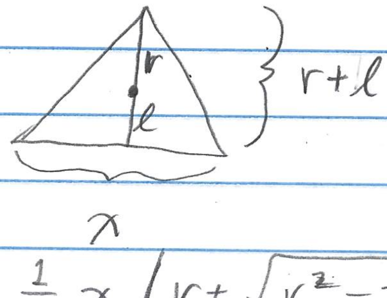
Express the area A within the circle, but outside the triangle, as a function of the length x of a side of the triangle.

Solution

First, we introduce a little right triangle using the radius and half of the lower edge of the equilateral triangle:



By the Pythagorean Theorem, we have $r^2 = l^2 + (x/2)^2$ or $l^2 = r^2 - x^2/4$ or $l = \sqrt{r^2 - x^2/4}$ (we take a "positive square root" since l is a distance). The area of a triangle of base b and height h is $\frac{1}{2}bh$. The equilateral triangle has base $b = x$ and height $h = r + l$:



So the area of the equilateral triangle is

$$\frac{1}{2}bh = \frac{1}{2}x(r+l) = \frac{1}{2}x\left(r + \sqrt{r^2 - x^2/4}\right).$$

The are of the circle is πr^2, so the area within the circle but outside the triangle is

$$\boxed{A = \pi r^2 - \frac{1}{2}x\left(r + \sqrt{r^2 - x^2/4}\right).}$$

□