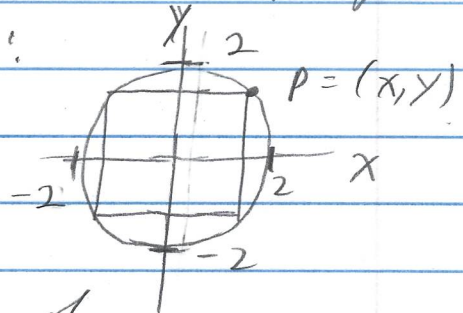


Exercise 2.6.9 a, b A rectangle is inscribed in a circle of radius 2. Let  $P = (x, y)$  be the point in quadrant I that is a vertex of the rectangle on the circle:

(a) Express the area  $A$  of the rectangle as a function of  $x$ .



(b) Express the perimeter  $p$  of the rectangle as a function of  $x$ .

Solution

(a) The area of a rectangle of width  $w$  and height  $h$  is  $A = wh$ . We see from the picture that  $w = 2x$  and  $h = 2y$ . Therefore  $A = (2x)(2y) = 4xy$ . Now point  $(x, y)$  lies on the circle of center  $(0, 0)$  and radius  $r = 2$ , so  $x^2 + y^2 = 2^2 = 4$ ; or  $y = \sqrt{4 - x^2}$  (notice that  $y > 0$  since  $(x, y)$  is in the first quadrant, so we take the "positive square root").

Hence  $A = 4x\sqrt{4 - x^2}$ .

(b) The perimeter of a rectangle of width  $w$  and height  $h$  is  $p = 2w + 2h$ . Since  $w = 2x$  and  $h = 2y$ , we have  $p = 2(2x) + 2(2y) = 4x + 4y$ .

Since  $y = \sqrt{4 - x^2}$ , then  $p = 4x + 4\sqrt{4 - x^2}$ .  $\square$